Research Article

Fuzzy-Approximation-Based Novel Back-Stepping Control of Flexible Air-Breathing Hypersonic Vehicles with Nonaffine Models

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This article investigates a novel fuzzy-approximation-based nonaffine control strategy for a flexible air-breathing hypersonic vehicle (FHV). Firstly, the nonaffine models are decomposed into an altitude subsystem and a velocity subsystem, and the nonaffine dynamics of the subsystems are processed by using low-pass filters. For the unknown functions and uncertainties in each subsystem, fuzzy approximators are used to approximate the total uncertainties, and norm estimation approach is introduced to reduce the computational complexity of the algorithm. Aiming at the saturation problem of actuator, a saturation auxiliary system is designed to transform the original control problem with input constraints into a new control problem without input constraints. Finally, the superiority of the proposed method is verified by simulation.

1. Introduction

Air-breathing hypersonic vehicle (AHV) is a kind of high-velocity aircraft flying at more than 5 times the speed of sound and cruising at an altitude in the near space. With the increasing extension of human activities to space, high-velocity and high-altitude vehicles represented by air-breathing hypersonic vehicles will play an important role in the field of aerospace. The control system is the “nerve center” of an aircraft, and the design of the AHV control system has become one of the frontier issues in the field of control science [1].

Compared with traditional aircrafts, the flight environment of flexible air-breathing hypersonic vehicles is more complex, the flight envelope span is larger, and the aerodynamic characteristic change is more complex. These problems bring difficulties to the design of the FHV control system. At the same time, in the control theory, FHV has the characteristics of strong nonlinearity, strong coupling, fast time-varying, and uncertainty, which make FHV extremely sensitive to flight attitude in dynamic characteris-

tics, and the design of the FHV control system is also subject to many constraints [2–4].

For the control of FHV, previous research has achieved fruitful results, for example, sliding mode variable structure control [5–8], neural network control [9–11], prescribed performance control [12–14], and fuzzy control [15, 16]. Hu et al. designed the FHV robust controller and the observers by means of sliding mode control in [6]. But in practical application, chattering problem exists in sliding mode control. In [7], the method of multimode switching between ordinary sliding surface and integral sliding surface is proposed to solve the chattering problem, but the switching between sliding surfaces is still relatively complex. In [8], Song et al. studied the double-loop sliding mode control method of HFV, unknown disturbances are observed by sliding mode observers, which enhanced the robustness of the control law. Bu et al. proposed an adaptive neural control method in [10]. The simulation results show that the control method can still guarantee the stable tracking of flight velocity and altitude to their respective reference inputs. In [11], the altitude subsystem of HFV is rewritten to a pure feedback

form, and an adaptive neural control law is designed to ensure the realizability of the control law. By constructing the prescribed performance functions in [14], the velocity tracking error and altitude tracking error of FHV are guaranteed, and the prescribed transient performance and steady-state accuracy of the system are satisfied. In [16], unknown functions of FHV models are approximated online by using fuzzy systems, but the algorithm requires that the unknown functions be strictly positive and bounded. There are a lot of unknown functions in each subsystem, and computational costs are heavy.

Most of the above studies focus on the affine models of FHV, which require simplifying the nonaffine models of FHV to the complete affine forms of the control input under certain assumptions, and then conducting research [17]. However, in fact, many of the dynamic performances of FHV are nonlinear functions of rudder deflection, angle of attack, velocity, and altitude. Particularly in the hypersonic flight process of FHV, these nonaffine dynamic characteristics cannot be affine, so it is necessary to strengthen the study of nonaffine models of FHV. [18] designed a control law of FHV based on the Mean Value Theorem [19], which has achieved certain effects. In [20], for a class of high-order nonaffine nonlinear systems with completely unknown dynamics, Meng et al. developed a novel system transformation to transform the nonaffine system into an affine system. According to the idea expressed in [20, 21] combined the system transformation with prescribed performance control scheme, the simulation results verified the effectiveness of the method. In [22], the novel system transformation with a low-pass filter is applied to constrained systems.

Based on above research, a fuzzy-approximation-based novel back-stepping control scheme of FHV with nonaffine models is proposed in this article. The FHV's nonaffine dynamics is decomposed into velocity subsystem and altitude subsystem to be controlled separately, and then nonaffine dynamics of FHV is processed by low-pass filters. A fuzzy approximator is introduced to approximate unknown function in each subsystem, and norm estimation approach [23] is introduced in the approximation of unknown functions. A novel back-stepping control method without virtual control law is designed. Considering the constraints of actuators, a new auxiliary system is designed to transform the control constraints into unconstrained ones. Finally, based on the simulation experiments, the performance of the control method is verified. The special contributions of this article are summarized as follows:

- (1) The traditional affine models need to simplify the nonaffine dynamics into fully affine forms. In this article, low-pass filters are used to solve the nonaffine dynamics of FHV
- (2) For each subsystem of FHV, only a fuzzy approximator is introduced to approximate the total uncertainties in the subsystem, which reduces the computational complexity of the system on the basis of ensuring the robustness of the controller. At the same time, the norm estimation approach is introduced in this

article to ensure the computational speed compared with the previous fuzzy control method

- (3) Compared with the traditional back-stepping control method, all the virtual controllers proposed in this article are only used as intermediate variables for the stability analysis of the system and do not participate in the final solution and execution
- (4) An auxiliary system is introduced to transform the original control problem with input constraints into a new control problem without input constraints to solve the transient saturation problem in the control input

2. FHV Model and Preliminaries

2.1. FHV Model. Taking X-43A as the modeling objective, the longitudinal integration analytical model of FHV was established by Bolender and Doman [24]; Parker et al. ignored the weak coupling in the dynamic model and established the parameter fitting model of FHV [17].

$$\dot{V} = \frac{T \cos(\theta - \gamma) - D}{m} - g \sin \gamma, \quad (1)$$

$$\dot{h} = V \sin \gamma, \quad (2)$$

$$\dot{\gamma} = \frac{L + T \sin(\theta - \gamma)}{mV} - \frac{g \cos \gamma}{V}, \quad (3)$$

$$\dot{\theta} = Q, \quad (4)$$

$$\dot{Q} = \frac{M + \tilde{\Psi}_1 \ddot{\eta}_1 + \tilde{\Psi}_2 \ddot{\eta}_2}{I_{yy}}, \quad (5)$$

$$k_1 \dot{\eta}_1 = -2\zeta_1 \omega_1 \dot{\eta}_1 - \omega_1^2 \eta_1 + N_1 - \tilde{\Psi}_1 \frac{M}{I_{yy}} - \frac{\tilde{\Psi}_1 \tilde{\Psi}_2 \ddot{\eta}_2}{I_{yy}}, \quad (6)$$

$$k_2 \dot{\eta}_2 = -2\zeta_2 \omega_2 \dot{\eta}_2 - \omega_2^2 \eta_2 + N_2 - \tilde{\Psi}_2 \frac{M}{I_{yy}} - \frac{\tilde{\Psi}_2 \tilde{\Psi}_1 \ddot{\eta}_1}{I_{yy}}, \quad (7)$$

where

$$\begin{aligned} k_1 &= 1 + \frac{\tilde{\Psi}_1}{I_{yy}}, \\ k_2 &= 1 + \frac{\tilde{\Psi}_2}{I_{yy}}, \end{aligned} \quad (8)$$

$$\tilde{\Psi}_1 = \int_{-L_f}^0 \hat{m}_f \xi \varphi_f(\xi) d(\xi),$$

$$\tilde{\Psi}_2 = \int_0^{L_a} \hat{m}_a \xi \varphi_a(\xi) d(\xi),$$

where $\varphi_f(\bullet)$ and $\varphi_a(\bullet)$ are structural mode shapes; other variables and functions are given in [24].

