Research Article

Guaranteeing Prescribed Performance Control for Gyrostabilized Platform with Unknown Control Direction Preceded by Hysteresis

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This paper investigates the problem of precise and quick tracking for gyrostabilized platform (GSP) with unknown hysteresis, unknown control directions, and unknown compound disturbance. Firstly, the dynamic model of GSP is transformed into a strict feedback formulation by designed FD to facilitate the backstepping control system. Secondly, performance functions are constructed at each step of backstepping design to force tracking errors to fall within the prescribed boundaries. Besides, through ingenious transformation, radial basis function neural network (RBFNN) is applied to estimate the unknown control gains preceded by hysteresis. Hence, the problem of prescribed performance control with unknown compound disturbances, unknown hysteresis, and unknown control directions is creatively solved. Furthermore, the exploited controllers are accurate model independent, which guarantees satisfactory robustness of control laws against unknown uncertainties. Finally, the stability of the closed-loop control system is confirmed via Lyapunov stability theory, and numerical simulations are given for a GSP to validate the effectiveness of the proposed controller.

1. Introduction

Gyrostabilized platform (GSP) is a kind of precise servo tracking system, which is usually mounted on a mobile carrier for stable tracking of moving targets. Varieties of detectors are mounted on GSP to isolate the motion of carrier as well as to get high-performance information of targets. GSP is originally used in weapons including nonstrapdown seekers [1], aerial shooting, and airborne remote sensing system [2]. Recently, it is also widely used in robotics [3], deep space exploration, and other high-precision tracking systems. Therefore, it has broad application prospects, and it is worth to investigate further.

GSP is a complex time-varying nonlinear system with compound disturbances [4, 5]. The internal disturbances of system include hysteresis nonlinearity of motor and the perturbation of model, while the external disturbances of system include the motion of basement and friction torque between the shafts. Faced with such problems, researches have tried varieties of approaches to realize better dynamic response performance and stronger robustness of GSP. For most actual GSP system, proportional-integral-derivative (PID) control or modified PID has been widely used because of its reliability and simple control structure [6].

Recently, to solve the problem of compound disturbances, many control strategies have been designed [7]. Active disturbance rejection control technique is commonly used to compensate for disturbance because of its strong robustness against various disturbances, X. Y. Zhou and et al. combine the feedforward control with friction observer to compensate for friction disturbance, and the friction is eliminated in large scale because of the precise estimation. Besides, they designed a backstepping integral adaptive compensator to compensate for disturbance [8, 9]. While the shortcoming is that the parameters of friction are estimated offline. Moreover, RBFNN is commonly used to estimate the disturbance for its approximation ability [10, 11]. In reference [11], an adaptive RBFNN was proposed to generate the feedback control parameters online, while the extended state observer is used to compensate
for composite disturbances. The control strategy eliminates disturbances in large scale, while the overshoot of response is large. In reference [12], an adaptive neural network is applied to estimate the uncertain disturbances as well as eliminating “chattering phenomenon,” the strategy is simple and efficient.

Though satisfying robustness and high precision can be achieved through the abovementioned methods, there are still some shortcomings to these methods. A fatal one is that most researchers focus on the characteristic of controlled object, while the nonlinearity of actuator is ignored. The servo motor plays a role of actuator, its precision is mainly restrained by trigger deadzone and hysteresis, while the deadzone can be seen as external disturbance to eliminate. Many researchers have drawn a common conclusion that that hysteresis of motor will deteriorate its response characteristic and tracking precision [13, 14]. Therefore, to promote the performance of GSP, hysteresis problem has to be solved. Besides, the existing research focuses on improving steady performance of GSP. The robustness and precision of GSP are enhanced, but the dynamic performance of GSP cannot be guaranteed.

The study of hysteresis nonlinearities has been drawing much attention in the control community for a long time [15, 16]. For several classes of deterministic nonlinear system with unknown backlash-like hysteresis, adaptive control was proposed by some researchers, while the method has received little attention [17]. Nevertheless, few researchers investigate the problem with both unknown backlash-like hysteresis and unknown control directions [18]. In reference [18], Yu et al. adopt backstepping control to control the mentioned system. By designing a state observer, the states of transformed system are estimated, besides, a RBFNN is adopted to acquire estimated unknown functions, while the control strategy brings in two additional states, which makes controller design complex.

Motivated by previous investigations, this paper will concentrate on solving the problem of compound disturbances and unknown control directions as well as guaranteeing prescribed performance for tracking errors. The compound disturbances include two parts. One part is friction torque, which is a strong nonlinear disturbance that affects the tracking performance at angular velocity “crossing zero” point sharply [2]. And the other is the movement of basement. The disturbances are all considered unknown and estimated online by RBFNN. RBFNN plays a crucial part in disturbance compensation and enhancing the robustness. The key point of application of RBFNN is the model transformation. It is model transformation that normalizes the disturbances. Meanwhile, the designed FD enables transformation by estimating newly defined states. Furthermore, because of the high dynamic performance of GSP, prescribed performance control is adopted. Performance functions are defined under the structure of backstepping control. At the last step of backstepping control, there exists unknown control gain; therefore, a Nussbaum gain function (NGF) is adopted to solve the problem. Thus, prescribed performance control under compound disturbances and unknown control directions is creatively achieved.

Special contributions of this paper are summarized as follows:

1. This paper focuses on the problem of compound disturbances for GSP. Different from existing researches on GSP, in this paper, hysteresis problem is creatively taken into account. Different from most investigations into hysteresis, the hysteresis is completely unknown. Especially, compared with reference [18], input-driven observer is not needed

2. Prescribed output quality is guaranteed for GSP through prescribed performance control; the control strategy is creatively proposed and applied to GSP

3. The presented control approach is independent of accurate models. Thus, its disturbance rejection ability is fine and the computational cost is relatively low

The paper is organised as follows: in Section 2, the model of GSP is built and its working preliminaries are presented. In Section 3, necessary control techniques are introduced, and the controller is designed along with the stability analysis. In Section 4, simulations are carried out to demonstrate the effectiveness.

