

Research Article

Robust Smooth Sliding-Mode-Based Controller with Fixed-Time Convergence for Missiles considering Aerodynamic Uncertainty

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This paper designed a smooth fixed-time-convergent sliding mode controller for a missile flight system considering aerodynamic uncertainties. Fixed-time convergence theory is incorporated with the sliding mode control technique to ensure that the system tracks desired commands within uniform bounded time under different initial conditions. Unlike previous terminal sliding mode approaches, not only is the bound of settling time independent of initial state, indicating that performance metrics like convergence rate can be predicted beforehand, but the control input is designed to be smooth based on adaptive estimations and some mathematical results without introducing any discontinuous items like the signum function, which avoids the problem of chattering consequently. A cascade control structure is employed with the derived control algorithm, and therein, the control input signal is obtained. Finally, a number of simulations are carried out and demonstrate the effectiveness of the designed controller.

1. Introduction

Increasing demand for high accuracy and system reliability has stimulated the development of control techniques in the past decades. For the missile flight system, a well-behaved controller can not only track the command signal swiftly but also exhibit appropriate robustness against disturbances and uncertainties which exist throughout the fickle flight regime. From the perspective of overall design, it is also preferable to evaluate the control performances as much as possible once the preliminary controller design is accomplished.

The conventional missile autopilot focused on improving the poor underdamped dynamics of the missile airframe. By adding feedback loops, the PI regulator [1], root locus and frequency domain technique [2], and pole placement method [3] can be applied conveniently, all constructing a new control circuit with moderate dynamics, wherein the missile body itself becomes part of the control loop. However, these methods fail to interact directly with uncertainties or disturbances. They normally designed the controller without caring

for uncertainties at the beginning. After independent tuning of designed parameters, the influence of uncertainties are alleviated through adding the conservativeness of control gains. To improve this, some modern control techniques are introduced. Reference [4] divided the missile attitude dynamic and kinematic model into fast and slow loops according to time scalar separations, which is the common idea that is widely adopted by almost all the control works for missile controller design. Variable structure approach is then applied in the loop design. In [5], an acceleration autopilot design methodology was developed based on optimal control theory, which revealed better performance and stability robustness compared with the classic three-loop controller. Reference [6] utilized the dynamic inversion method to design the control input, together with the extended-mean technique dealing with system uncertainties. References [7, 8] finished a similar work through incorporating fuzzy logic to PID control, constituting an adaptive hybrid control strategy, so that online self-tuning of the controller gains was achieved. Further, [9, 10] deliberated the fusion of a sliding

mode controller and the fuzzy logic networks, comprising the advantages of both systems. Simulations revealed the effectiveness of the designed method.

Among the myriad control approaches for missile controller design, the sliding mode control (SMC) technique is widely used and performs remarkably against uncertainties and disturbances without exact modeling of the uncertainties or disturbance estimation [11–13]. This enables the system with good robustness and interference immunity. SMC method involves a sliding manifold that should be chosen based on control objectives and errors, and then appropriate reaching laws are designed to maintain the surface “on sliding mode.” Conventional SMC algorithm usually prescribes the switching manifolds as linear combinations of system state and desired command with linear reaching law [11, 14, 15], which could only achieve the asymptotic or exponential convergence in state movement towards the equilibrium point. Then, the concept of “terminal sliding mode control” (TSMC) is put forward, which indicates the finite convergence property of the sliding phase [16–18]. As the sliding manifolds and reaching law of TSMC are nonlinear functions with higher order, a singularity problem is possibly found and prohibits the practical application when the error being controlled becomes very small. In [19], the sliding manifold of TSMC has been improved through substitutions of state and its derivative variables, together with the change of exponential coefficient range. Therefore, the singularity problem has been eliminated when taking the derivatives of the sliding states. Similarly, [20] applied a non-singular TSMC method to the integrated guidance and control system design incorporated with adaptive method estimating the bound of disturbances, so that not only system states converge to the desired values in finite time but also the missile can finally hit the target in finite time.

However, in the abovementioned works, where finite-time convergence characteristic is incorporated, exhaustive estimations of settling time are not addressed. Only theoretical results are derived in the citation of lemmas. From the expression of theoretical settling time, prior knowledge of the initial system state must be known to estimate the convergence rate. Different initial conditions correspond to various settling time, of which the maximum value depends on the maximum initial errors related to the Lyapunov function. In the preliminary design of a missile control system, it would be very helpful if the settling time can be predicted without information of the initial conditions. Thus, in contrast with existing finite-time controllers, the upper bound of the settling time could be estimated in the sense of the fixed-time convergence concept [21], of which the bounded convergence time is independent of system initial conditions. With the newly constructed sliding manifold, [22] achieved a non-singular fixed-time consensus tracking for second-order multi-agent networks. Carrying the idea further, [23] expanded the fixed-time terminal sliding mode control methods for a class of second-order nonlinear systems. In [24], a controller in the sense of the fixed-time concept had been constructed for rigid spacecraft and the attitude of the spacecraft converged to the equilibrium within the defined fixed time even with actuator saturation and faults.

Further, variable structure items, which comprise discontinuous parts like the signum function, are normally introduced in the SMC control methodologies mentioned above to suppress the disturbances. The discontinuous signum function induces the well-known chattering problem as the switching gain must be chosen larger than the bound of uncertainty, which results in a degradation of system performance to some considerable extent under specific occasions. Some works replaced the discontinuous item with saturation function or sigmoid function [25], whereas control effectiveness is sacrificed to a certain degree. References [26, 27] utilized the extended state observer (ESO) to estimate uncertainties. Despite the control burden being reduced with ESO's output incorporated into the control algorithm as a feed forward scheme, the discontinuous variable structure item is unavoidable and the system suffers from the chattering phenomenon inevitably. To alleviate chattering effectively, [28] proposed a smooth terminal sliding mode control algorithm for robotic manipulators, where a kind of continuous reaching condition was elaborately devised.

Inspired by the previous discussion, this paper designed a new robust smooth sliding-mode-based controller with fixed-time convergence. An adaptive estimation and some primary mathematical results are utilized to alleviate the effect of uncertainties, as well as deducing the control algorithm. Unlike existing works, the designed smooth fixed-time-convergent sliding mode controller motivates missile control variables to converge to the equilibrium point before the uniform bounded settling time in the presence of aerodynamic uncertainties with its input inherently continuous without using any discrete items, like the signum function. The upper bound of settling time here is simply a function of designed parameters, and therefore, prior knowledge of the convergence rate can be evaluated in advance without any information of system initial conditions. With a uniform bounded convergence time, the controller can track the desired command in the presence of aerodynamic uncertainties under different initial state situations, which is very helpful for both preliminary design and performance evaluation. The smooth input avoids the problem of singularity and chattering, which ensures better performance of the missile control system and exhibits superior availability for practical application. Comprehensive simulations considering a nominal missile dynamic model with aerodynamic uncertainties are carried out to demonstrate the effectiveness of the designed attitude controller. The fixed-time convergence characteristic is fully reflected under the cases of various controller gains and initial states, and the smooth input exhibits nice continuity through comparison with the conventional terminal sliding mode method.

The rest of this paper is organized as follows. In the forthcoming section, a missile dynamic model with aerodynamic uncertainties is presented. Next, some preliminaries involving primary mathematical results and the fixed-time convergence theory are introduced, and then a smooth fixed-time-convergent sliding mode controller is designed in Section 3. In Section 4, detailed control property and system stability analysis are given. Finally, various simulations are performed

in the last section and demonstrate the satisfactory performance of the designed controller.

2. Problem Formulation

Typical missile control embodies the tracking of commanded attitude angle. To begin with, some fundamental assumptions are introduced.