The FHV models contain five rigid body states and two elastic modes. The five rigid body states are flight velocity

V , flight altitude h , track angle γ , pitching angle θ , and pitch rate Q ; η_1 and η_2 represent the two elastic modes of FHV; the distance between the vehicle and the earth's center is expressed by r ; m is the mass of FHV; moment of inertia is I_{yy} ; define the flight angle of attack $\alpha = \theta - \gamma$. ς_1 and ς_2 represent the damping ratio, ω_1 and ω_2 represent the natural frequency for elastic modes, respectively; N_1 and N_2 are generalized forces; T , D , L , and M represent the thrust, resistance, lift, and pitching moment of FHV, respectively; where the approximations of T , D , L , M , N_1 , and N_2 are designed as follows [17]:

$$\begin{aligned}
T &\approx C_T^{\alpha^3} \alpha^3 + C_T^{\alpha^2} \alpha^2 + C_T^{\alpha} \alpha + C_T^0, \\
D &\approx \bar{q}S \left(C_D^{\alpha^2} \alpha^2 + C_D^{\alpha} \alpha + C_D^{\delta_e^2} \delta_e^2 + C_D^{\delta_e} \delta_e + C_D^0 \right), \\
L &\approx \bar{q}S \left(C_L^{\alpha} \alpha + C_L^{\delta_e} \delta_e + C_L^0 \right), \\
M &\approx z_T T + \bar{q}S \bar{c} \left[C_{M,\alpha}^{\alpha^2} \alpha^2 + C_{M,\alpha}^{\alpha} \alpha + C_{M,\alpha}^0 + c_e \delta_e \right], \\
N_1 &\approx N_1^{\alpha^2} \alpha^2 + N_1^{\alpha} \alpha + N_1^0, \\
N_2 &\approx N_2^{\alpha^2} \alpha^2 + N_2^{\alpha} \alpha + N_2^{\delta_e} \delta_e + N_2^0, \\
\bar{q} &= \frac{1}{2} \rho V^2, \\
\rho &= \rho_0 \exp \left(-\frac{h-h_0}{h_s} \right),
\end{aligned} \tag{9}$$

with

$$\begin{aligned}
C_T^{\alpha^3} &= \beta_1(h, \bar{q})\Phi + \beta_2(h, \bar{q}), \\
C_T^{\alpha^2} &= \beta_3(h, \bar{q})\Phi + \beta_4(h, \bar{q}), \\
C_T^{\alpha} &= \beta_5(h, \bar{q})\Phi + \beta_6(h, \bar{q}), \\
C_T^0 &= \beta_7(h, \bar{q})\Phi + \beta_8(h, \bar{q}).
\end{aligned} \tag{10}$$

The control inputs are fuel-air ratio Φ and elevator deflection angle δ_e ; the average aerodynamic chord length of FHV is $\bar{c}$; the reference area is S ; $\bar{q}$ represents the dynamic pressure of FHV; the air density at h is ρ ; z_T is the coupling coefficient of thrust and moment; c_e represents rudder deflection coefficient; h_0 represents nominal altitude; the air density at nominal altitude is ρ_0 ; $1/h_s$ is the decay rate of air density.

Remark 1. In the following controller design, the rigid body states can be measured, and the elastic modes are usually regarded as the unknown dynamics, which are treated as the robustness of the controller [10, 25].

Remark 2. It can be seen from equation (10) that the FHV models are nonaffine for the control inputs, because the parameter fitting form of resistance D contains δ_e^2 [17]. Therefore, in the subsequent control system design, based on the back-stepping control framework, in the case of uncertainties and nonaffine dynamics, fuzzy-approximation-based non-

linear tracking control laws Φ and δ_e are designed to achieve robust tracking of velocity and altitude commands V_{ref} and h_{ref} .

Remark 3. Because the fuel-air ratio Φ is related to thrust T , so the velocity V is mainly controlled by Φ . The elevator angle δ_e controls the change of altitude h by affecting the pitch rate Q , pitch angle θ , and track angle γ . Therefore, the FHV models can be decomposed into velocity subsystem (equation (1)) and altitude subsystem (equations (2)-(5)), and then the control laws are designed separately [26].

2.2. Theory of Fuzzy Approximation. In recent years, the universal approximation characteristics of the fuzzy system have been gradually discovered, proposed, and applied. Under sufficient rules of the fuzzy system, the fuzzy system can approximate the given nonlinear continuous function with arbitrary precision [27, 28]. The complete fuzzy system includes inference engine, knowledge base, fuzzification interface, and defuzzification interface. Compared with neural network and polynomial function approximation, the advantage of the fuzzy system is that it can effectively control based on fuzzy logic. Where the input of the system $\mathbf{X} = [x_1, x_2, \dots, x_n]^T \in \mathbf{R}^n$, the output is $\hat{F}(\mathbf{X})$.

$$\hat{F}(\mathbf{X} | \boldsymbol{\theta}) = \boldsymbol{\theta}^{*T} \boldsymbol{\xi}(\mathbf{X}). \tag{11}$$

In the fuzzy system, $\boldsymbol{\theta}^* = [\theta_1^*, \theta_2^*, \dots, \theta_p^*]^T \in \mathbf{R}^p$ represents the parameter vector of ideal weight coefficient, fuzzy basis function $\boldsymbol{\xi}(\mathbf{X}) = [\xi_1(\mathbf{X}), \xi_2(\mathbf{X}), \dots, \xi_p(\mathbf{X})]^T \in \mathbf{R}^p$. Fuzzy basis functions are expressed as

$$\xi_j(\mathbf{X}) = \frac{\prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{l=1}^N \left(\prod_{i=1}^n \mu_{F_i^l}(x_i) \right)}. \tag{12}$$

In equation (12), F_i^j is the j th ($j=1, \dots, N$) fuzzy rule corresponding to the i th ($i=1, \dots, n$) input point, N is the number of fuzzy rules, $\mu_{F_i^j}$ is the membership function, $0 \leq \mu_{F_i^j} \leq 1$. In this article, the Gauss basis function is chosen as the membership function.

Lemma 1 [27]. Suppose that the real continuous function $F(\mathbf{X})$ is an arbitrary function defined on compact set Ω . For any $\mu > 0$, the fuzzy system $\hat{F}(\mathbf{X}) = \boldsymbol{\theta}^{*T} \boldsymbol{\xi}(\mathbf{X})$ can be obtained, and the following conditions are guaranteed:

$$\sup_{\mathbf{x} \in \Omega} |F(\mathbf{X}) - \boldsymbol{\theta}^{*T} \boldsymbol{\xi}(\mathbf{X})| \leq \mu. \tag{13}$$

2.3. Saturation Auxiliary System. In order to maintain the normal working mode of scramjet and the physical constraint of FHV by deflection angle, the actuators of control inputs Φ and δ_e are limited to some extent. The saturation problem of control input is described as follows:

$$\Phi_c = \begin{cases} \Phi_{\max}, & \Phi > \Phi_{\max}, \\ \Phi, & \Phi_{\min} \leq \Phi \leq \Phi_{\max}, \\ \Phi_{\min}, & \Phi < \Phi_{\min}, \end{cases} \quad (14)$$

$$\delta_{ec} = \begin{cases} \delta_{emax}, & \delta_e > \delta_{emax}, \\ \delta_e, & \delta_{emin} \leq \delta_e \leq \delta_{emax}, \\ \delta_{emin}, & \delta_e < \delta_{emin}, \end{cases} \quad (15)$$

where Φ_c denotes the fuel-air ratio in the case of input saturation, δ_{ec} denotes the elevator deflection in the case of input saturation, $\Phi_{\min}$ and $\Phi_{\max}$ are the lower and upper bounds of Φ , and δ_{emin} and δ_{emax} are the lower and upper bounds of δ_e . In this article, a saturation auxiliary system is designed under the condition of input saturation. The input constraints of the control system are transformed into unconstrained ones.

When the inputs of the saturated auxiliary system are Φ and δ_e , the sigmoid functions are used to approximate the outputs Φ_c and δ_{ec} of the auxiliary system.

$$\begin{aligned} \Phi_c &= \frac{\Phi_{\max} - \Phi_{\min}}{2} \text{sigmoid}\left(\frac{2\Phi - \Phi_{\max} + \Phi_{\min}}{\Phi_{\max} - \Phi_{\min}}\right) \\ &\quad + \frac{\Phi_{\max} + \Phi_{\min}}{2}, \\ \delta_{ec} &= \frac{\delta_{emax} - \delta_{emin}}{2} \text{sigmoid}\left(\frac{2\delta_e - \delta_{emin} + \delta_{emax}}{\delta_{emax} - \delta_{emin}}\right) \\ &\quad + \frac{\delta_{emax} + \delta_{emin}}{2}. \end{aligned} \quad (16)$$

According to equation (16) and the characteristic of the sigmoid function, whether Φ and δ_e take any value, the saturation auxiliary system can make Φ_c and δ_{ec} meet the constraints of equation (15).

Remark 4. In the controller design, the original control objective can be converted to design unconstrained Φ and δ_e such that velocity V and altitude h track their reference commands V_{ref} and h_{ref} .

2.4. Controller Design. In the following section, the altitude subsystem and velocity subsystem of the controller are designed with the new fuzzy-approximation-based control scheme in this article.

2.4.1. Controller Design of the Altitude Subsystem. In the altitude subsystem, we design an altitude controller based on nonaffine dynamics and a fuzzy approximator to achieve stable tracking of reference altitude h_{ref} .