2. System Modeling and Problem Formulation

2.1. Constitution and Operating Principle of Two-Axis GSP. Figure 1 shows the schematic diagram of two-axis GSP. We can see that stabilized platform consists of two gimbals, which are pitch gimbal and yaw gimbal, respectively. The system is driven by two servo motors; the detective sensor is placed in the inner frame.

From Figure 1, we can see the relationships between two gimbals: gyroscopes measuring the angular rate of pitch and yaw gimbals, angle sensors measuring the angle of pitch and yaw gimbals, and current sensors measuring the current of pitch and yaw motors.

GSP is fixed at the projectile body to track the target [19]. Because of the low coupling and similar characteristic of pitch and yaw channels [20], we choose pitch channel to analyze.

2.2. Dynamic Model of the GSP. Figure 2 shows the pitch channel block diagram of GSP. The block within the red imaginary line stands for the servo motor, while the block within the blue imaginary line stands for the friction disturbance; \( \theta_d \) represents the angle conference signal of the system; \( k_{PWM} \) is the power amplifier coefficient; \( \hat{\theta} \) is the angular rate of stabilized platform in inertial space; \( \hat{\theta} \) is the disturbance of basement movement; \( T_{turb} \) is the disturbance moment; \( k_g \) is the simplified transfer function of rate gyroscope; \( T_c \) is the moment output of servo motor; \( i \) is the electric current of the servo motor; \( C_m \) is the moment coefficient of motor; \( C_e \) is the coefficient of counter electromotive force.
Combining with the dynamic equation of stable platform and the dynamic equation of motor, mathematical model of GSP is acquired as follows \[21\]:

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{C_m x_3}{J_L} - \frac{T_{\text{turb}}}{J_L}, \\
\dot{x}_3 &= -\frac{C_e}{L_a} x_2 - \frac{R_a}{L_a} x_3 + \frac{k_{\text{PWM}}}{L_a} u - \frac{C_e}{L_a} \dot{\theta},
\end{align*}
\]

(1)

where \(x_1, x_2, \) and \(x_3\) are state variables, which represent \(\theta, \dot{\theta},\) and \(i,\) respectively. \(u(v)\) is the system input and the output of the following backlash-like hysteresis described as \[22\]

\[
\dot{u}(v) = a|v|(cv - u(v)) + B\dot{v},
\]

(2)

where \(v\) is the input of the backlash-like hysteresis, \(a\) and \(B\) are constants, and \(c > 0\) is the slope of lines satisfying \(c > B.\)

This paper assumes that the parameters of (2) are unknown.

In the light of the analysis in \[18\], (2) can be solved as

\[
\begin{align*}
\dot{u}(t) &= cv(t) + d(v),
\end{align*}
\]

(3)

where

\[
\begin{align*}
d(v) &= (u_0 - cv_0)e^{-av(v)} \text{sgn}(v) \\
&+ d(v) = (u_0 - cv_0)e^{-av(v)} \text{sgn}(v) \\
&+ e^{-av(v)} \text{sgn}(v) \int_{v_0}^{v} (B - c)e^{ac} \text{sgn}(\xi) d\xi,
\end{align*}
\]

(4)

where \(u_0\) and \(v_0\) are the initial values of \(u\) and \(v,\) respectively. As is shown in \[21\], \(d(v)\) is bounded by an unknown constant value \(d_{\text{max}}.\)
The disturbance torque mainly results from friction between gimbals [23]. Striebeck friction model is chosen as the torque disturbance model in this paper. The mathematical formulation is given as follows:

\[ T_{\text{turb}} = \left[ F_c + (F_s - F_c)e^{-(\pi \omega t)^2} \right] \text{sgn}(v) + B_s \omega \sin(v), \]

where \( F_c \) and \( F_s \) are Coulomb friction and static friction, respectively.

**Remark 1.** The controller output is \( v \) in Figure 2, while there exists unknown hysteresis, which can be expressed as \( u(v) = a(v)(cv - u(v)) + B_s \omega \), is regarded as a completely unknown function. Thus, \( u \) in Figure 2 is regarded as the newly defined output of controller that drives the motor directly. The hysteresis makes the control directions unknown.

**Remark 2.** In this paper, “unknown hysteresis” means that in (3), the coefficient \( c \) and function \( d(v) \) are unknown. “Unknown control directions” result from unknown hysteresis, as is shown in (1). In (1), the designed controller output is \( v \), while the control item is \( (k_{PWM}/L_s)u(v) \). The map from \( v \) to \( u(v) \) is unknown, so the problem is called “problem with unknown control directions.” “Unknown compound disturbances” mean that \( T_{\text{turb}}, \bar{\theta} \) are completely unknown.

**Remark 3.** To enhance the control precision, some researchers identify the parameters of friction model [2, 24]. While the process of identification is complex owing to its complex dynamic behavior, besides, identification has to be carried out repeatedly for different systems. In this paper, friction torque is regarded unknown, which simplifies the process of controller design, and the algorithm is more universal to apply.