- The variation of static parameters like mass and moment of inertial through entire flight is not taken into consideration.
- Trustworthy measurements of attitude angles and angular velocities are available with high precision sensors.
- Aerodynamic uncertainties and their derivatives are bounded.
- In the controller design, missile velocity and flight altitude are assumed to be constant.

Regardless of gravity, the longitudinal model of the missile in the presence of aerodynamic uncertainties is considered here:

$$\begin{aligned}\dot{\alpha} &= \omega_z - b_\alpha(1 + d_{\alpha\alpha})\alpha - b_{\delta_z}(1 + d_{\alpha\delta_z})\delta_z, \\ \dot{\omega}_z &= -a_{\omega_z}(1 + d_{\omega_z\omega_z})\omega_z - a_\alpha(1 + d_{\omega_z\alpha})\alpha - a_{\delta_z}(1 + d_{\omega_z\delta_z})\delta_z.\end{aligned}\quad (1)$$

In the above equation, α denotes the angle of attack, ω_z stands for the pitch rate, and δ_z is the elevator deflection of the missile. Denotations like b_α , b_{δ_z} , a_α , a_{δ_z} , and a_{ω_z} are defined as

$$\begin{aligned}b_\alpha &= \frac{P + C_y^\alpha qs}{mV}, \\ b_{\delta_z} &= \frac{C_y^{\delta_z} qs}{mV}, \\ a_\alpha &= \frac{-m_z^\alpha qsL}{J_z}, \\ a_{\delta_z} &= \frac{-m_z^{\delta_z} qsL}{J_z}, \\ a_{\omega_z} &= \frac{-m_z^{\omega_z} qsL}{J_z},\end{aligned}\quad (2)$$

where m , V , P , s , and L represent the missile weight, velocity, thrust, reference area, and length, respectively. q denotes the dynamic pressure and can be calculated by $q = 0.5\rho V^2$. C_y^* and m_z^* are coefficients of aerodynamic force and moment, respectively. Thus, in (1), $d_{\alpha\alpha}$ and $d_{\alpha\delta_z}$ describe the aerodynamic uncertainties in terms of lift coefficient generated by angle of attack and elevator deflection. $d_{\omega_z\omega_z}$, $d_{\omega_z\alpha}$, and $d_{\omega_z\delta_z}$ depict the aerodynamic uncertainties of moment coefficient in pitch direction resulting from pitch rate, angle of attack, and elevator deflection, respectively. As assumed at the

beginning, all of the coefficients representing aerodynamic perturbations are bounded.

For our convenience, (1) can be rearranged to form the following dynamic system:

$$\dot{\mathbf{x}} = \mathbf{f}_x + \mathbf{g}_x \mathbf{u} + \mathbf{d}, \quad (3)$$

where

$$\begin{aligned}\mathbf{x} &= [x_1, x_2]^T = [\alpha, \omega_z]^T, \\ \mathbf{f}_x &= \begin{bmatrix} \omega_z - b_\alpha\alpha - b_{\delta_z}\delta_z \\ -a_\alpha\alpha - a_{\omega_z}\omega_z \end{bmatrix}, \\ \mathbf{g}_x &= \begin{bmatrix} 0 \\ -a_{\delta_z} \end{bmatrix}, \\ \mathbf{d} &= \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} -b_\alpha d_{\alpha\alpha}\alpha - b_{\delta_z} d_{\alpha\delta_z}\delta_z \\ -a_{\omega_z} d_{\omega_z\omega_z}\omega_z - a_\alpha d_{\omega_z\alpha}\alpha - a_{\delta_z} d_{\omega_z\delta_z}\delta_z \end{bmatrix}.\end{aligned}\quad (4)$$

System (3) well represents the complex behavior of missiles steering by elevator deflection in the presence of aerodynamic uncertainties. With measurable state variables x_1 and x_2 , items like \mathbf{f}_x and \mathbf{g}_x can be calculated.

3. Controller Design

The dynamic model developed in Section 2 is suitable to design a controller based on robust stabilizing and tracking control law. In this section, some primary mathematical results and the concept of fixed-time stability are first introduced. Afterwards, a robust smooth adaptive sliding-mode-based controller has been designed with fixed-time convergence characteristics, wherein the influence of uncertainties is alleviated without using any discontinuous items (like the signum function).

3.1. Preliminary. Two fundamental equations are presented here. For any $a, b, c, d \in \mathbf{R}$ and arbitrary positive value $\mu, \zeta > 0$, one can get

$$\frac{(ab)^2}{4\mu} + \mu \geq ab, \quad (5)$$

$$\zeta c^2 + \frac{d^2}{4\zeta} \geq cd. \quad (6)$$

Equations (5) and (6) are mathematically correct and are helpful to prove the stability of the designed controller in the subsequent section.

Another important synthesis in this paper is the concept of fixed-time stability, which can be assumed as the extension of finite-time stability.

Consider the nonlinear system [17]

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t), \quad \mathbf{f}(\mathbf{0}, t) = \mathbf{0}, \quad \mathbf{x} \in \mathbf{R}^n, \quad (7)$$

where $\mathbf{f} : \mathbf{U}_0 \times \mathbf{R} \rightarrow \mathbf{R}^n$ is continuous on $\mathbf{U}_0 \times \mathbf{R}$ and \mathbf{U}_0 is an open set of the origin.

The origin is a finite-time stable equilibrium if it is Lyapunov stable and for any given initial time t_0 and initial state $\mathbf{x}(t_0) = \mathbf{x}_0 \in \mathcal{U} \subset \mathcal{U}_0$, there exists a settling time $T \geq 0$ which is dependent on \mathbf{x}_0 , such that for every solution of system (7), $\mathbf{x}(t) = \boldsymbol{\varphi}(t : t_0, \mathbf{x}_0) \in \mathcal{U} \setminus \{\mathbf{0}\}$ satisfies

$$\begin{aligned} \lim_{t \rightarrow T(\mathbf{x}_0)} \boldsymbol{\varphi}(t : t_0, \mathbf{x}_0) &= \mathbf{0}, \quad t \in [t_0, T(\mathbf{x}_0)], \\ \boldsymbol{\varphi}(t : t_0, \mathbf{x}_0) &= \mathbf{0}, \quad t > T(\mathbf{x}_0). \end{aligned} \quad (8)$$

Moreover, if the origin is finite-time stable with $\mathbf{U} = \mathbf{R}^n$, then the origin is a global finite-time stable equilibrium. The following lemma [16] provides sufficient condition for the finite convergence control design.

Lemma 1. Consider the differential equation of system (7). Suppose there exists a continuous positive-definite function $V(\mathbf{x}, t): \hat{\mathbf{U}} \rightarrow \mathbf{R}$, where $\hat{\mathbf{U}}$ is a neighborhood of the origin, such that there are real numbers $c > 0$ and $0 < a < 1$ satisfying $\dot{V}(\mathbf{x}, t) + cV^a(\mathbf{x}, t) \leq 0$ on $\hat{\mathbf{U}}$. Then the zero solution of system (7) is finite-time stable. The settling time is given by

$$T(\mathbf{x}_0) \leq \frac{V^{1-\alpha}(\mathbf{x}_0)}{c(1-\alpha)}. \quad (9)$$

In addition, if $\hat{\mathbf{U}} = \mathbf{R}^n$, and V is radially unbounded, then the origin is globally finite-time stable.

From Lemma 1, the finite convergence time depends on the system initial condition. Diverse initial states result in varied settling times, which means no uniform bound of the convergence time can be obtained in advance without the knowledge of maximum $V(\mathbf{x}_0)$. If prior information of system initial states is not available, preevaluations of controller performance may be restricted to some extent. For preliminary design, it would be much helpful if system performance metrics, for example, the bound of settling time, can be estimated in advance regardless of initial system states. Moreover, for those with strict positive dwelling time, for example, a hybrid system control, it is preferable to stabilize the controlled system exactly before the next switching occurs [29]. Thus, moving further on, the concept of fixed-time convergence is addressed.