Define altitude tracking error $\tilde{h} = h - h_{\text{ref}}$, the reference command of γ is chosen as

$$\gamma_d = \arcsin\left(\frac{-k_{h,1}\tilde{h} - k_{h,2}\int_0^t \tilde{h}d\tau + \dot{h}_{\text{ref}}}{V}\right), \quad (17)$$

where $k_{h,1} \in R^+$ and $k_{h,2} \in R^+$ are design parameters.

If the design parameters $k_{h,1} > 0$, $k_{h,2} > 0$ and $\gamma \rightarrow \gamma_d$, then $\tilde{h}$ can be regulated to zero exponentially [29]. In this way, the control objective of the altitude subsystem can be transformed to $\gamma \rightarrow \gamma_d$ by designing the appropriate control law δ_e .

According to the idea expressed in [20, 21], we define a set of newly state variables; the control input δ_e is also viewed as the newly defined state to deal with the nonaffine features. Define new state variables

$$\begin{aligned} \psi_{h,1} &= \gamma, \\ \psi_{h,2} &= \theta, \\ \psi_{h,3} &= Q, \\ \psi_{h,4} &= \delta_e. \end{aligned} \quad (18)$$

Original nonaffine systems (3)-(5) can be rewritten to the following equivalent general nonaffine form according to equation (18).

$$\begin{aligned} \dot{\psi}_{h,1} &= \mathcal{K}_1(\psi_{h,1}, \psi_{h,2}), \\ \dot{\psi}_{h,2} &= \psi_{h,3}, \\ \dot{\psi}_{h,3} &= \mathcal{K}_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}), \\ \dot{\psi}_{h,4} &= u_h - \tau_h \psi_{h,4}, \\ \gamma_h &= \psi_{h,1} = \gamma, \end{aligned} \quad (19)$$

where $\mathcal{K}_1(\psi_{h,1}, \psi_{h,2})$ and $\mathcal{K}_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})$ are continuous differentiable nonlinear functions; u_h is an introduced variable, and it is regarded as the virtual control input of (19); the control inputs δ_e are generated by a low-pass filter “ $\dot{\psi}_{h,4} = u_h - \tau_h \psi_{h,4}$ ” driven by u_h . “ $\dot{\psi}_{h,4} = u_h - \tau_h \psi_{h,4}$ ” is input-to-state stable. γ_h is the output of (19); $\tau_h \in R^+$ is the design parameter.

From equations (3)-(5) together with previous studies [17], we have

$$\begin{aligned} \frac{\partial \mathcal{K}_1(\psi_{h,1}, \psi_{h,2})}{\partial \psi_{h,1}} &\approx \frac{\bar{q}SC_L^\alpha + T \cos(\theta - \gamma)}{mV} > 0, \\ \frac{\partial \mathcal{K}_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,4}} &\approx \frac{\bar{q}S\bar{c}c_e + \bar{\zeta}_2 N_2^{\delta_e}/k_2}{I_{yy}} > 0. \end{aligned} \quad (20)$$

Next, transform the nonaffine models.

Define $v_1 = \psi_{h,1} = \gamma$, $v_2 = \dot{v}_1 = \mathcal{K}_1(\psi_{h,1}, \psi_{h,2})$, combined with equation (19), the time derivative of v_2 is derived as

$$\begin{aligned} \dot{v}_2 &= \frac{\partial \mathcal{K}_1(\psi_{h,1}, \psi_{h,2})}{\partial \psi_{h,1}} \dot{\psi}_{h,1} + \frac{\partial \mathcal{K}_1(\psi_{h,1}, \psi_{h,2})}{\partial \psi_{h,2}} \dot{\psi}_{h,2} \\ &= \frac{\partial \mathcal{K}_1(\psi_{h,1}, \psi_{h,2})}{\partial \psi_{h,1}} \mathcal{K}_1(\psi_{h,1}, \psi_{h,2}) + \frac{\partial \mathcal{K}_1(\psi_{h,1}, \psi_{h,2})}{\partial \psi_{h,2}} \psi_{h,3} \\ &\triangleq \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}). \end{aligned} \quad (21)$$

It can be noted that

$$\frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,3}} = \frac{\partial \mathcal{K}_1(\psi_{h,1}, \psi_{h,2})}{\partial \psi_{h,2}} > 0. \quad (22)$$

We further define $v_3 = \dot{v}_2 = Y_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})$. Taking time derivative of v_3 and substituting (19)

$$\begin{aligned} \dot{v}_3 &= \frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,1}} \dot{\psi}_{h,1} \\ &+ \frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,2}} \dot{\psi}_{h,2} \\ &+ \frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,3}} \dot{\psi}_{h,3} \\ &= \frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,1}} \mathcal{K}_1(\psi_{h,1}, \psi_{h,2}) \\ &+ \frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,2}} \psi_{h,3} \\ &+ \frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,3}} \mathcal{K}_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}) \\ &\triangleq \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}). \end{aligned} \quad (23)$$

Finally, define $v_4 = \dot{v}_3 = Y_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})$. Invoking (19), the time derivative of v_4 is derived as

$$\begin{aligned} \dot{v}_4 &= \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,1}} \dot{\psi}_{h,1} \\ &+ \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,2}} \dot{\psi}_{h,2} \\ &+ \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,3}} \dot{\psi}_{h,3} \\ &+ \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,4}} \dot{\psi}_{h,4} \\ &= \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,1}} \mathcal{K}_1(\psi_{h,1}, \psi_{h,2}) \\ &+ \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,2}} \psi_{h,3} \\ &+ \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,3}} \mathcal{K}_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}) \\ &- \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,4}} \tau_h \psi_{h,4} \\ &+ \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,4}} u_h \\ &\triangleq \Upsilon_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}) + K_h u_h. \end{aligned} \quad (24)$$

where

$$\begin{aligned} K_h &= \frac{\partial \Upsilon_2(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,4}} \\ &= \frac{\partial \Upsilon_1(\psi_{h,1}, \psi_{h,2}, \psi_{h,3})}{\partial \psi_{h,3}} \frac{\partial \mathcal{K}_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4})}{\partial \psi_{h,4}} > 0. \end{aligned} \quad (25)$$

Through model transformation, the nonaffine system (19) can be converted into affine form.

$$\begin{aligned} \dot{v}_1 &= v_2, \\ \dot{v}_2 &= v_3, \\ \dot{v}_3 &= v_4, \\ \dot{v}_4 &= \Upsilon_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}) + K_h u_h, \\ y_h &= v_1 = \gamma. \end{aligned} \quad (26)$$

The fuzzy-approximation-based altitude subsystem controller is designed by using the transformed affine form.

Remark 5. According to the theory of input-to-state stability, when the input is bounded, the state of the system is also bounded. So we need to prove the boundedness of u_h .

Theorem 1. *If the control inputs u_h are bounded, there must be fixed values u_{hM} , such that*

$$|u_h| \leq u_{hM}. \quad (27)$$

Proof. Define the Lyapunov function

$$L_{\psi_{h,4}} = \frac{\psi_{h,4}^2}{2}. \quad (28)$$

Taking time derivative along (28).

$$\begin{aligned} \dot{L}_{\psi_{h,4}} &= \psi_{h,4} \dot{\psi}_{h,4} = \psi_{h,4} (u_h - \tau_h \psi_{h,4}) \\ &= -\tau_h \psi_{h,4}^2 + u_h \psi_{h,4} \leq -\tau_h \psi_{h,4}^2 + \frac{u_{hM}^2}{2} + \frac{u_{h,4}^2}{2} \\ &= -\left(\tau_h - \frac{1}{2}\right) \frac{u_{h,4}^2}{2} + \frac{u_{hM}^2}{2} \end{aligned} \quad (29)$$

If $\tau_h > 1/2$, $\psi_{h,4}$ will ultimately converge to the compact $\Omega_{\psi_{h,4}}$, where $\Omega_{\psi_{h,4}} = \{\psi_{h,4} \mid |\psi_{h,4}| \leq \sqrt{(u_{hM}^2/2)/(\tau_h - (1/2))}\}$. That is, $\psi_{h,4}$ is bounded and the low-pass filter “ $\dot{\psi}_{h,4} = u_h - \tau_h \psi_{h,4}$ ” is stable when u_h is bounded.

Define tracking error $e_{h,1}$ as

$$e_{h,1} = v_1 - v_{1d}. \quad (30)$$

Taking time derivative along (30) and employing (26), it leads to

$$\dot{e}_{h,1} = \dot{v}_1 - \dot{v}_{1d} = v_2 - \dot{v}_{1d}. \quad (31)$$

Define intermediate variable v_{2d} as

$$v_{2d} = -\lambda_{h,1}e_{h,1} + \dot{v}_{1d}, \quad (32)$$

where $\lambda_{h,1} \in R^+$ is the design parameter.