### 2.3. Control Problems for GSP

There are some troublesome characteristics in the GSP:

1. When there exist unknown disturbances, high-performance angle tracking is hard to guarantee
2. The unknown hysteresis results in unknown control direction problem for GSP, which increases the difficulty of controller design

### 3. Controller Design

#### 3.1. Finite-Time-Convergent Tracking Differentiator Design

**Assumption 1** (see [25]). The input signal \( x(t) \) is continuous and piecewise \( n \)-order derivable with the following characteristics. The derivatives of \( x(t) \) up to order \( n - 2 \) exist on the whole time domain and \( x(t) \) is not \( n - 1 \) order at some instants \( t_j, j = 1, 2, \ldots, k \) may hold.

Based on the assumption, to estimate newly defined states, a high-order tracking differentiator (FD) is designed. The FD is formulated as follows:

\[
\begin{align*}
\dot{\chi}_1 &= \chi_2 \\
\dot{\chi}_2 &= \chi_3 \\
& \quad \vdots \\
\dot{\chi}_n &= R^n \left[ -a_1 \tanh(\chi_1 - x(t)) - a_2 \tanh(\chi_2) \\
& \quad - \ldots - a_n \tanh(\chi_n) \left( \frac{\chi_n}{R} \right) \right].
\end{align*}
\]

Based on reference [26], there exist \( \phi > 0 \) and \( \bar{\phi} > n \) such that \( \chi_i - x^{(i-1)}(t) = O((1/R)^{\phi-i+1}), i = 1, 2, \ldots, n \) where \( R > 0 \), \( a_i > 0, i = 1, 2, \ldots, n \) are positive constants to be designed; \( O((1/R)^{\bar{\phi}-i+1}) \) means the approximation of \((1/R)^{\bar{\phi}-i+1}\) order between \( \chi_i \) and \( x^{(i-1)}(t) \); \( \phi = (1 - \bar{\phi})/\bar{\phi} \) and \( x(t) \); \( \omega \in (0, \infty) \). The corresponding estimation errors are defined as follows:

\[
\begin{align*}
s_1 &= \chi_1 - x(t), s_2 = \chi_2 - \dot{x}(t), \ldots, s_n &= \chi_n - x^{(n-1)}(t).
\end{align*}
\]

The designed FD is proven finite-time-convergence and stable in reference [20], and the estimation error is bounded.

#### 3.2. Neural Network

To guarantee the controller’s robustness, an adaptive RBFNN is introduced to approximate the uncertain functions. The adaptive RBFNN is defined as the mapping relationship between input vector \( X = [x_1, x_2, \ldots, x_m]^T \in \mathbb{R}^m \) and the output \( y \in \mathbb{R} \) [26, 27].

\[
y = W^T h(X),
\]

where \( W = [w_1, w_2, \ldots, w_m]^T \in \mathbb{R}^m \) stands for weight vector; \( m \) and \( n \) represent the node number and input number, respectively; \( h(X) = [h_1(X), h_2(X), \ldots, h_m(X)]^T \in \mathbb{R}^m \) with \( h_j(X) \) is defined as follows:

\[
h_j(X) = \exp \left( -\frac{||X - c||}{\sigma_j} \right) \in \mathbb{R}^m, \quad j = 1, 2, \ldots, m,
\]

where \( c = [c_1, c_2, \ldots, c_m]^T \in \mathbb{R}^m \) and \( \sigma = [\sigma_{12}, \sigma_{13}, \ldots, \sigma_{jm}]^T \in \mathbb{R}^m \) mean a center and a width vector of \( h_j(X) \), respectively [28, 29].

For an arbitrary continuous unknown function \( F(X) \), it has to be proven that there exists an ideal weight vector \( W = [w_1, w_2, \ldots, w_m]^T \in \mathbb{R}^m \) such that

\[
F(X) = W^T h(X) + \varepsilon,
\]
where $\varepsilon$ is approximate error, which satisfies that $|\varepsilon| \leq \varepsilon_{\text{max}}$. It should be noted that $W^*$ is unknown, its elements $w_1^*, w_2^*, \ldots, w_m^*$ are required to be adjusted adaptively.

Define the error between the ideal weight vector $W^*$ and the estimated weight vector $\tilde{W}$ as

$$\tilde{W} = \tilde{W} \cdot W^*. \quad (11)$$

The adaptation laws of $\tilde{W}$ is designed in next section.

3.3. Prescribed Performance. By prescribed performance, we mean that the tracking error $e$ evolves strictly within the prescribed decaying bounds as follows:

$$-\delta h(t) < e(t) < \bar{\delta} h(t), \quad (12)$$

where the performance function $h(t) = (h_0 - h_{\infty}) e^{-\delta t} + h_{\infty}$ is bounded and strictly decreasing with $h_{\infty} \leq h(t) \leq h_0$. And $h_0 > h_{\infty} > 0$, $t > 0$, $0 < \delta < 1$, and $0 < \bar{\delta} < 1$ are design parameters [26].

If $e$ remains within the prescribed bound of (12), the maximum overshoot of $e$ is restrained less than max $\{ \delta h(t), \bar{\delta} h(t) \}$, and the steady value is no more than max $\{ \delta h_{\infty}, \bar{\delta} h_{\infty} \}$. Thus, the transient performance and steady performance of $e$ are guaranteed by choosing appropriate parameters for (12).

Considering that it is unable to devise controller directly based on (12), an error transfer function $\mu(\varepsilon(t)) = \bar{\delta} e(\varepsilon(t)) - \delta e^{-\varepsilon(t)} e^{\varepsilon(t)} + e^{-\varepsilon(t)}$ is applied to transfer (12) into the following formulation [28].

$$\varepsilon(t) = \mu(\varepsilon(t)) h(t), \quad (13)$$

where $\varepsilon(t)$ is transfer error.

Appropriately, (13) is equivalent illustrated by (12). Furthermore, $\mu(\varepsilon(t))$ is bounded and strictly increasing.