Considering system (7), the origin is said to be a ‘‘fixed-time stable’’ equilibrium point if it is globally finite-time stable and the settling time function $T(\mathbf{x}_0)$ is bounded, that is, there exists $T_{\max} > 0$ such that $T(\mathbf{x}_0) < T_{\max}$, $\forall \mathbf{x}_0 \in \mathbf{R}^n$. The following lemma [21] presents a helpful result for the design of a controller with fixed-time convergence.

Lemma 2. Consider a scalar system

$$\dot{y} = -\alpha y^{m/n} - \beta y^{p/q}, \quad y(0) = y_0, \quad (10)$$

where $\alpha > 0$ and $\beta > 0$. m, n, p , and q are positive odd integers satisfying $m > n$ and $p < q$. Then the origin of system (10) is fixed-time stable, and the settling time T is bounded by

$$T < T_{\max} \triangleq \frac{1}{\alpha} \frac{n}{m-n} + \frac{1}{\beta} \frac{q}{q-p}. \quad (11)$$

As denoted in (11), the upper bound of the settling time is independent of system initial condition and is basically determined by designed parameters, namely, m, n, p, q, α , and β . Therefore, prior knowledge of ‘‘fixed’’ settling time can be known in advance with appropriate selection of parameters on the basis of system performance metrics.

3.2. Smooth Adaptive Fixed-Time-Convergent Sliding Mode Controller. This part derived a robust smooth sliding-mode-based controller with fixed-time convergence for system (5). In the light of time scale separation principal and cascade control strategy, system (3) is decomposed into outer loop and inner loop architecture. The former accounts for generating proper angular rate command to trace the reference attitude command, while the latter for operating on the tail deflection to track the reference angular rate command obtained from the angle tracking loop. System (3) is expressed in a scalar form as

$$\begin{aligned} \dot{x}_1 &= f_{x1} + x_2 + d_1, \\ \dot{x}_2 &= f_{x2} + g_{x2}u + d_2, \end{aligned} \quad (12)$$

where $f_{x1} = -b_\alpha \alpha - b_{\delta_z} \delta_z$ and $f_{x2} = -a_\alpha \alpha - a_{\omega_z} \omega_z$. d_1 and d_2 are bounded by $|d_1| \leq \sigma_1$ and $|d_2| \leq \sigma_2$, respectively.

3.2.1. Outer Loop. Suppose the desired command is x_c . The objective of the outer loop control is to design the angular rate defined as a virtual control input x_{2c} to make x_1 converge to x_c within a fixed bounded time. The first equation of system (12) is rewritten here:

$$\dot{x}_1 = f_{x1} + x_{2c} + d_1. \quad (13)$$

Define a sliding manifold as

$$s_1 = x_1 - x_c = \alpha - \alpha_c. \quad (14)$$

Differentiating s_1 with respect to time yields

$$\dot{s}_1 = f_{x1} + x_{1c} + d_1 - \dot{\alpha}_c. \quad (15)$$

To achieve fixed-time convergence, the reaching law is designed in the form

$$\dot{s}_1 = -\beta_{11} s_1^{m_1/n_1} - \beta_{21} s_1^{p_1/q_1} - \beta_{31} s_1 - \frac{\hat{\chi}_1 s_1}{4\mu_1} + d_1, \quad (16)$$

where $\beta_{11}, \beta_{21}, \beta_{31}$, and μ_1 are nonnegative. m_1, n_1, p_1 , and q_1 are positive odd constants satisfying $m_1 > n_1$ and $p_1 < q_1$. $\hat{\chi}_1$ is determined by the updating law

$$\dot{\hat{\chi}}_1 = -\beta_{41} \hat{\chi}_1 + \frac{s_1^2}{4\mu_1}, \quad (17)$$

where $\beta_{41} > 0$. $\hat{\chi}_1$ is the estimation of χ_1 and $\chi_1 = \sigma_1^2$.

Substituting (16) into (15), the virtual control signal for outer loop can be designed as

$$x_{2c} = -f_{x1} - \beta_{11} s_1^{m_1/n_1} - \beta_{21} s_1^{p_1/q_1} - \beta_{31} s_1 - \frac{\hat{\chi}_1 s_1}{4\mu_1} + \dot{\alpha}_c. \quad (18)$$

Next, with the obtained virtual control input, the control law for the inner loop can be designed under the same baseline.

3.2.2. Inner Loop. Similar procedures are conducted in the inner loop. The objective is to deduce the actual control input such that tracking error between the virtual control signal x_{2c} and the actual angular rate variable x_2 converges to zero within a fixed bounded time. Rewrite the second equation of system (12)

$$\dot{x}_2 = f_{x_2} + g_{x_2}u + d_2. \quad (19)$$

Another sliding manifold is defined as

$$s_2 = x_2 - x_{2c}. \quad (20)$$

Taking the derivative of s_2 gives

$$\dot{s}_2 = f_{x_2} + g_{x_2}u + d_2 - \dot{x}_{2c}. \quad (21)$$

In a similar way, the reaching law for inner loop is designed as

$$\dot{s}_2 = -\beta_{12}s_2^{m_2/n_2} - \beta_{22}s_2^{p_2/q_2} - \beta_{32}s_2 - \frac{\hat{\chi}_2 s_2}{4\mu_2} + d_2, \quad (22)$$

where β_{12} , β_{22} , β_{32} , and μ_2 are nonnegative. m_2 , n_2 , p_2 , and q_2 are positive odd constants satisfying $m_1 > n_1$ and $p_1 < q_1$. $\hat{\chi}_2$ is the estimation of $\chi_2 = \sigma_2^2$ and is determined by

$$\dot{\hat{\chi}}_2 = -\beta_{42}\hat{\chi}_2 + \frac{s_2^2}{4\mu_2}, \quad (23)$$

where $\beta_{42} > 0$.

Considering (21) and (22), the actual control input for stabilizing the tracking error in inner loop is derived as

$$u = \delta_z = g_{x_2}^{-1} \left(-f_{x_2} - \beta_{12}s_2^{m_2/n_2} - \beta_{22}s_2^{p_2/q_2} - \beta_{32}s_2 - \frac{\hat{\chi}_2 s_2}{4\mu_2} + \dot{x}_{2c} \right). \quad (24)$$

It can be observed from (23) and (24) that there is no discontinuous item like the signum function, and the control input is inherently smooth and consequently avoids the problem of chattering.

4. Stability Analysis

In this section, closed form stability analysis of system (12) is given with the deduced controller. Preliminaries introduced in the last section are helpful to the proof of the main results.

Theorem 1. *For the dynamic system (12), system states can track the desired command with the designed controller within fixed bounded time and converge to a neighborhood around the sliding manifold. Further, the fixed bound of settling time is independent of initial conditions.*

Proof. For the outer loop, consider the Lyapunov function

$$V_1 = \frac{1}{2}s_1^2 + \frac{1}{2}\tilde{\chi}_1^2, \quad (25)$$

where $\tilde{\chi}_1 = \chi_1 - \hat{\chi}_1$.