Then, define tracking error $e_{h,2}$ as

$$e_{h,2} = v_2 - v_{2d}. \quad (33)$$

Applying (26), the time derivative of $e_{h,2}$ is derived as

$$\dot{e}_{h,2} = \dot{v}_2 - \dot{v}_{2d} = v_3 - \dot{v}_{2d}. \quad (34)$$

Define a new intermediate state variable v_{3d} as

$$v_{3d} = -\lambda_{h,2}e_{h,2} - e_{h,1} + \dot{v}_{2d}, \quad (35)$$

where $\lambda_{h,2} \in R^+$ is the design parameter.

Define tracking error $e_{h,3}$ as

$$e_{h,3} = v_3 - v_{3d}. \quad (36)$$

Differentiating along (36) with respect to time and applying (26), we get

$$\dot{e}_{h,3} = \dot{v}_3 - \dot{v}_{3d} = v_4 - \dot{v}_{3d}. \quad (37)$$

Then, the intermediate variable v_{4d} is designed as

$$v_{4d} = -\lambda_{h,3}e_{h,3} - e_{h,2} + \dot{v}_{3d}, \quad (38)$$

where $\lambda_{h,3} \in R^+$ is the design parameter.

Finally, define tracking error $e_{h,4}$ as

$$e_{h,4} = v_4 - v_{4d}. \quad (39)$$

Taking the time derivative of $e_{h,4}$ and invoking (26) lead to

$$\begin{aligned} \dot{e}_{h,4} &= \dot{v}_4 - \dot{v}_{4d} = Y_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}) + K_h u_h - \dot{v}_{4d} \\ &= K_h \left[\frac{Y_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}) - \dot{v}_{4d}}{K_h} + u_h \right]. \end{aligned} \quad (40)$$

Define unknown function F_h as

$$F_h = \frac{Y_3(\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}) - \dot{v}_{4d}}{K_h} + u_h = F_h(\Psi_h), \quad (41)$$

where $\Psi_h = [\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}]^T$. Since the function F_h is an unknown function, the fuzzy function approximator designed in the above part is used to approximate it. The

estimated value $\hat{F}_h$ of the nonlinear function F_h can be expressed as

$$\hat{F}_h(\Psi_h | \Theta_h) = \Theta_h^{*T} \xi_h(\Psi_h), \quad (42)$$

where $\Psi_h = [\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}]^T \in R^+$ is the input vector to the unknown function, fuzzy basis function $\xi_h(\Psi_h) = [\xi_{h,1}(\Psi_h), \xi_{h,2}(\Psi_h), \dots, \xi_{h,p1}(\Psi_h)]^T \in R^{p1}$, $\Theta_h^* = [\theta_{h,1}^*, \theta_{h,2}^*, \dots, \theta_{h,p1}^*]^T \in R^{p1}$ is the ideal weight coefficient parameter vector, and the unknown function F_h can be expressed as

$$F_h(\Psi_h) = \Theta_h^{*T} \xi_h(\Psi_h) + \mu_h, \quad (43)$$

where μ_h represents the estimated error, and $|\mu_h| \leq \mu_{hM}$; μ_{hM} is the upper bound of the estimated error.

Finally, the control law is devised as

$$u_h = -\lambda_{h,4}e_{h,4} - e_{h,3} - \frac{\hat{\varphi}_1}{2} \xi_h^T(\Psi_h) \xi_h(\Psi_h), \quad (44)$$

where $\lambda_{h,4} \in R^+$ is the design parameter. $\hat{\varphi}_1$ denotes the estimation of $\varphi_1 = \|\Theta_h^*\|^2$; its adaptive law is selected as

$$\dot{\hat{\varphi}}_1 = \frac{l_1}{2} e_{h,1} \xi_h^T(\Psi_h) \xi_h(\Psi_h) - 2\hat{\varphi}_1, \quad (45)$$

where $l_1 \in R^+$ is the design parameter.

Substitute equation (32) into equation (33) to obtain

$$\begin{aligned} e_{h,2} &= v_2 - (-\lambda_{h,1}e_{h,1} + \dot{v}_{1d}) \\ &= v_2 + \lambda_{h,1}e_{h,1} - \dot{v}_{1d} \\ &= \dot{e}_{h,1} + \lambda_{h,1}e_{h,1}. \end{aligned} \quad (46)$$

Taking time derivative along (46),

$$\dot{e}_{h,2} = \ddot{e}_{h,1} + \lambda_{h,1}\dot{e}_{h,1}. \quad (47)$$

By combining equations (26) and (35), equation (36) can be obtained.

$$\begin{aligned} e_{h,3} &= v_3 - (-\lambda_{h,2}e_{h,2} - e_{h,1} + \dot{v}_{2d}) \\ &= v_3 + \lambda_{h,2}e_{h,2} + e_{h,1} - \dot{v}_{2d} = \dot{e}_{h,2} + \lambda_{h,2}e_{h,2} + e_{h,1}. \end{aligned} \quad (48)$$

Substituting equations (46) and (47) into equation (48), it yields

$$\begin{aligned} e_{h,3} &= \ddot{e}_{h,1} + \lambda_{h,1}\dot{e}_{h,1} + \lambda_{h,2}(\dot{e}_{h,1} + \lambda_{h,1}e_{h,1}) + e_{h,1} \\ &= \ddot{e}_{h,1} + (\lambda_{h,1} + \lambda_{h,2})\dot{e}_{h,1} + (\lambda_{h,1}\lambda_{h,2} + 1)e_{h,1}. \end{aligned} \quad (49)$$

Taking time derivative along (49), we obtain

$$\dot{e}_{h,3} = \ddot{e}_{h,1}(\lambda_{h,1} + \lambda_{h,2}) + (\lambda_{h,1}\lambda_{h,2} + 1)\dot{e}_{h,1}. \quad (50)$$

Substituting (26) and (38) into (39), $e_{h,4}$ is formulated as

$$\begin{aligned} e_{h,4} &= v_4 - (-\lambda_{h,3}e_{h,3} - e_{h,2} + \dot{v}_{3d}) \\ &= v_4 + \lambda_{h,3}e_{h,3} + e_{h,2} - \dot{v}_{3d} \\ &= \dot{e}_{h,3} + \lambda_{h,3}e_{h,3} + e_{h,2}. \end{aligned} \quad (51)$$

Combining equations (46), (49), and (50), we can obtain

$$\begin{aligned} e_{h,4} &= \ddot{e}_{h,1} + (\lambda_{h,1} + \lambda_{h,2})\dot{e}_{h,1} + (\lambda_{h,1}\lambda_{h,2} + 1)\dot{e}_{h,1} \\ &\quad + \lambda_{h,3}[\ddot{e}_{h,1} + (\lambda_{h,1} + \lambda_{h,2})\dot{e}_{h,1} + (\lambda_{h,1}\lambda_{h,2} + 1)e_{h,1}] \\ &\quad + \dot{e}_{h,1} + \lambda_{h,1}e_{h,1} = \ddot{e}_{h,1} + (\lambda_{h,1} + \lambda_{h,2} + \lambda_{h,3})\dot{e}_{h,1} \\ &\quad + (\lambda_{h,1}\lambda_{h,2} + \lambda_{h,1}\lambda_{h,3} + \lambda_{h,2}\lambda_{h,3} + 2)\dot{e}_{h,1} \\ &\quad + (\lambda_{h,1} + \lambda_{h,3} + \lambda_{h,1}\lambda_{h,2}\lambda_{h,3})e_{h,1}. \end{aligned} \quad (52)$$

Substituting (49) and (52) into (44) leads to

$$\begin{aligned} u_h &= -\lambda_{h,4}\ddot{e}_{h,1} - (\lambda_{h,4}\lambda_{h,1} + \lambda_{h,4}\lambda_{h,2} + \lambda_{h,4}\lambda_{h,3} + 1)\ddot{e}_{h,1} \\ &\quad - (\lambda_{h,4}\lambda_{h,1}\lambda_{h,2} + \lambda_{h,4}\lambda_{h,2}\lambda_{h,3} + \lambda_{h,4}\lambda_{h,3}\lambda_{h,1} \\ &\quad + 2\lambda_{h,4} + \lambda_{h,1} + \lambda_{h,2})\dot{e}_{h,1} - (\lambda_{h,4}\lambda_{h,1}\lambda_{h,2}\lambda_{h,3} + \lambda_{h,4}\lambda_{h,3} \\ &\quad + \lambda_{h,4}\lambda_{h,1} + \lambda_{h,1}\lambda_{h,2} + 1)e_{h,1} - \frac{\hat{\varphi}_1}{2}\xi_h^T(\Psi_h)\xi_h(\Psi_h). \end{aligned} \quad (53)$$

Remark 6. Unlike the back-stepping control method proposed by Bu et al. [30], the presented altitude controller is exploited using an improved back-stepping framework, in which the virtual controller is transformed into intermediate variables and only the final actual control laws are needed to be implemented, which avoids repeated calculation and greatly reduces the computational pressure of the system.