From (13), we have

$$\varepsilon(t) = \mu^{-1}(\varepsilon) = \frac{1}{2} \ln \left( \frac{\varepsilon h + \bar{\delta}}{\delta - \varepsilon h} \right). \quad (14)$$

Furthermore, $\dot{\varepsilon}(t)$ is acquired as

$$\dot{\varepsilon}(t) = \frac{\varepsilon - \bar{\delta} \dot{h}(t)}{h(t)} \quad (15)$$

With

$$\dot{h}(t) = -l(h_0 - h_{\infty}) e^{-\delta t} \in [-l(h_0 - h_{\infty}), 0],$$

$$r = \frac{1}{2h(t)} \left[ \frac{1}{\varepsilon h(t) + \bar{\delta}} - \frac{1}{\varepsilon h(t) - \delta} \right] \quad (16)$$

$$= \frac{1}{2h(t)} \left[ \frac{1}{\mu(\varepsilon(t)) + \bar{\delta}} - \frac{1}{\mu(\varepsilon(t)) - \delta} \right].$$

It is obvious that $r$ is bounded.

3.4. Backstepping Prescribed Performance Controller Design for GSP

3.4.1. Model Transformation. The first step of controller design is to transfer the mathematical model in (1). The mathematical model will be transferred into pure-feedback formation.

Define $y_1 = x_1$, $y_2 = x_2$, and $y_3 = (C_m x_3/I_L) - (T_{\text{turb}}/I_L)$, then we have

$$\dot{y}_1 = y_2, \quad \dot{y}_2 = y_3, \quad \dot{y}_3 = f_1(y_2, y_3) + f_2(\nu) + f_d, \quad (17)$$

where $f_1(y_2, y_3) = -(C_m x_3/I_2) - (R_a I_2/L_a) y_2) C_m/I_L$, and $f_2(\nu) = (C_m k_{\text{PWM}}/I_L) \nu(t) + d(\nu)$ is control item, while $f_d = (C_m/I_L) (-C_m/I_a) \delta - (T_{\text{turb}}/C_m) - (R_a/C_m L_a) T_{\text{turb}}$ represents transferred disturbance.

Then, separating the linear part of control item from nonlinear part, the following equation is acquired.

$$\dot{y}_1 = y_2, \quad \dot{y}_2 = y_3, \quad \dot{y}_3 = f_1(y_2, y_3) + A c(t) + f_{2d}(\nu) + f_d, \quad (18)$$

where the constant $A = C_m k_{\text{PWM}}/I_a, \nu$ is unknown gain, and $f_{2d} = (C_m k_{\text{PWM}}/I_L) d(\nu)$ is unknown function regarded as disturbance to be estimated.

Remark 4. Aiming at the problem of unknown complex disturbances, a RBFNN will be adopted. Because of unknown control directions, RBFNN cannot be applied directly. Through equivalent transformation, unknown gain $c$ becomes a part of unknown functions to be estimated.

Thus, the model of GSP in (1) is transferred into pure-feedback system in (18).

3.4.2. Control Law Design

Assumption 2. We assume that the angle reference signal $\theta_d$ is limited, while its derivative and its second-order derivative are also limited. Furthermore, the disturbance of basement $\delta$ is also limited.

Step 1. Define the angle tracking error $e_1$.

$$e_1 = y_1 - y_{1d}. \quad (19)$$

Define a performance function $h_1(t) = (h_0 - h_{\infty}) e^{-\varepsilon_1 t} + h_{\infty}$ to restrain $e_1$.

$$-\delta h_1(t) < e_1(t) < \bar{\delta} h_1(t), \quad (20)$$
where \( h_{01} > 0, h_{1oc} > 0, l_1 > 0, 0 < \delta_1 < 1 \), and \( 0 < \delta_2 < 1 \) are design parameters and satisfying \( h_{01} > h_{1oc}, \delta_1 h_1(t) < \delta_1 \) \( 0 < \delta_3 h_2(t), \) and \( h_{1oc} < h_2(t) < h_{10} \).

We convert (20) into the following formulation:

\[
ed_1 = \mu_1(e_1(t)h_1(t)),
\]

where \( \mu_1(e_1(t)) = (\delta_1 e^{-\delta_1 t} - \delta_1 e^{-\delta_1 t}/e^{e_1(t)} + e^{-\delta_1 t}) \in (-\delta_1, \delta_1) \) is a transformed function and \( e_1(t) \) is transformed error. From (14), we get the formulation of \( e_1(t) \)

\[
e_1(t) = \mu_1^{-1}(e_1(t)) = \frac{1}{2} \ln \left( \frac{e_1/h_1(t) + \delta_1}{e_1/h_1(t) - \delta_1} \right). \tag{22}
\]

Furthermore, \( \dot{e}_1(t) \) is derived as

\[
\dot{e}_1(t) = r_1 \left[ \dot{h}_1(t) - e_1 h_1(t) \right] = r_1 \left[ \dot{y}_2 - \dot{y}_2d - e_1 h_1(t) \right], \tag{23}
\]

where \( r_1 = 1/2h_2(t)\left[ (1/\delta_1 e_1/h_1(t) + \delta_1) - (1/\delta_1 e_1/h_1(t) - \delta_1) \right] \geq (1/2\delta_2 h_2(t)) > 0. \)

The virtual control law is designed as

\[
eg_3 = -k_1 e_1(t) + \dot{y}_2 - e_1 h_1(t), \tag{24}
\]

where \( k_1 \) is a design parameter, \( \dot{h}_1(t) = -l_1(h_{10} - h_{1oc})e^{-l_1 t}. \)

**Step 2.** Define angular velocity tracking error \( e_2 \).

\[
e_2(t) = y_2 - y_2d. \tag{25}
\]