Differentiating V_1 with respect to time and substituting (17) and (18) into it yield

$$\begin{aligned} \dot{V}_1 &= s_1\dot{s}_1 + \tilde{\chi}_1\dot{\tilde{\chi}}_1 = -\beta_{11}s_1^{(m_1+n_1)/n_1} - \beta_{21}s_1^{(p_1+q_1)/q_1} - \beta_{31}s_1^2 \\ &\quad + s_1d_1 - \frac{\tilde{\chi}_1 s_1^2}{4\mu_1} + \beta_{41}\tilde{\chi}_1\tilde{\chi}_1 - \frac{\tilde{\chi}_1 s_1^2}{4\mu_1} \leq -\beta_{31}s_1^2 + |s_1|\sigma_1 \\ &\quad - \frac{\tilde{\chi}_1 s_1^2}{4\mu_1} + \beta_{41}\tilde{\chi}_1\tilde{\chi}_1 - \frac{\tilde{\chi}_1 s_1^2}{4\mu_1} = -\beta_{31}s_1^2 + |s_1|\sigma_1 - \frac{\chi_1 s_1^2}{4\mu_1} \\ &\quad + \beta_{41}\chi_1\tilde{\chi}_1 - \beta_{41}\tilde{\chi}_1^2. \end{aligned} \quad (26)$$

Considering (5) and (6), one gets

$$\begin{aligned} \dot{V}_1 &\leq -\beta_{31}s_1^2 + \mu_1 + \beta_{41}\zeta_1\tilde{\chi}_1^2 + \beta_{41}\frac{\chi_1^2}{4\zeta_1} - \beta_{41}\tilde{\chi}_1^2 \\ &= -\beta_{31}s_1^2 - \beta_{41}(1-\zeta_1)\tilde{\chi}_1^2 + \mu_1 + \beta_{41}\frac{\chi_1^2}{4\zeta_1} \leq -\eta_1 V_1 + \varsigma_1, \end{aligned} \quad (27)$$

where $\eta_1 = \min \{2\beta_{31}, 2\beta_{41}(1-\zeta_1)\}$, $\varsigma_1 = \mu_1 + \beta_{41}\chi_1^2/4\zeta_1$.

According to the boundedness theorem, s_1 and $\tilde{\chi}_1$ are uniformly ultimately bounded. Assume that $|\tilde{\chi}_1| \leq \xi_1$, where ξ_1 is positive.

Similarly, define another Lyapunov function of inner loop as

$$V_2 = \frac{1}{2}s_2^2 + \frac{1}{2}\tilde{\chi}_2^2, \quad (28)$$

where $\tilde{\chi}_2 = \chi_2 - \hat{\chi}_2$.

Taking the derivative of V_2 , it follows from (23) and (24) that

$$\begin{aligned} \dot{V}_2 &= s_2\dot{s}_2 + \tilde{\chi}_2\dot{\tilde{\chi}}_2 = -\beta_{12}s_2^{(m_2+n_2)/n_2} - \beta_{22}s_2^{(p_2+q_2)/q_2} - \beta_{32}s_2^2 \\ &\quad + s_2d_2 - \frac{\tilde{\chi}_2 s_2^2}{4\mu_2} + \beta_{42}\tilde{\chi}_2\tilde{\chi}_2 - \frac{\tilde{\chi}_2 s_2^2}{4\mu_2} \leq -\beta_{32}s_2^2 + |s_2|\sigma_2 \\ &\quad - \frac{\tilde{\chi}_2 s_2^2}{4\mu_2} + \beta_{42}\tilde{\chi}_2\tilde{\chi}_2 - \frac{\tilde{\chi}_2 s_2^2}{4\mu_2} = -\beta_{32}s_2^2 + |s_2|\sigma_2 - \frac{\chi_2 s_2^2}{4\mu_2} \\ &\quad + \beta_{42}\chi_2\tilde{\chi}_2 - \beta_{42}\tilde{\chi}_2^2 \leq -\beta_{32}s_2^2 + \mu_2 + \beta_{42}\zeta_2\tilde{\chi}_2^2 + \beta_{42}\frac{\chi_2^2}{4\zeta_2} \\ &\quad - \beta_{42}\tilde{\chi}_2^2 = -\beta_{32}s_2^2 - \beta_{42}(1-\zeta_2)\tilde{\chi}_2^2 + \mu_2 \\ &\quad + \beta_{42}\frac{\chi_2^2}{4\zeta_2} \leq -\eta_2 V_2 + \varsigma_2, \end{aligned} \quad (29)$$

where $\eta_2 = \min \{2\beta_{32}, 2\beta_{42}(1-\zeta_2)\}$ and $\varsigma_2 = \mu_2 + \beta_{42}\chi_2^2/4\zeta_2$. Thus, s_2 and $\tilde{\chi}_2$ are uniformly ultimately bounded. Assume that $|\tilde{\chi}_2| \leq \xi_2$, where ξ_2 is positive.

For the integral system, consider the Lyapunov function

$$V_3 = \frac{1}{2}s_1^2 + \frac{1}{2}s_2^2. \quad (30)$$

Combined with (17), (18), (23), and (24), the derivative of V_3 is expressed in the following form

$$\begin{aligned} \dot{V}_3 = & s_1 \dot{s}_1 + s_2 \dot{s}_2 = -\beta_{11}s_1^{(m_1+n_1)/n_1} - \beta_{21}s_1^{(p_1+q_1)/q_1} - \beta_{31}s_1^2 \\ & + s_1 d_1 - \frac{\tilde{\chi}_1 s_1^2}{4\mu_1} - \beta_{12}s_2^{(m_2+n_2)/n_2} - \beta_{22}s_2^{(p_2+q_2)/q_2} - \beta_{32}s_2^2 \\ & + s_2 d_2 - \frac{\tilde{\chi}_2 s_2^2}{4\mu_2} \leq -\beta_{11}s_1^{(m_1+n_1)/n_1} - \beta_{21}s_1^{(p_1+q_1)/q_1} - \beta_{31}s_1^2 \\ & + \frac{\xi_1 s_1^2}{4\mu_1} + \mu_1 - \beta_{12}s_2^{(m_2+n_2)/n_2} - \beta_{22}s_2^{(p_2+q_2)/q_2} - \beta_{32}s_2^2 \\ & + \frac{\xi_2 s_2^2}{4\mu_2} + \mu_2 = -\beta_{11}s_1^{(m_1+n_1)/n_1} - \beta_{21}s_1^{(p_1+q_1)/q_1} \\ & - \left(\beta_{31} - \frac{\xi_1}{4\mu_1} \right) s_1^2 + \mu_1 - \beta_{12}s_2^{(m_2+n_2)/n_2} \\ & - \beta_{22}s_2^{(p_2+q_2)/q_2} - \left(\beta_{32} - \frac{\xi_2}{4\mu_2} \right) s_2^2 + \mu_2. \end{aligned} \quad (31)$$

Denote

$$\begin{aligned} \beta_{31}^* &= \beta_{31} - \frac{\xi_1}{4\mu_1}, \\ \beta_{32}^* &= \beta_{32} - \frac{\xi_2}{4\mu_2}. \end{aligned} \quad (32)$$

Equation (31) yields

$$\begin{aligned} \dot{V}_3 \leq & -\beta_{11}s_1^{(m_1+n_1)/n_1} - \beta_{21}s_1^{(p_1+q_1)/q_1} - \left(\beta_{31}^* - \frac{\mu_1}{s_1^2} \right) s_1^2 \\ & - \beta_{12}s_2^{(m_2+n_2)/n_2} - \beta_{22}s_2^{(p_2+q_2)/q_2} - \left(\beta_{32}^* - \frac{\mu_2}{s_2^2} \right) s_2^2 \\ \leq & -\beta_{11}s_1^{(m_1+n_1)/n_1} - \beta_{21}s_1^{(p_1+q_1)/q_1} - \beta_{12}s_2^{(m_2+n_2)/n_2} \\ & - \beta_{22}s_2^{(p_2+q_2)/q_2}. \end{aligned} \quad (33)$$