Remark 7. Compared with the method in [16, 31, 32], each weight update requires a lot of calculation. In this article, only one online learning parameter is needed by using the norm estimation approach, which makes the online calculation of the system significantly reduced.

Theorem 2. Considering the closed-loop subsystem consisting of plant (26), with controller (53), and adaptive law (45). Then all the signals involved are semiglobally uniformly ultimately bounded.

Proof. Consider the following Lyapunov function:

$$W_h = \frac{e_{h,1}^2}{2} + \frac{e_{h,2}^2}{2} + \frac{e_{h,3}^2}{2} + \frac{e_{h,4}^2}{2K_h} + \frac{\tilde{\varphi}_1^2}{2l_1} \quad (54)$$

with $\tilde{\varphi}_1 = \hat{\varphi}_1 - \varphi_1$.

Applying (31), (34), (37), (40), (41), and (45), $\dot{W}_h$ is derived as

$$\begin{aligned} \dot{W}_h &= e_{h,1}\dot{e}_{h,1} + e_{h,2}\dot{e}_{h,2} + e_{h,3}\dot{e}_{h,3} + \frac{e_{h,4}\dot{e}_{h,4}}{K_h} + \frac{\tilde{\varphi}_1\dot{\tilde{\varphi}}_1}{l_1} \\ &= e_{h,1}(v_2 - \dot{v}_{1d}) + e_{h,2}(v_3 - \dot{v}_{2d}) + e_{h,3}(v_4 - \dot{v}_{3d}) \\ &\quad + e_{h,4}\left[\frac{Y_3(\Psi_{h,1}, \Psi_{h,2}, \Psi_{h,3}, \Psi_{h,4}) - \dot{v}_{4d}}{K_h} + u_h\right] \\ &\quad + \frac{\tilde{\varphi}_1}{l_1}\left(\frac{l_1}{2}e_{h,1}\hat{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) - 2\hat{\varphi}_1\right) \\ &= e_{h,1}(v_{2d} + e_{h,2} - \dot{v}_{1d}) + e_{h,2}(v_{3d} + e_{h,3} - \dot{v}_{2d}) \\ &\quad + e_{h,3}(v_{4d} + e_{h,4} - \dot{v}_{3d}) + \frac{1}{2}e_{h,1}\tilde{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) \\ &\quad + e_{h,4}\left[\theta_h^{*T}\xi_h(\Psi_h) + \mu_h + u_h\right] - \frac{2\tilde{\varphi}_1\hat{\varphi}_1}{l_1}. \end{aligned} \quad (55)$$

Considering (32), (35), and (38), $\dot{W}_h$ becomes

$$\begin{aligned} \dot{W}_h &= e_{h,1}(-\lambda_{h,1}e_{h,1} + \dot{v}_{1d} + e_{h,2} - \dot{v}_{1d}) \\ &\quad + e_{h,2}(-\lambda_{h,2}e_{h,2} - e_{h,1} + \dot{v}_{2d} + e_{h,3} - \dot{v}_{2d}) \\ &\quad + e_{h,3}(-\lambda_{h,3}e_{h,3} - e_{h,2} + e_{h,4}) + \frac{1}{2}e_{h,1}\tilde{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) \\ &\quad + e_{h,4}\left[\theta_h^{*T}\xi_h(\Psi_h) + \mu_h + u_h\right] - \frac{2\tilde{\varphi}_1\hat{\varphi}_1}{l_1}. \end{aligned} \quad (56)$$

Substituting (44) into (56) yields, we can get

$$\begin{aligned} \dot{W}_h &= -\lambda_{h,1}e_{h,1}^2 - \lambda_{h,2}e_{h,2}^2 - \lambda_{h,3}e_{h,3}^2 + e_{h,3}e_{h,4} \\ &\quad + \frac{1}{2}e_{h,1}\tilde{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) + e_{h,4}\theta_h^{*T}\xi_h(\Psi_h) \\ &\quad + e_{h,4}\mu_h + e_{h,4}\left[-\lambda_{h,4}e_{h,4} - e_{h,3} - \frac{\hat{\varphi}_1}{2}\xi_h^T(\Psi_h)\xi_h(\Psi_h)\right] \\ &\quad - \frac{2\tilde{\varphi}_1\hat{\varphi}_1}{l_1} = -\lambda_{h,1}e_{h,1}^2 - \lambda_{h,2}e_{h,2}^2 - \lambda_{h,3}e_{h,3}^2 - \lambda_{h,4}e_{h,4}^2 \\ &\quad + \frac{1}{2}e_{h,1}\tilde{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) + e_{h,4}\theta_h^{*T}\xi_h(\Psi_h) \\ &\quad - e_{h,4}\frac{\hat{\varphi}_1}{2}\xi_h^T(\Psi_h)\xi_h(\Psi_h) + e_{h,4}\mu_h - \frac{2\tilde{\varphi}_1\hat{\varphi}_1}{l_1}. \end{aligned} \quad (57)$$

From the fact that

$$\tilde{\varphi}_1^2 + 2\tilde{\varphi}_1(\hat{\varphi}_1 - \tilde{\varphi}_1) + \varphi_1^2 = \tilde{\varphi}_1^2 + 2\tilde{\varphi}_1\varphi_1 + \varphi_1^2 = (\tilde{\varphi}_1 + \varphi_1)^2 \geq 0, \quad (58)$$

we obtain

$$2\tilde{\varphi}_1\hat{\varphi}_1 \geq \tilde{\varphi}_1^2 - \varphi_1^2. \quad (59)$$

Thereby, equation (57) becomes

$$\begin{aligned} \dot{W}_h &\leq -\lambda_{h,1}e_{h,1}^2 - \lambda_{h,2}e_{h,2}^2 - \lambda_{h,3}e_{h,3}^2 - \lambda_{h,4}e_{h,4}^2 \\ &\quad + \frac{1}{2}e_{h,1}\tilde{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) + \frac{1}{2}e_{h,1}\tilde{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) \\ &\quad + e_{h,4}\theta_h^{*T}\xi_h(\Psi_h) + e_{h,4}\mu_h - e_{h,4}\frac{\tilde{\varphi}_1}{2}\xi_h^T(\Psi_h)\xi_h(\Psi_h) \\ &\quad - e_{h,4}\frac{\varphi_1}{2}\xi_h^T(\Psi_h)\xi_h(\Psi_h) - \frac{\tilde{\varphi}_1^2}{l_1} + \frac{\varphi_1^2}{l_1}. \end{aligned} \quad (60)$$

In this article, since $0 \leq \mu_{F_i}(x) \leq 1$, and applying (12), we know that $0 \leq \xi_j(\mathbf{X}) \leq 1$. There exists a constant $\bar{\xi}_h \in R^+$, such that $\|\xi_h(\Psi_h)\| \leq \bar{\xi}_h$. Hence, we get

$$\begin{aligned} \frac{1}{2}e_{h,1}\tilde{\varphi}_1\xi_h^T(\Psi_h)\xi_h(\Psi_h) &\leq \frac{1}{2}\bar{\xi}_h^2|e_{h,1}||\tilde{\varphi}_1| \\ &\leq \frac{2l_1}{2}\left(\frac{1}{2}\bar{\xi}_h^2|e_{h,1}|\right)^2 + \frac{\tilde{\varphi}_1^2}{2(2l_1)} \\ &= \frac{l_1\bar{\xi}_h^4}{4}e_{h,1}^2 + \frac{\tilde{\varphi}_1^2}{4l_1}, \\ e_{h,4}\theta_h^{*T}\xi_h(\Psi_h) &\leq |e_{h,4}|\sqrt{\varphi_1\bar{\xi}_h} \leq \sqrt{\varphi_1\bar{\xi}_h}\left(\frac{e_{h,4}^2}{2} + \frac{1}{2}\right) \\ &= \frac{\sqrt{\varphi_1\bar{\xi}_h}}{2}e_{h,4}^2 + \frac{\sqrt{\varphi_1\bar{\xi}_h}}{2}, \end{aligned}$$

$$\begin{aligned} e_{h,4}\mu_h &\leq \mu_{hM}|e_{h,4}| \leq \mu_{hM}\left(\frac{e_{h,4}^2}{2} + \frac{1}{2}\right) \\ &= \frac{\mu_{hM}e_{h,4}^2}{2} + \frac{\mu_{hM}}{2} - e_{h,4}\frac{\tilde{\varphi}_1}{2}\xi_h^T(\Psi_h)\xi_h(\Psi_h) \\ &\leq \frac{1}{2}\bar{\xi}_h^2|e_{h,4}||\tilde{\varphi}_1| \leq \frac{2l_1}{2}\left(\frac{1}{2}\bar{\xi}_h^2|e_{h,4}|\right)^2 + \frac{\tilde{\varphi}_1^2}{2(2l_1)} \\ &= \frac{l_1\bar{\xi}_h^4}{4}e_{h,4}^2 + \frac{\tilde{\varphi}_1^2}{4l_1} - e_{h,4}\frac{\varphi_1}{2}\xi_h^T(\Psi_h)\xi_h(\Psi_h) \\ &\leq \frac{1}{2}\bar{\xi}_h^2|e_{h,4}|\varphi_1 \leq \frac{1}{2}\varphi_1\bar{\xi}_h^2\left(\frac{e_{h,4}^2}{2} + \frac{1}{2}\right) \\ &= \frac{\varphi_1\bar{\xi}_h^2}{4}e_{h,4}^2 + \frac{\varphi_1\bar{\xi}_h^2}{4}. \end{aligned} \quad (61)$$