Construct a performance function \( h_2(t) = (h_{02} - h_{2oc})e^{-l_2 t} + h_{2oc} > 0 \) to restrain \( e_2 \):

\[
-\delta_2 h_2(t) < e_2(t) < \delta_2 h_2(t). \tag{26}
\]

Where \( h_{02} > 0, h_{2oc} > 0, l_2 > 0, 0 < \delta_2 < 1 \), and \( 0 < \delta_2 < 1 \) are design parameters and satisfying \( h_{02} > h_{2oc}, \delta_2 h_2(t) < e_2(0) \) \( 0 < \delta_3 h_2(t), \) and \( h_{2oc} < h_2(t) < h_{20}. \)

Equation (26) is further converted into the following formulation:

\[
e_2(t) = \mu_2(e_2(t)h_2(t)). \tag{27}
\]

where \( \mu_2(e_2(t)) = (\delta_2 e^{-\delta_2 t} - \delta_2 e^{-\delta_2 t}/e^{e_2(t)} + e^{-\delta_2 t}) \in (-\delta_2, \delta_2) \) is a transformed function and \( e_2(t) \) is transformed error. From (27), we can get

\[
e_2(t) = \mu_2^{-1}(e_2(t)) = \frac{1}{2} \ln \left( \frac{e_1/h_1(t) + \delta_2}{e_1/h_1(t) - \delta_2} \right). \tag{28}
\]

The time derivative of \( e_2(t) \) is

\[
\dot{e}_2(t) = r_2 \left[ \dot{h}_2(t) - e_2 h_2(t) \right] = r_2 \left[ \dot{y}_2d - e_2 h_2(t) \right] = r_2 \left[ \dot{y}_3 - \dot{y}_2d - e_2 h_2(t) \right], \tag{29}
\]

where \( r_2 = 1/2h_2(t)\left[ (1/\delta_2 e_2/h_2(t) + \delta_2) - (1/\delta_2 e_2/h_2(t) - \delta_2) \right] \geq (1/2\delta_2 h_2(t)) > 0. \)

The virtual control law is devised as

\[
ey_3 = -k_2 e_3(t) + \dot{y}_2d + e_3 h_2(t), \tag{30}
\]

where \( k_2 \) is a design parameter, \( \dot{h}_2(t) = -l_2(h_{20} - h_{2oc})e^{-l_2 t}. \)

**Step 3.** Define current tracking error \( e_3 \).

\[
e_3(t) = y_3 - y_3d. \tag{31}
\]

Construct a performance function \( h_3(t) = (h_{03} - h_{3oc})e^{-l_3 t} + h_{3oc} > 0 \) to restrain \( e_3 \):

\[
-\delta_3 h_3(t) < e_3(t) < \delta_3 h_3(t). \tag{32}
\]

Where \( h_{03} > 0, h_{3oc} > 0, l_3 > 0, 0 < \delta_3 < 1 \), and \( 0 < \delta_3 < 1 \) are design parameters and satisfying \( h_{03} > h_{3oc}, \delta_3 h_3(t) < e_3(0) < \delta_3 h_3(t), \) and \( h_{3oc} < h_3(t) < h_{30}. \)

Equation (28) is further converted into the following formulation:

\[
e_3(t) = \mu_3(e_3(t)h_3(t)), \tag{33}
\]

where \( \mu_3(e_3(t)) = (\delta_3 e^{-\delta_3 t} - \delta_3 e^{-\delta_3 t}/e^{e_3(t)} + e^{-\delta_3 t}) \in (-\delta_3, \delta_3) \) is a transformed function and \( e_3(t) \) is transformed error. From (33), we can get

\[
e_3(t) = \mu_3^{-1}(e_3(t)) = \frac{1}{2} \ln \left( \frac{e_3/h_3(t) + \delta_3}{e_3/h_3(t) - \delta_3} \right). \tag{34}
\]

The time derivative of \( e_3(t) \) is

\[
\dot{e}_3(t) = r_3 \left[ \dot{h}_3(t) - e_3 h_3(t) \right] = r_3 \left[ \dot{y}_3 - \dot{y}_3d - e_3 h_3(t) \right] = r_3 \left[ f_1(y_2, y_3) + Av(t) + f_2d(y) + f_d - \dot{y}_3d - e_3 h_3(t) \right], \tag{35}
\]
where \( r_3 = 1/2h_3(t)[1/(\delta_x e_j/h_3(t) + \delta_y)] - (1/2(h_3(t) - \delta_y)) \geq (1/2\delta_x h_3(t)) + (1/(2\delta_y h_3(t)) > 0 \).

Define Lyapunov function
\[
V = \frac{\varepsilon_1^2}{2} + \frac{\varepsilon_2^2}{2} + \frac{\varepsilon_3^2}{2} + \frac{1}{\gamma_1} \hat{W}^T \hat{W}.
\] (36)

Employing (23), (29), and (35), \( V \) is acquired as follows:
\[
\dot{V} = \varepsilon_1 \dot{\varepsilon}_1 + \varepsilon_2 \dot{\varepsilon}_2 + \varepsilon_3 \dot{\varepsilon}_3 + \frac{1}{\gamma_1} \hat{W}^T \hat{W} = \varepsilon_1 \dot{r}_1 \left[ y_2 - y_{1d} - \varepsilon_1 \dot{h}_1 \right]
+ \varepsilon_2 r_2 \left[ y_3 - y_{2d} - \varepsilon_2 \dot{h}_2 \right] + \varepsilon_3 r_3 \left[ f_1(y_2, y_3) + Acv(t) \right]
+ f_{2d}(v) + f_d - y_{3d} - \mu_3 \dot{h}_3(t) + \frac{1}{\gamma_1} \hat{W}^T \hat{W}
\]
\[
= \varepsilon_1 \dot{r}_1 \left[ y_2 - y_{1d} - \dot{h}_1 \right] + \varepsilon_2 r_2 \left[ y_3 - y_{2d} - \dot{h}_2 \right]
+ \varepsilon_3 r_3 Acv(t) + \varepsilon_3 r_3 \left[ f_1(y_2, y_3) - y_{3d} - \mu_3 \dot{h}_3(t) \right]
+ \frac{1}{\gamma_1} \hat{W}^T \hat{W}.
\] (37)