To ensure the stability of the integral system, designed parameters are chosen to satisfy $\beta_{31}^* \geq \mu_1/s_1^2$ and $\beta_{32}^* \geq \mu_2/s_2^2$. To simplify the design process and keep its uniformity, other parameters can be chosen conveniently in the form $\beta_{11} = \beta_{12} = \beta_1$, $\beta_{21} = \beta_{22} = \beta_2$, $m_1 = m_2 = m$, $n_1 = n_2 = n$, $p_1 = p_2 = p$, and $q_1 = q_2 = q$. Then, (33) is changed into

$$\begin{aligned} \dot{V}_3 = & \mathbf{s}_1^T \dot{\mathbf{s}}_1 + \mathbf{s}_2^T \dot{\mathbf{s}}_2 \leq -\beta_1 s_1^{(m+n)/n} - \beta_2 s_1^{(p+q)/q} - \beta_1 s_2^{(m+n)/n} \\ & - \beta_2 s_2^{(p+q)/q} = -\beta_1 \left[(s_{1i}^2)^{(m+n)/2n} + (s_{2i}^2)^{(m+n)/2n} \right] \\ & - \beta_2 \left[(s_{1i}^2)^{(p+q)/2q} + (s_{2i}^2)^{(p+q)/2q} \right] \\ \leq & -2^{(n-m)/2n} \beta_1 (2V_3)^{(m+n)/2n} - \beta_2 (2V_3)^{(p+q)/2q}. \end{aligned} \quad (34)$$

Considering Lemma 2, $V_3 = 0$ implies $s_1 = s_2 = 0$. If $V_3 \neq 0$, substituting $y = \sqrt{2V_3}$ into (34) yields

$$\dot{y} \leq -2^{(n-m)/2n} \beta_1 (y)^{m/n} - \beta_2 (y)^{p/q}, \quad (35)$$

which follows from Lemma 2 that system states reach the sliding bounded region within fixed-time $t < T_{\max}$. The fixed settling time is bounded and can be expressed as follows:

$$t < T_{\max} \triangleq \frac{1}{\beta_1} \frac{2^{(m-n)/2n} n}{m-n} + \frac{1}{\beta_2} \frac{q}{q-p}. \quad (36)$$

For the control variable α , decreasing V ultimately drives the state of trajectory to converge to the small region $|s_1| < (\mu_1/\beta_{31}^*)^{1/2}$ within fixed bounded convergence time. Eventually, the tracking of the desired command is guaranteed.

Further, (36) indicates that the uniform fixed bound of settling time is a function of designed parameters. It is independent of system initial conditions and can be known in advance.

It is noticed in (24) that the first derivative of x_{2c} should be calculated. Because of the fact that f_{x1} is a function of control input, it is difficult to deduce \dot{x}_{2c} from (18) analytically. In piratical application and engineering field, the use of some low pass filters to get the value of \dot{x}_{2c} is widely adopted [30–32]. In our proposed method, the following low pass filter is used.

$$\tau \dot{\bar{x}}_{2c} + \bar{x}_{2c} = x_{2c}, \quad \bar{x}_{2c}(0) := x_{2c}(0), \quad (37)$$

where τ is a small positive time constant of the filter. Denote the estimation of \dot{x}_{2c} by $\dot{\bar{x}}_{2c}$. Then, the control input is revised as

$$u = \delta_z = g_z^{-1} \left(-f_{x2} - \beta_{12}s_2^{m_2/n_2} - \beta_{22}s_2^{p_2/q_2} - \beta_{32}s_2 - \frac{\tilde{\chi}_2^* s_2^2}{4\mu_2^*} + \dot{\bar{x}}_{2c} \right), \quad (38)$$

$$\dot{\tilde{\chi}}_2^* = -\beta_{42}^* \tilde{\chi}_2^* + \frac{s_2^2}{4\mu_2^*}, \quad (39)$$

where β_{42}^* is positive and $\tilde{\chi}_2^*$ is the estimation of χ_2^* and will be explained in the lateral analysis.

Suppose the estimate error $\tilde{x}_{2c} - \dot{x}_{2c}$ is bounded by a positive constant [31, 32], then according to the assumption at the beginning, there exists a positive constant σ_2^* such that $|\tilde{x}_{2c} - \dot{x}_{2c} + d_2| \leq \sigma_2^*$. Denote $\chi_2^* = (\sigma_2^*)^2$; similarly, for the inner loop, define its Lyapunov function as

$$V_2^* = \frac{1}{2}s_2^2 + \frac{1}{2}(\tilde{\chi}_2^*)^2, \quad (40)$$

where $\tilde{\chi}_2^* = \chi_2^* - \tilde{\chi}_2^*$.

TABLE 1: The parameters for a dual-control missile.

Variable	Value
b_α	2.63
b_{δ_z}	0.45
a_{ω_z}	0.86
a_α	10.98
a_{δ_z}	7.47
δ_z	$\leq \delta_{\max} = 15 \text{ deg}$

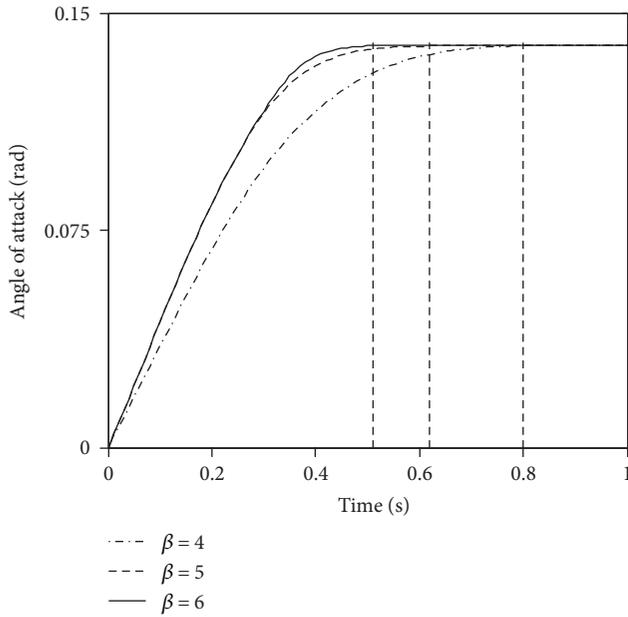


FIGURE 1: Response of the angle of attack.

Substituting (21), (38), and (39) into (40), one gets

$$\begin{aligned}
\dot{V}_2^* &= s_2 \dot{s}_2 + \tilde{\chi}_2^* \dot{\tilde{\chi}}_2^* \\
&= -\beta_{12} s_2^{(m_2+n_2)/n_2} - \beta_{22} s_2^{(p_2+q_2)/q_2} - \beta_{32} s_2^2 \\
&\quad + s_2 (d_2 + \dot{\tilde{x}}_{2c} - \dot{\tilde{x}}_{2c}) - \frac{\tilde{\chi}_2^* s_2^2}{4\mu_2^*} + \beta_{42}^* \tilde{\chi}_2^* \tilde{\chi}_2^* \\
&\quad - \frac{\tilde{\chi}_2^* s_2^2}{4\mu_2^*} \leq -\beta_{32} s_2^2 + |s_2| \sigma_2^* - \frac{\tilde{\chi}_2^* s_2^2}{4\mu_2^*} + \beta_{42}^* \tilde{\chi}_2^* \tilde{\chi}_2^* \quad (41) \\
&\quad - \frac{\tilde{\chi}_2^* s_2^2}{4\mu_2^*} \leq -\beta_{32} s_2^2 - \beta_{42}^* (1 - \zeta_2) (\tilde{\chi}_2^*)^2 + \mu_2^* \\
&\quad + \beta_{42}^* \frac{(\tilde{\chi}_2^*)^2}{4\zeta_2} \leq -\eta_2^* V_2^* + \varsigma_2^*,
\end{aligned}$$

where $\eta_2^* = \min \{2\beta_{32}, 2\beta_{42}(1 - \zeta_2)\}$, $\varsigma_2 = \mu_2 + \beta_{42} \tilde{\chi}_2^{*2} / 4\zeta_2$. Thus, s_2 and $\tilde{\chi}_2^*$ are uniformly ultimately bounded. Assume that $|\tilde{\chi}_2^*| \leq \zeta_2^*$ where ζ_2^* is positive.