Then, (60) becomes

$$\begin{aligned} \dot{W}_h &\leq -\lambda_{h,1}e_{h,1}^2 - \lambda_{h,2}e_{h,2}^2 - \lambda_{h,3}e_{h,3}^2 - \lambda_{h,4}e_{h,4}^2 + \frac{l_1\bar{\xi}_h^4}{4}e_{h,1}^2 \\ &\quad + \frac{\tilde{\varphi}_1^2}{4l_1} + \frac{\sqrt{\varphi_1\bar{\xi}_h}}{2}e_{h,4}^2 + \frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} + \frac{\mu_{hM}e_{h,4}^2}{2} + \frac{\mu_{hM}}{2} \\ &\quad + \frac{l_1\bar{\xi}_h^4}{4}e_{h,4}^2 + \frac{\tilde{\varphi}_1^2}{4l_1} + \frac{\varphi_1\bar{\xi}_h^2}{4}e_{h,4}^2 + \frac{\varphi_1\bar{\xi}_h^2}{4} - \frac{\tilde{\varphi}_1^2}{l_1} + \frac{\varphi_1^2}{l_1}, \\ \dot{W}_h &\leq -\left(\lambda_{h,1} - \frac{l_1\bar{\xi}_h^4}{4}\right)e_{h,1}^2 - \lambda_{h,2}e_{h,2}^2 - \lambda_{h,3}e_{h,3}^2 + \frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} \\ &\quad + \frac{\mu_{hM}}{2} + \frac{\varphi_1\bar{\xi}_h^2}{4} - \left(\lambda_{h,4} - \frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} - \frac{\mu_{hM}}{2} \right. \\ &\quad \left. - \frac{l_1\bar{\xi}_h^4}{4} - \frac{\varphi_1\bar{\xi}_h^2}{4}\right)e_{h,4}^2 + \frac{\varphi_1^2}{l_1} - \frac{\tilde{\varphi}_1^2}{2l_1}. \end{aligned} \quad (62)$$

Let $\lambda_{h,1} > (l_1\bar{\xi}_h^4/4)$, $\lambda_{h,4} > (\sqrt{\varphi_1\bar{\xi}_h}/2) + (\mu_{hM}/2) + (l_1\bar{\xi}_h^4/4) + (\varphi_1\bar{\xi}_h^2/4)$, $\lambda_{h,2} > 0$, $\lambda_{h,3} > 0$, and $l_1 > 0$. And define the following compact sets

$$\begin{aligned} \Omega_{e_{h,1}} &= \left\{ e_{h,1} \mid |e_{h,1}| \leq \sqrt{\left(\frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} + \frac{\mu_{hM}}{2} + \frac{\varphi_1^2}{l_1} + \frac{\varphi_1\bar{\xi}_h^2}{4}\right) / \left(\lambda_{h,1} - \frac{l_1\bar{\xi}_h^4}{4}\right)} \right\}, \\ \Omega_{e_{h,4}} &= \left\{ e_{h,4} \mid |e_{h,4}| \leq \sqrt{\left(\frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} + \frac{\mu_{hM}}{2} + \frac{\varphi_1^2}{l_1} + \frac{\varphi_1\bar{\xi}_h^2}{4}\right) / \left(\lambda_{h,4} - \frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} - \frac{\mu_{hM}}{2} - \frac{l_1\bar{\xi}_h^4}{4} - \frac{\varphi_1\bar{\xi}_h^2}{4}\right)} \right\}, \\ \Omega_{e_{h,2}} &= \left\{ e_{h,2} \mid |e_{h,2}| \leq \sqrt{\left(\frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} + \frac{\mu_{hM}}{2} + \frac{\varphi_1^2}{l_1} + \frac{\varphi_1\bar{\xi}_h^2}{4}\right) / \lambda_{h,2}} \right\}, \\ \Omega_{e_{h,3}} &= \left\{ e_{h,3} \mid |e_{h,3}| \leq \sqrt{\left(\frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} + \frac{\mu_{hM}}{2} + \frac{\varphi_1^2}{l_1} + \frac{\varphi_1\bar{\xi}_h^2}{4}\right) / \lambda_{h,3}} \right\}, \\ \Omega_{\tilde{\varphi}_1} &= \left\{ \tilde{\varphi}_1 \mid |\tilde{\varphi}_1| \leq \sqrt{\left(\frac{\sqrt{\varphi_1\bar{\xi}_h}}{2} + \frac{\mu_{hM}}{2} + \frac{\varphi_1^2}{l_1} + \frac{\varphi_1\bar{\xi}_h^2}{4}\right) / \left(\frac{1}{2l_1}\right)} \right\}. \end{aligned} \quad (63)$$

If $e_{h,1} \notin \Omega_{e_{h,1}}$ or $e_{h,4} \notin \Omega_{e_{h,4}}$ or $e_{h,2} \notin \Omega_{e_{h,2}}$ or $e_{h,3} \notin \Omega_{e_{h,3}}$ or $\tilde{\varphi}_1 \notin \Omega_{\tilde{\varphi}_1}$, $\dot{W}_h$ will be negative. It is clear that $e_{h,1}$, $e_{h,4}$, $e_{h,2}$, $e_{h,3}$ and $\tilde{\varphi}_1$ are semiglobally uniformly ultimately bounded. This completes the proof.

2.5. Controller Design of Velocity Subsystem. For the velocity subsystem, its tracking target is to achieve the stable tracking of velocity V to reference velocity V_{ref} .

Define new state variables $\psi_{V,1} = V$ and $\psi_{V,2} = \Phi$. In order to ensure the control effect, the velocity subsystem should be changed into an equivalent general nonaffine form with $\psi_{V,1}$ and $\psi_{V,2}$.

$$\begin{aligned}\dot{\psi}_{V,1} &= \Gamma(\psi_{V,1}, \psi_{V,2}), \\ \dot{\psi}_{V,2} &= u_V - \tau_V \psi_{V,2}, \\ y_V &= \psi_{V,1},\end{aligned}\quad (64)$$

where the transformation consists of a low-pass filter “ $\dot{\psi}_{V,2} = u_V - \tau_V \psi_{V,2}$ ” and 2 newly defined state variables. $\Gamma(\psi_{V,1}, \psi_{V,2})$ is an unknown continuous differentiable nonlinear function, u_V is the input of (64), and y_V is the output of (64); $\tau_V \in \mathbb{R}^+$ is the design parameter.