Here, a RBFNN is applied to offset the disturbance \( f_d \) as well as the unknown input function \( f_{2d}(v) \). Considering that the input of the system contains an unknown gain \( c \), RBFNN cannot be applied directly. Through an equivalent transformation, the following equation is acquired.

**Remark 5.** The unknown compound disturbance is \( D = \left(f_d + f_{2d}(v)\right)/Ac \), where \( f_d = C_m/L_1\left((-C_m/L_2)d - (T_{\text{turb}}/C_m) - (R_d/C_m)L_0\right)T_{\text{turb}} \). \( f_{2d} = (C_m/k_{\text{PWM}}/L_1)d_v \). \( T_{\text{turb}} \) is friction disturbance, and it is one part of unknown compound disturbance. Besides, the unknown function of unknown hysteresis \( d(v) \) is estimated too. The compound disturbance will be estimated in whole.

\[
\dot{V} = \varepsilon_1 \dot{r}_1 \left[ y_2 - y_{1d} - \dot{h}_1 \right] + \varepsilon_2 r_2 \left[ y_3 - y_{2d} - \dot{h}_2 \right] + \varepsilon_3 r_3 Acv(t) + \varepsilon_3 r_3 \left[ f_1(y_2, y_3) - y_{3d} - \mu_3 \dot{h}_3(t) \right]
+ \frac{1}{\gamma_1} \hat{W}^T \hat{W} = \varepsilon_1 \dot{r}_1 \left[ y_2 - y_{1d} - \dot{h}_1 \right]
+ \varepsilon_2 r_2 \left[ y_3 - y_{2d} - \dot{h}_2 \right]
+ \varepsilon_3 r_3 Acv(t) + \varepsilon_3 r_3 \left[ f_1(y_2, y_3) - y_{3d} - \mu_3 \dot{h}_3(t) \right]
+ \frac{1}{\gamma_1} \hat{W}^T \hat{W}.
\] (38)

Substituting (19), (25), and (31) into (38) leads to
\[
\dot{V} = \varepsilon_1(t)\dot{r}_1\left[ y_2 - k_1 e_1 \right] + \varepsilon_2(t)\dot{r}_2\left[ y_3 - k_2 e_2(t) \right]
+ \varepsilon_3(t)\dot{r}_3 Ac\left( v_1(t) - \hat{W}h(x) \right)
+ \varepsilon_3(t)\dot{r}_3 \left[ f_1(y_2, y_3) - y_{3d} - \mu_3 \dot{h}_3(t) \right]
+ \varepsilon_3(t)\dot{r}_3 \left[ f_{2d}(v) + f_d \right] + \frac{1}{\gamma_1} \hat{W}^T \hat{W}
\]
\[
= r_1 \left[ \mu_2(\varepsilon_2)(h_2(t) \varepsilon_1(t) - k_1 \varepsilon_1(t)) \right]
+ r_2 \left[ \mu_3(\varepsilon_3)(h_3(t) \varepsilon_2(t) - k_2 \varepsilon_2(t)) \right]
+ \varepsilon_3 r_3 Ac\left( v_1(t) - \hat{W}h(x) \right)
+ \varepsilon_3 r_3 \left[ f_1(y_2, y_3) - y_{3d} - \mu_3 \dot{h}_3(t) \right]
+ \varepsilon_3 r_3 \left[ f_{2d}(v) + f_d \right] + \frac{1}{\gamma_1} \hat{W}^T \hat{W}.
\] (39)

Define a total disturbance \( D = (f_d + f_{2d}(v))/Ac \), we mean that \( D = \hat{W}h(x) + \varepsilon \), where \( x = [x_1, x_2, x_3] \). That is to say, when \( \varepsilon \) is small enough, \( \hat{W}h(x) \) will converge to \( D \) in high precision [26]. Thus, \( v(t) = v_1(t) - \hat{W}h(x) \).

To analyze disturbance item individually, define a new function as follows.
\[
\dot{V}_d = \varepsilon_3 r_3 Ac\left( -\hat{W}h(x) + \frac{f_{2d}(v) + f_d}{A} \right) + \frac{1}{\gamma_1} \hat{W}^T \hat{W}
\]
\[
\leq \varepsilon_3 r_3 Ac\left[ (\hat{W}h(x) + \varepsilon_{\text{max}}) + \frac{1}{\gamma_1} \hat{W}^T \hat{W} \right]
\] (40)

Define a positive constant \( \tau \) satisfying \( |\varepsilon| \leq \tau \). Applying Young’s inequality [24], we can acquire that
\[
\dot{V}_d \leq \frac{(\hat{W}h(x) + \varepsilon_{\text{max}})^2}{2} + \frac{(\varepsilon_3 r_3 Ac)^2}{2} + \frac{1}{\gamma_1} \hat{W}^T \hat{W} = \frac{\varepsilon_{\text{max}}^2}{2}
\]
\[
+ \frac{(\varepsilon_3 r_3 Ac)^2}{2} + \frac{1}{\gamma_1} \hat{W}^T \hat{W}
\]
\[
= \frac{(\hat{W}h(x))^2 + \varepsilon_{\text{max}}^2 + \varepsilon_{\text{max}} h(x) + \hat{W}^T \hat{W}}{\gamma_1}.
\] (41)