Similar with the previous process, for the integral system, the derivative of Lyapunov function V_3 can be deduced in the

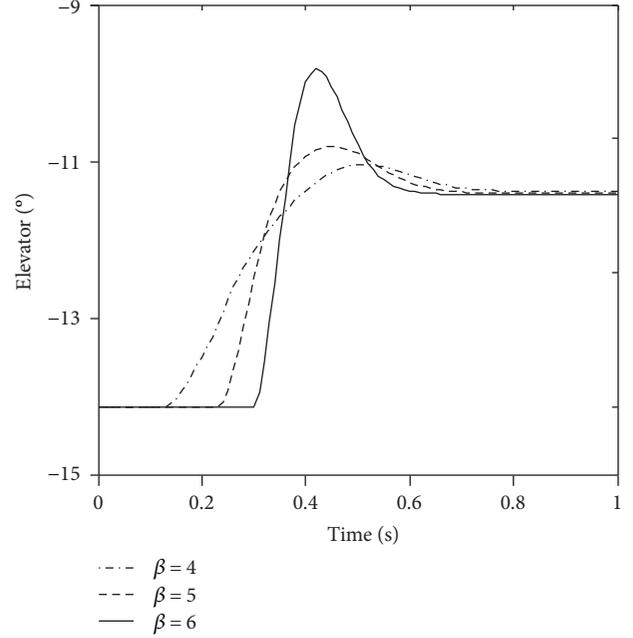


FIGURE 2: Time history of elevator deflection.

same form with (35); thus, the fixed bounded convergence time is expressed identical with (36).

5. Simulation

In this section, a missile control problem in the longitudinal plane in the presence of aerodynamic uncertainties is considered. The dynamic model of (5) and the designed smooth fixed-time-convergent sliding mode controller of (17), (18), (38), and (39) are used in the simulation. To validate the effectiveness of the designed controller, simulations are divided into 2 subsections. Two kinds of maneuvers are pursued with varying aerodynamic uncertainties, namely, tracking fixed constant command and tracking continuous turn command. For the former, the fixed-time convergence characteristic is first validated, and afterwards, Monte Carlo simulations are carried out with different initial conditions to further demonstrate the convergence performance of the designed controller. For the latter, a continuous command is tracked under the condition of diverse aerodynamic uncertainties, which reflects the robustness of the designed controller adequately. From comparison with that using a conventional terminal sliding mode control algorithm, the superiority of the designed approach is revealed.

The flight speed and altitude are Mach 5 and 25 km, respectively. Parameters of a generic tail fin controlled missile are given in Table 1. Elevator deflection is limited to 15 degrees.

Control parameters are chosen to be $\beta_1 = \beta_2 = \beta$, $m_1 = m_2 = m = 5$, $n_1 = n_2 = n = 3$, $p_1 = p_2 = p = 3$, $q_1 = q_2 = q = 5$, $\beta_{41} = 0.03$, $\mu_1 = 0.005$, $\beta_{42}^* = 0.15$, $\mu_2^* = 0.05$, and $\tau = 0.1$

5.1. Case 1: Fixed Convergence Time with Different β and Initial Conditions. This case simply takes the tracking ability

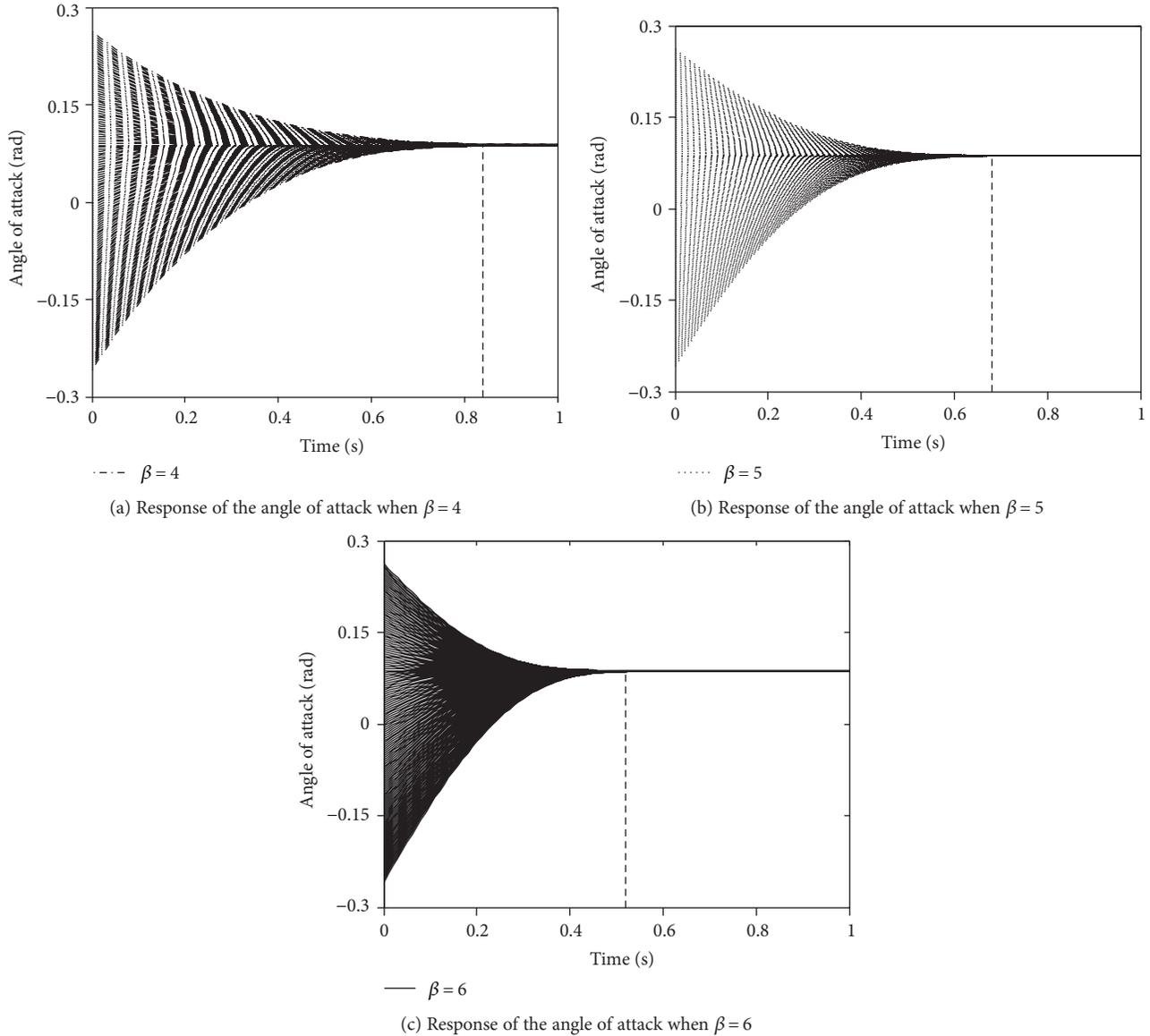


FIGURE 3: System response with the designed controller.

of fixed constant command into consideration. The coefficients of aerodynamic forces and moments are all increased by 30% of their respective nominal values. First, the initial system state is set to be $\alpha_0 = 0$ rad and $\alpha_c = 0.09$ rad. Figure 1 illustrates the time histories of system response in the longitudinal plane with the designed controller. Elevator deflections in different scenarios is depicted in Figure 2.