Then, define state variables ε_1 and ε_2 . Where $\varepsilon_1 = y_V$, $\varepsilon_2 = \dot{\varepsilon}_1 = \dot{\psi}_{V,1} = \Gamma(\psi_{V,1}, \psi_{V,2})$, we can get

$$\begin{aligned}\dot{\varepsilon}_2 &= \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,1}} \dot{\psi}_{V,1} + \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,2}} \dot{\psi}_{V,2} \\ &= \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,1}} \Gamma(\psi_{V,1}, \psi_{V,2}) \\ &\quad + \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,2}} (u_V - \tau_V \psi_{V,2}) \\ &= \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,1}} \Gamma(\psi_{V,1}, \psi_{V,2}) + \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,2}} u_V \\ &\quad - \tau_V \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,2}} \psi_{V,2} \triangleq Y_V(\psi_{V,1}, \psi_{V,2}) \\ &= \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,1}} \Gamma(\psi_{V,1}, \psi_{V,2}) + K_V u_V,\end{aligned}\quad (65)$$

Where nonlinear continuous functions $Y_V(\psi_{V,1}, \psi_{V,2})$ and K_V are expressed as

$$\begin{aligned}Y_V(\psi_{V,1}, \psi_{V,2}) &= \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,1}} \Gamma(\psi_{V,1}, \psi_{V,2}) \tau_V \\ &\quad + \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,2}} \psi_{V,2}, \\ K_V &= \frac{\partial \Gamma(\psi_{V,1}, \psi_{V,2})}{\partial \psi_{V,2}} = \frac{\partial [T \cos \alpha - D/m - g \sin \gamma]}{\partial \Phi} \\ &= \frac{\cos \alpha}{m} [\beta_1(h, \bar{q}) \alpha^3 + \beta_3(h, \bar{q}) \alpha^2 \\ &\quad + \beta_5(h, \bar{q}) \alpha + \beta_7(h, \bar{q})].\end{aligned}\quad (66)$$

Remark 8. Noting that $\alpha \in [-8.7\text{rad}, 8.7\text{rad}] \times 10^{-2}$, $\alpha \in [-8.7\text{rad}, 8.7\text{rad}] \times 10^{-2}$, so we can get the definitions of α^2 and α^3 as $\alpha^2 \in [0\text{rad}^2, 7.57\text{rad}^2] \times 10^{-3}$ and $\alpha^3 \in [-6.5850\text{rad}^3, 6.5850\text{rad}^3] \times 10^{-4}$, respectively. And we can calculate that $K_V > 0$.

According to the above analysis, we can express the non-affine model in the form of the affine model, as shown in equation (67).

$$\begin{aligned}\dot{\varepsilon}_1 &= \varepsilon_2, \\ \dot{\varepsilon}_2 &= Y_V(\psi_{V,1}, \psi_{V,2}) + K_V u_V, \\ y_V &= \psi_{V,1}.\end{aligned}\quad (67)$$

Next, we need to prove the boundedness of u_V .

Theorem 3. If the control input u_V is bounded, there must be a fixed value u_{VM} , such that

$$|u_V| \leq u_{VM}. \quad (68)$$

Proof. Construct the Lyapunov function

$$L_{\psi_{V,2}} = \frac{\psi_{V,2}^2}{2}. \quad (69)$$

Taking time derivative along (69)

$$\begin{aligned}\dot{L}_{\psi_{V,2}} &= \psi_{V,2} \dot{\psi}_{V,2} = \psi_{V,2} (u_V - \tau_V \psi_{V,2}) \\ &= -\tau_V \psi_{V,2}^2 + u_V \psi_{V,2} \leq -\tau_V \psi_{V,2}^2 + \frac{u_{VM}^2}{2} + \frac{\psi_{V,2}^2}{2} \\ &= -\left(\tau_V - \frac{1}{2}\right) \frac{\psi_{V,2}^2}{2} + \frac{u_{VM}^2}{2}.\end{aligned}\quad (70)$$

If $\tau_V > (1/2)$, $\psi_{V,2}$ will ultimately converge to the compact $\Omega_{\psi_{V,2}}$. Where $\Omega_{\psi_{V,2}} = \{\psi_{V,2} \mid |\psi_{V,2}| \leq \sqrt{(u_{VM}^2/2)/(\tau_V - (1/2))}\}$. As a result, the boundedness of u_h leads to the newly introduced system “ $\dot{\psi}_{V,2} = u_V - \tau_V \psi_{V,2}$ ” which is stable.

Define tracking error $e_{V,1}$ as

$$e_{V,1} = \varepsilon_1 - \varepsilon_{1d} = \varepsilon_1 - V_{\text{ref}}. \quad (71)$$

Taking time derivative along (71) and employing (67), it leads to

$$\dot{e}_{V,1} = \dot{\varepsilon}_1 - \dot{\varepsilon}_{1d} = \varepsilon_2 - \dot{\varepsilon}_{1d}. \quad (72)$$

Define intermediate variable ε_{2d} as

$$\varepsilon_{2d} = -\lambda_{V,1} e_{V,1} + \dot{\varepsilon}_{1d}, \quad (73)$$

where $\lambda_{V,1} \in \mathbb{R}^+$ is the design parameter.

Define tracking error $e_{V,2}$ as

$$e_{V,2} = \varepsilon_2 - \varepsilon_{2d}. \quad (74)$$

Utilizing (67) and (73), the time derivative of $e_{V,2}$ is derived as

$$\begin{aligned} \dot{e}_{V,2} &= \dot{\varepsilon}_2 - \dot{\varepsilon}_{2d} = Y_V(\psi_{V,1}, \psi_{V,2}) + K_V u_V - \dot{\varepsilon}_{2d} \\ &= K_V \left[\frac{Y_V(\psi_{V,1}, \psi_{V,2}) - \dot{\varepsilon}_{2d}}{K_V} + u_V \right] \\ &\triangleq K_V F_V(\Psi_V), \end{aligned} \quad (75)$$

where $\Psi_V = [\psi_{V,1}, \psi_{V,2}]^T$. Since the function F_V is an unknown function, the fuzzy function approximator is used to approximate it. The estimated value $\hat{F}_V$ of the nonlinear function F_V can be expressed as

$$\hat{F}_V(\Psi_V | \theta_V) = \theta_V^{*T} \xi_V(\Psi_V). \quad (76)$$

$\Psi_V = [\psi_{V,1}, \psi_{V,2}]^T \in \mathbf{R}^+$ is the input vector of the unknown function; $\theta_V^* = [\theta_{V,1}^*, \theta_{V,2}^*, \dots, \theta_{V,p2}^*]^T \in \mathbf{R}^{p2}$ is the parameter vector of ideal weight coefficient, fuzzy basis function $\xi_V(\Psi_V) = [\xi_{V,1}(\Psi_V), \xi_{V,2}(\Psi_V), \dots, \xi_{V,p2}(\Psi_V)]^T \in \mathbf{R}^{p2}$. The unknown function F_V can be expressed as

$$F_V(\Psi_V) = \theta_V^{*T} \xi_V(\Psi_V) + \mu_V, \quad (77)$$

where μ_V stands for estimation error, $|\mu_V| \leq \mu_{VM}$; μ_{VM} represents the upper bound of estimation error.

The velocity control law is designed as

$$u_V = -\lambda_{V,2} e_{V,2} - e_{V,1} - \frac{\hat{\varphi}_2}{2} \xi_V^T(\Psi_V) \xi_V(\Psi_V), \quad (78)$$

where $\hat{\varphi}_2 = \|\theta_V^*\|^2$, and its adaptive law is chosen to be

$$\dot{\hat{\varphi}}_2 = \frac{l_2}{2} e_{V,1} \xi_V^T(\Psi_V) \xi_V(\Psi_V) - 2\hat{\varphi}_2, \quad (79)$$

where $l_2 \in \mathbf{R}^+$ is the design parameter.

Substituting equations (67) and (73) into equation (74), we can obtain

$$\begin{aligned} e_{V,2} &= \varepsilon_2 - \varepsilon_{2d} = \dot{\varepsilon}_1 - \varepsilon_{2d} = \dot{\varepsilon}_1 - (-\lambda_{V,1} e_{V,1} + \dot{\varepsilon}_{1d}) \\ &= \dot{\varepsilon}_1 - \dot{\varepsilon}_{1d} + \lambda_{V,1} e_{V,1} = e_{V,1} + \lambda_{V,1} e_{V,1}. \end{aligned} \quad (80)$$

Substitute equation (80) into equation (78) to get

$$\begin{aligned} u_V &= -\lambda_{V,2} (e_{V,1} + \lambda_{V,1} e_{V,1}) - e_{V,1} - \frac{\hat{\varphi}_2}{2} \xi_V^T(\Psi_V) \xi_V(\Psi_V) \\ &= -\lambda_{V,2} \dot{e}_{V,1} - (\lambda_{V,1} \lambda_{V,2}) e_{V,1} - \frac{\hat{\varphi}_2}{2} \xi_V^T(\Psi_V) \xi_V(\Psi_V). \end{aligned} \quad (81)$$

Theorem 4. Considering the closed-loop subsystem consisting of plant (67), with controller (81) and adaptive law (79). Then, all the signals involved are semiglobally uniformly ultimately bounded.

Proof. Consider the following Lyapunov function candidate:

$$W_V = \frac{e_{V,1}^2}{2} + \frac{e_{V,2}^2}{2K_V} + \frac{\tilde{\varphi}_2^2}{2l_2} \quad (82)$$

with $\tilde{\varphi}_2 = \hat{\varphi}_2 - \varphi_2$.