Since \( W^* \) is a constant, it should be noted that \( \hat{W} = \hat{W} \). According to (41), adaptation law \( \hat{W} \) is designed as follows:
\[
\dot{\hat{W}} = -\gamma_1 \left( \frac{(\hat{W}h(x))^2}{2} + \varepsilon_{\text{max}} h(x) \right).
\] (42)

Then, (41) becomes
\[
\dot{V}_d \leq \frac{\varepsilon_{\text{max}}^2}{2} + \frac{(\varepsilon_3 r_3 Ac)^2}{2}.
\] (43)

Combining with (39), the following inequality is acquired.
\[ V \leq r_1 |e_1(t)| \left[ \overline{\theta}_2 h_2(t) - k_1 |e_1(t)| \right] \\
+ r_2 |e_2(t)| \left[ \overline{\theta}_3 h_3(t) - k_2 |e_2(t)| \right] \\
+ |e_3| r_3 \sum_{k=0}^{2} e_{3}^{k+2} (t) \left( k_{31} e_{3}(t) + \frac{k_{32}^2 e_{3}^2(t)}{2} \right) \\
+ \epsilon_{max}^2 + \epsilon_{max}^2. \]  

Then (47) becomes
\[ V \leq r_1 |e_1(t)| \left[ \overline{\theta}_2 h_2(t) - k_1 |e_1(t)| \right] \\
+ r_2 |e_2(t)| \left[ \overline{\theta}_3 h_3(t) - k_2 |e_2(t)| \right] \\
+ \frac{k_{31} r_3 e_3^2(t)}{2} \sum_{k=0}^{2} e_{3}^{k+2} (t) \\
+ \epsilon_{max}^2 + \epsilon_{max}^2 + \frac{(e_{3} r_{3} A \tau)^2}{2}. \]  

Finally, the actual control law is chosen as
\[ \begin{align*}
    v_1(t) &= N_3(\xi_3) \left[ k_{31} e_{3}(t) + \frac{k_{32}^2 e_{3}^2(t)}{2} \right] - \mathbf{Wh}(x), \\
    \dot{\xi}_3 &= k_{31} r_3 e_3^2(t) + \frac{k_{32}^2 e_{3}^2(t)}{2},
\end{align*} \]

where \( N_3(\xi_3) = e^{\xi_3^2} \cos(\pi \xi_3/2) \) is a Nussbaum function [30]; \( k_{31} \) and \( k_{32} \) are design parameters. 

Invoking (41), (44) becomes
\[ \begin{align*}
    V &\leq r_1 |e_1(t)| \left[ \overline{\theta}_2 h_2(t) - k_1 |e_1(t)| \right] \\
    &+ r_2 |e_2(t)| \left[ \overline{\theta}_3 h_3(t) - k_2 |e_2(t)| \right] \\
    &+ |e_3| r_3 \sum_{k=0}^{2} e_{3}^{k+2} (t) \left( k_{31} e_{3}(t) + \frac{k_{32}^2 e_{3}^2(t)}{2} \right) \\
    &+ \epsilon_{max}^2 + \epsilon_{max}^2. \]

Thus, (48) becomes
\[ \begin{align*}
    V &\leq r_1 |e_1(t)| \left[ \overline{\theta}_2 h_2(t) - k_1 |e_1(t)| \right] \\
    &+ r_2 |e_2(t)| \left[ \overline{\theta}_3 h_3(t) - k_2 |e_2(t)| \right] \\
    &+ \frac{k_{31} r_3 e_3^2(t)}{2} \sum_{k=0}^{2} e_{3}^{k+2} (t) \\
    &+ \epsilon_{max}^2 + \epsilon_{max}^2 + \frac{(e_{3} r_{3} A \tau)^2}{2}. \]

Based on Young’s inequality, it is easy to get
\[ \begin{align*}
    &\sum_{k=0}^{2} e_{3}^{k+2} (t) + (\sum_{k=0}^{2} e_{3}^{k+2} (t)/2) \\
    \end{align*} \]
With \( t_1 = \min \{ 2k_3, 2k_4, \kappa_3 r_3 - ((r_3/\delta)^2/2), \kappa_3 > (r_3/\delta)^2 \} \).

Multiplying \( \epsilon^{t_1} \) on both sides of (49) leads to

\[
\frac{d}{dx} \left( V \epsilon^{t_1} \right) \leq AcN_3(\xi_3) \dot{\xi}_3^{t_1} + \dot{\xi}_3^{t_1} + \sum_0^2 \frac{\epsilon_{max}^2}{2} + \frac{\epsilon_{max}^2}{2}. \tag{50}
\]

Integrating (45) over \([0, t]\), we obtain

\[
0 \leq V \leq V(0) + e^{-t_1} \int_0^t AcN_3(\xi_3) \dot{\xi}_3^{t_1} dt + e^{-t_1} \int_0^t \dot{\xi}_3^{t_1} dt + \int_0^t \left( \sum_0^2 \frac{\epsilon_{max}^2}{2} \right) e^{-t_1} dt. \tag{51}
\]