As visualized in Figure 1, with the proposed controller, the missile can track the desired command with satisfactory performance in the presence of aerodynamic uncertainties. The settling times are approximately 0.51 s, 0.62 s, and 0.8 s, which are all less than theoretical bounded fixed convergence time calculated from (36) with control parameters, namely, 0.73 s for $\beta = 6$, 0.88 s for $\beta = 5$, and 1.1 s for $\beta = 4$. At the beginning, deviations between command signal and initial state variable are considerably large, so that the tail fin goes beyond the deflection constraint promptly for the sake of

tracking, as shown in Figure 2. Full aerodynamic fin deflection continuously contributes to the control efforts, propelling the required control quantities rapidly to the scope of available control load. The burden of aerodynamic actuator is reduced and gradually actuated smoothly to ensure the tracking precision. As a result, fast tracking within fixed bounded convergence time is achieved for the missile flight system with the designed controller.

Next, Monte Carlo simulations are carried out to verify the fixed-time convergence property of the designed controller, which is independent of system initial conditions. In this occasion, $\alpha_c = 0.087$ rad. The initial state varies from -0.26 to approximately 0.26 rad. Other parameters are identical with the previous occasion.

Figure 3 depicts the tracking performance of a tail controlled missile with the designed controller under different initial conditions.

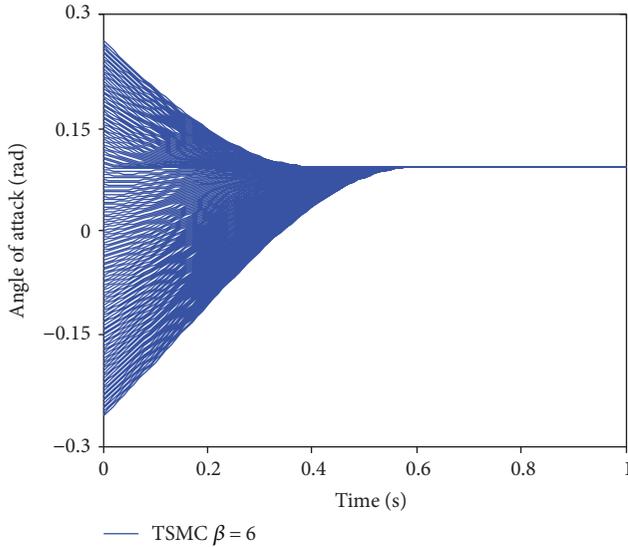


FIGURE 4: System response with TSMC controller.

As calculated above, theoretical bounds of settling time with the designed controller are 1.1 s, 0.88 s, and 0.73 s, respectively. Performance diagrams in Figure 3 demonstrate that under different initial conditions, the missile can track the command signal before the “fixed” predefined bound of convergence time. The time history of system response exhibits a nice uniform convergent characteristic when diverse initial states are given. Simulation results of this situation fully reflect the fixed-time convergence characteristic of the designed controller, which is independent of initial conditions. That is also what separates the designed method with conventional TSMC controllers.

A terminal sliding mode controller (TSMC) is used here with identical designed parameters for comparison. The expressions are given in (42) and (43).

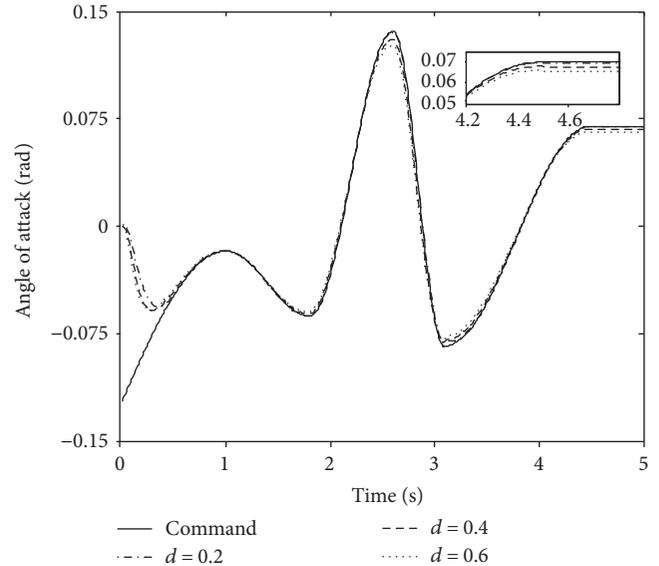
$$x_{2c} = -f_{x1} - \beta s_1^\eta \operatorname{sgn}(s_1) - \varepsilon_1 \operatorname{sgn}(s_1), \quad (42)$$

$$u = \delta_z = g_{x2}^{-1}(-f_{x2} - \beta s_2^\eta \operatorname{sgn}(s_2) - \varepsilon_2 \operatorname{sgn}(s_2) + \dot{x}_{2c}), \quad (43)$$

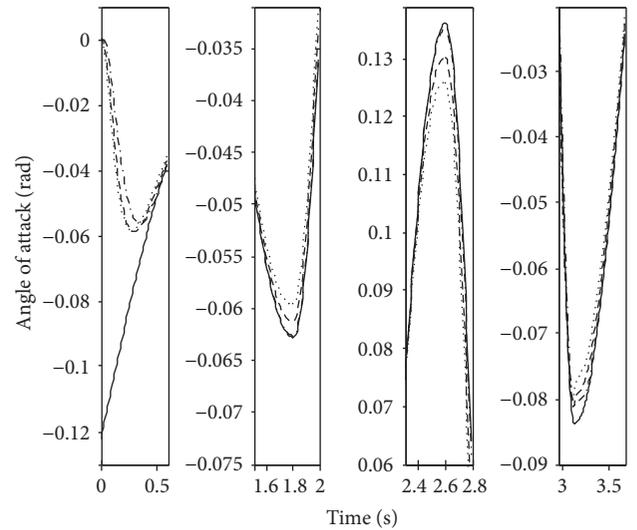
where $\varepsilon_1 > \sigma_1$ and $\varepsilon_2 > \sigma_2$. Here, let $\beta = 6$, for various initial conditions, and time histories of system response are plotted in Figure 4.

Figure 4 depicts the fact that the missile with TSMC controller is able to track the command in an acceptable settling time. However, no uniform boundary of convergence time can be defined in advance since the settling time of the TSMC algorithm is in connection with initial states, which is very different from that in Figure 3(c). Various initial conditions correspond to diverse convergence rates, without which it is almost impossible to predict the bounded settling time when conducting preliminary design and performance evaluation. That is different with the designed method.

5.2. Case 2: Continuous Command under Different Aerodynamic Uncertainty Scenarios. In this case, it is assumed that aerodynamic uncertainty coefficients are increased by



(a) Response of the angle of attack



(b) Details of response in the pitch plane

FIGURE 5: Time history of the angle of attack.

20%, 40%, and 60%. A continuous signal is generated as turn commands for the missile to maneuver, of which $\beta = 5$, and the initial angle of attack equals to zero. Time histories of system response with the designed controller in the presence of various uncertainties are depicted in Figure 5, where the continuous turn commands are illustrated as a reference. Figure 6 gives the diagram of corresponding tail deflections. Details of distinctions between diverse uncertainty conditions are supplemented in Figures 5(b) and 6(b).