Noticing \( \int_0^1 (\sum_0^2/2\kappa_{32}) + (\epsilon_{max}^2/2) e^{-t_1} dt = (\sum_0^2/2\kappa_{32}) + (\epsilon_{max}^2/2) (1 - e^{-t_1}) \), we know that \( \int_0^t (\sum_0^2/2\kappa_{32}) + (\epsilon_{max}^2/2) e^{-t_1} dt \) is bounded. Furthermore, (46) becomes

\[
0 \leq V \leq V(0) + \sum_0^2 \frac{\epsilon_{max}^2}{2} + e^{-t_1} \int_0^t AcN_3(\xi_3) \dot{\xi}_3^{t_1} dt + e^{-t_1} \int_0^t \dot{\xi}_3^{t_1} dt. \tag{52}
\]

Invoking lemmas 1 and 2 presented in [30, 31], we know that \( V \), \( e^{-t_1} \int_0^t AcN_3(\xi_3) \dot{\xi}_3^{t_1} dt \), and \( e^{-t_1} \int_0^t \dot{\xi}_3^{t_1} dt \) are all bounded. Thus, all the signals involved are bounded. From the boundedness of \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \), we know that there exist positive constants \( \epsilon_1, \epsilon_2, \) such that \( |\epsilon_1| \leq \epsilon_1, |\epsilon_2| \leq \epsilon_2, \) and \( |\epsilon_3| \leq \epsilon_3. \) Furthermore, \( -\dot{\xi}_3 h(t) < (\delta \epsilon_1^{t_1}) + (\delta \epsilon_2^{t_1}) + (\delta \epsilon_3^{t_1}) h(t) \), and \( \dot{\epsilon}_j(t) < (\delta \epsilon_1^{t_1}) + (\delta \epsilon_2^{t_1}) + (\delta \epsilon_3^{t_1}) h(t) \), \( j = 1, 2, 3. \) It is obvious that the prescribed performance for \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \) is guaranteed.

The design procedure of GSP is completed.

**Remark 6.** It should be noted that the controller design is not relying on model of GSP, which guarantees the robustness against model uncertain and unknown disturbance.

**Remark 7.** Though similar control framework is designed as reference [31], compared with [31], the robustness against compound disturbances and unknown control directions is enhanced in large scale. Especially, the stability of system under compound disturbances is proven. However, in [31], the stability analysis as well as controller design did not consider any disturbances, which was not rigorous.

### 4. Simulation and Analysis

To verify the effectiveness of the proposed control scheme, a seeker servo system is chosen as simulation case.

The parameters of the control plant are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_L )</td>
<td>1.2 \times 10^{-5} \text{kg} \cdot \text{m}^{-2}</td>
<td>( C_y )</td>
<td>0.75</td>
</tr>
<tr>
<td>( C_m )</td>
<td>0.625N \cdot \text{m/A}</td>
<td>( v_y )</td>
<td>0.2</td>
</tr>
<tr>
<td>( l_a )</td>
<td>0.0062H</td>
<td>( F_c )</td>
<td>0.03</td>
</tr>
<tr>
<td>( R_s )</td>
<td>5.1\Omega</td>
<td>( F_s )</td>
<td>0.05</td>
</tr>
<tr>
<td>( k_{PWM} )</td>
<td>3.75kg</td>
<td>( B_p )</td>
<td>0.017</td>
</tr>
</tbody>
</table>

The control parameters of controller are chosen as follows: \( k_1 = 20, k_2 = 15, k_3 = -6, k_3 = 3, \delta_1 = 1, \delta_2 = 1, h_01 = 5, h_10 = 0.1, l_1 = 0.6, \delta_2 = 1, h_02 = 15, h_20 = 0.15, l_2 = 0.5, \delta_3 = 1, \delta_0 = 1, h_03 = 30, h_30 = 5, \) and \( l_3 = 0.3. \) Moreover, all the model coefficients in (1) are assumed to be uncertain by defining \( C = C_0 (1 + 0.3 \cos 2t) \), where \( C \) is the uncertain coefficient and \( C_0 \) is the nominal value of \( C \). While the parameters of RBFNN are chosen as \( \gamma_1 = 100, \epsilon_{max} = 0.001, \) the number of node is 100, and the center of RBFNN is evenly spaced in \( \epsilon \) \([-5, 4.9]\), the width is chosen as \( b = 10. \)

We choose \( \sin t \) as angular reference signal, while the initial disalignment angle is 0.5rad. Because the GSP possesses great isolation towards high-frequency signals [32], the basement disturbance is \( \dot{\delta} = 0.5 \cos (2t) \text{rad/s) \). The tracking performance of the presented control approach is depicted in Figures 3–6.

Figures 4–6 show that all tracking errors are forced within the prescribed bounds in the presence of compound disturbances. Thus, the control object is achieved. Furthermore, we can observe from Figures 4 and 5 that the system output of the proposed method converges faster and the precision is higher. Results show that the steady error of angle and angular velocity are no more than 0.02rad and 0.12rad/\( s \), respectively. It indicates that better transient and steady performance can be achieved through designed controller. Besides, the controller output is shown in Figure 7; because of nonlinear friction disturbance, the controller output is nonsmooth at velocity zero point. Especially, the designed controller is independent of precise model, and the method is able to achieve off-line estimation of unknown certain disturbance and the method enables great robustness against uncertain disturbance.

### 5. Conclusion

In this paper, a prescribed performance controller based on RBFNN is exploited for GSP with the backstepping framework. The problem of unknown hysterisis, unknown friction, and unknown compound disturbance is solved through a model transformation and RBFNN estimation of compound disturbances. Furthermore, Nussbaum function is adopted in this paper, which overcomes the problem of GSP with "unknown control directions". By constructing performance functions, angle and angular velocity tracking errors are restrained within the prescribed boundaries. The stability of the proposed method is proved by Lyapunov stability synthesis. Finally, the tracking performance and
superiority of the designed method are proved through simulations. Especially, the research of GSP can be improved by considering interactions between gimbals, which will be more meaningful in practical systems.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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References