It can be seen from Figure 5 that continuous command could be tracked with the proposed smooth fixed-time-convergent sliding mode controller. Within theoretical predefined settling time, which is 0.88 s, the trajectory of the missile attitude angle is driven closely to the desired value and follows the continuous command effectively afterwards. As shown in Figure 6, the deflection of elevator first exceeds

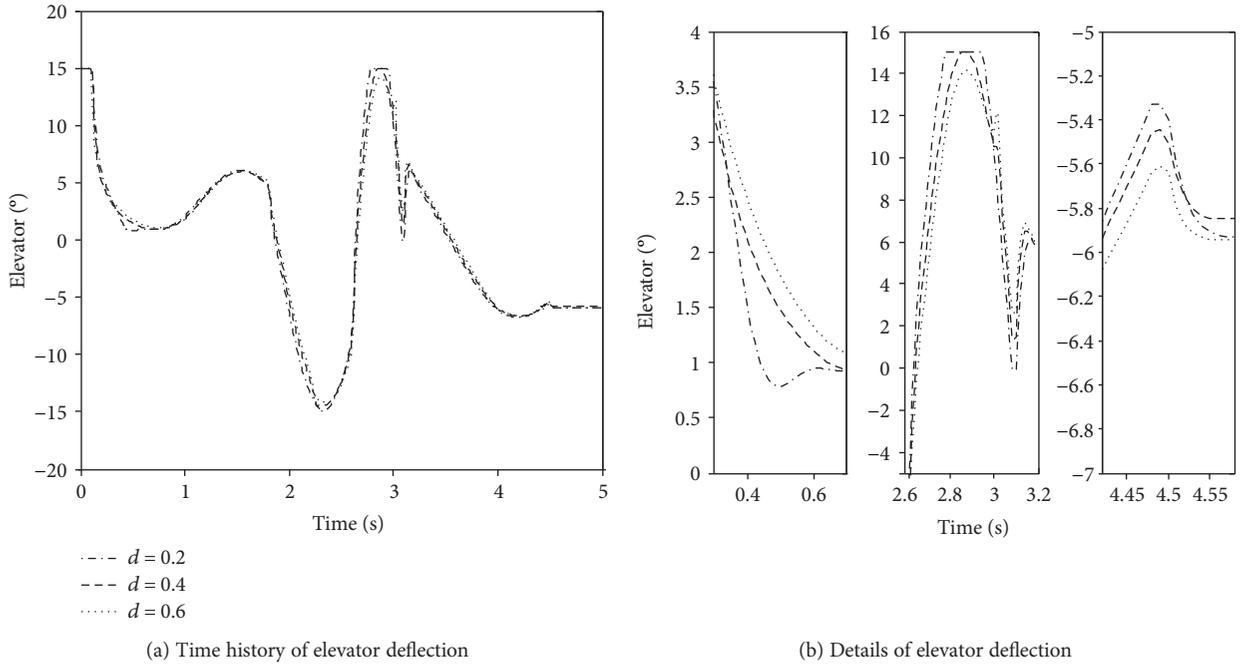


FIGURE 6: Elevator deflection.

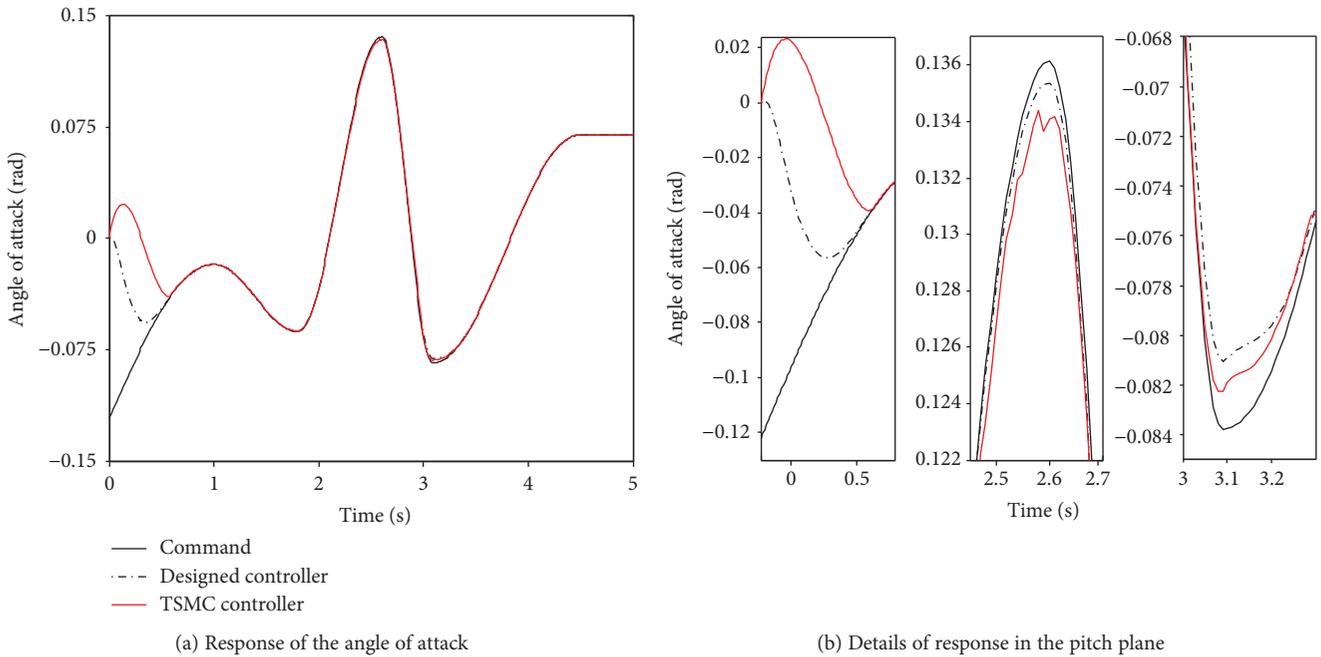


FIGURE 7: Time history of the angle of attack.

its physical constraint because of the large deviation between the initial condition and the desired command. When an initial error is swiped out to a small extent, effective control efforts becomes available, and consequently, smooth elevator deflection plays a crucial part consecutively. Although different degrees of aerodynamic uncertainty exist throughout the flight regime, the designed approach can ensure good tracking precision of the missile flight system and demonstrate satisfactory robustness against uncertainties.

To show the effectiveness of the proposed smooth controller further, the TSMC controller is simulated with identical designed parameters for comparison.

On this occasion, aerodynamic uncertainty is assumed to increase by 20%. Figures 7 and 8 illustrate the system response and control input, respectively.

Although it seems both controllers can track the continuous command, the control quality of the designed controller is superior to that of the TSMC method. It can be observed

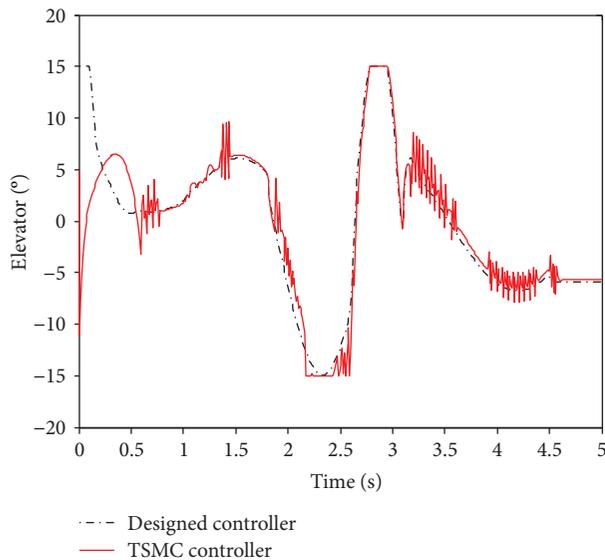


FIGURE 8: Elevator deflection.

from the above figures that the chattering problem exists in the performance of the TSMC controller. With the designed smooth controller, the input signal is smooth without singularity and chattering, which illustrates better tracking performance than the TSMC approach.

6. Conclusions

This paper designed a smooth fixed-time-convergent sliding mode controller for a missile flight system in the presence of aerodynamic uncertainties. Based on fixed-time stability theorem, some primary mathematical results, and adaptive estimations, the derived control algorithm guarantees that control variables converge to the desired command within fixed uniform bounded time regardless of initial conditions with a smooth control input. The bounded convergence rate can be predicted in advance without knowledge of system original states, which is of significant value in system preliminary design, as well as performance evaluation. Meanwhile, the input signal is designed to be continuous without introducing any discrete items like the signum function, thus eliminating the chattering phenomenon and facilitating practical application significantly. In the end, extensive simulations are carried out and validate the effectiveness and robustness of the designed controller.

Conflicts of Interest

The authors declare that there is no conflict of interest.

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