Applying (73), (74) into (72), $\dot{W}_V$ is expressed as

$$\begin{aligned} \dot{W}_V &= e_{V,1} \dot{e}_{V,1} + \frac{e_{V,2} \dot{e}_{V,2}}{K_V} + \frac{\tilde{\varphi}_2 \dot{\tilde{\varphi}}_2}{l_2} \\ &= e_{V,1} (e_{V,2} - \lambda_{V,1} e_{V,1}) + \frac{e_{V,2} \dot{e}_{V,2}}{K_V} + \frac{\tilde{\varphi}_2 \dot{\tilde{\varphi}}_2}{l_2}. \end{aligned} \quad (83)$$

Considering (75), (77), and (79), $\dot{W}_V$ becomes

$$\begin{aligned} \dot{W}_V &= e_{V,1} (e_{V,2} - \lambda_{V,1} e_{V,1}) \\ &\quad + e_{V,2} \left[\frac{Y_V(\psi_{V,1}, \psi_{V,2}) - \dot{\varepsilon}_{2d}}{K_V} + u_V \right] + \frac{\tilde{\varphi}_2 \dot{\tilde{\varphi}}_2}{l_2} \\ &= e_{V,1} e_{V,2} - \lambda_{V,1} e_{V,1}^2 + e_{V,2} \left[\theta_V^{*T} \xi_V(\Psi_V) + \mu_V + u_V \right] \\ &\quad + \frac{1}{2} e_{V,1} \tilde{\varphi}_2 \xi_V^T(\Psi_V) \xi_V(\Psi_V) - \frac{2\tilde{\varphi}_2 \dot{\tilde{\varphi}}_2}{l_2}. \end{aligned} \quad (84)$$

To simplify equation (84) and substituting (78) into (84) yields, we can get

$$\begin{aligned} \dot{W}_V &= e_{V,1} e_{V,2} - \lambda_{V,1} e_{V,1}^2 + e_{V,2} \left[\theta_V^{*T} \xi_V(\Psi_V) + \mu_V \right. \\ &\quad \left. - \lambda_{V,2} e_{V,2} - e_{V,1} - \frac{\hat{\varphi}_2}{2} \xi_V^T(\Psi_V) \xi_V(\Psi_V) \right] \\ &\quad + \frac{1}{2} e_{V,1} \tilde{\varphi}_2 \xi_V^T(\Psi_V) \xi_V(\Psi_V) - \frac{2\tilde{\varphi}_2 \dot{\tilde{\varphi}}_2}{l_2} \\ &= -\lambda_{V,1} e_{V,1}^2 - \lambda_{V,2} e_{V,2}^2 + \frac{1}{2} e_{V,1} \tilde{\varphi}_2 \xi_V^T(\Psi_V) \xi_V(\Psi_V) \\ &\quad + e_{V,2} \theta_V^{*T} \xi_V(\Psi_V) + e_{V,2} \mu_V - e_{V,2} \frac{\tilde{\varphi}_2}{2} \xi_V^T(\Psi_V) \xi_V(\Psi_V) \\ &\quad - e_{V,2} \frac{\varphi_2}{2} \xi_V^T(\Psi_V) \xi_V(\Psi_V) - \frac{2\tilde{\varphi}_2 \dot{\tilde{\varphi}}_2}{l_2}. \end{aligned} \quad (85)$$

From the fact that

$$\begin{aligned} \tilde{\varphi}_2^2 + 2\tilde{\varphi}_2(\hat{\varphi}_2 - \tilde{\varphi}_2) + \varphi_2^2 \\ = \tilde{\varphi}_2^2 + 2\tilde{\varphi}_2\varphi_2 + \varphi_2^2 = (\tilde{\varphi}_2 + \varphi_2)^2 \geq 0, \end{aligned} \quad (86)$$

TABLE 1: Initial trim conditions.

Item	Value	Units
V	2500	m/s
h	27000	m
γ	0	deg
θ	1.5295	deg
Q	0	deg/s
η_1	0.2857	—
η_2	0.2857	—

we obtain

$$2\tilde{\varphi}_2\hat{\varphi}_2 \geq \tilde{\varphi}_2^2 - \varphi_2^2. \quad (87)$$

Thereby, equation (85) becomes

$$\begin{aligned} \dot{W}_V \leq & -\lambda_{V,1}e_{V,1}^2 - \lambda_{V,2}e_{V,2}^2 + \frac{1}{2}e_{V,1}\tilde{\varphi}_2\xi_V^T(\Psi_V)\xi_V(\Psi_V) \\ & + e_{V,2}\theta_V^{*T}\xi_V(\Psi_V) - e_{V,2}\frac{\tilde{\varphi}_2}{2}\xi_V^T(\Psi_V)\xi_V(\Psi_V) \\ & - e_{V,2}\frac{\varphi_2}{2}\xi_V^T(\Psi_V)\xi_V(\Psi_V) + e_{V,2}\mu_V - \frac{\tilde{\varphi}_2^2}{l_2} + \frac{\varphi_2^2}{l_2}. \end{aligned} \quad (88)$$

In this article, since $0 \leq \mu_{F_i}(x) \leq 1$, and applying (12), we know that $0 \leq \xi_j(\mathbf{X}) \leq 1$. There exists a constant $\bar{\xi}_V \in R^+$, such that $\|\xi_V(\Psi_V)\| \leq \bar{\xi}_V$. Hence, we get

$$\begin{aligned} \frac{1}{2}e_{V,1}\tilde{\varphi}_2\xi_V^T(\Psi_V)\xi_V(\Psi_V) & \leq \frac{1}{2}\bar{\xi}_V^2|e_{V,1}||\tilde{\varphi}_2| \leq \frac{2l_2}{2}\left(\frac{1}{2}\bar{\xi}_V^2|e_{V,1}|\right)^2 \\ & + \frac{\tilde{\varphi}_2^2}{2(2l_2)} = \frac{l_2\bar{\xi}_V^4}{4}e_{V,1}^2 + \frac{\tilde{\varphi}_2^2}{4l_2}, \\ e_{V,2}\theta_V^{*T}\xi_V(\Psi_V) & \leq |e_{V,2}|\sqrt{\varphi_2}\bar{\xi}_V \leq \sqrt{\varphi_2}\bar{\xi}_V\left(\frac{e_{V,2}^2}{2} + \frac{1}{2}\right) \\ & = \frac{\sqrt{\varphi_2}\bar{\xi}_V}{2}e_{V,2}^2 + \frac{\sqrt{\varphi_2}\bar{\xi}_V}{2}, \end{aligned}$$

$$\begin{aligned} e_{V,2}\mu_V & \leq \mu_{hM}|e_{V,2}| \leq \mu_{VM}\left(\frac{e_{V,2}^2}{2} + \frac{1}{2}\right) \\ & = \frac{\mu_{VM}e_{V,2}^2}{2} + \frac{\mu_{VM}}{2} - e_{V,2}\frac{\tilde{\varphi}_2}{2}\xi_V^T(\Psi_V)\xi_V(\Psi_V) \\ & \leq \frac{1}{2}\bar{\xi}_V^2|e_{V,2}||\tilde{\varphi}_2| \leq \frac{2l_2}{2}\left(\frac{1}{2}\bar{\xi}_V^2|e_{V,2}|\right)^2 + \frac{\tilde{\varphi}_2^2}{2(2l_2)} \\ & = \frac{l_2\bar{\xi}_V^4}{4}e_{V,2}^2 + \frac{\tilde{\varphi}_2^2}{4l_2} - e_{V,2}\frac{\varphi_2}{2}\xi_V^T(\Psi_V)\xi_V(\Psi_V) \\ & \leq \frac{1}{2}\bar{\xi}_V^2|e_{V,2}|\varphi_2 \leq \frac{1}{2}\varphi_2\bar{\xi}_V^2\left(\frac{e_{V,2}^2}{2} + \frac{1}{2}\right) \\ & = \frac{\varphi_2\bar{\xi}_V^2}{4}e_{V,2}^2 + \frac{\varphi_2\bar{\xi}_V^2}{4}. \end{aligned} \quad (89)$$

Then, (88) becomes

$$\begin{aligned} \dot{W}_V \leq & -\lambda_{V,1}e_{V,1}^2 - \lambda_{V,2}e_{V,2}^2 + \frac{l_2\bar{\xi}_V^4}{4}e_{V,1}^2 + \frac{\tilde{\varphi}_2^2}{4l_2} \\ & + \frac{\sqrt{\varphi_2}\bar{\xi}_V}{2}e_{V,2}^2 + \frac{\sqrt{\varphi_2}\bar{\xi}_V}{2} + \frac{\mu_{VM}e_{V,2}^2}{2} + \frac{\mu_{VM}}{2} \\ & + \frac{l_2\bar{\xi}_V^4}{4}e_{V,2}^2 + \frac{\tilde{\varphi}_2^2}{4l_2} + \frac{\varphi_2\bar{\xi}_V^2}{4}e_{V,2}^2 + \frac{\varphi_2\bar{\xi}_V^2}{4} \\ & - \frac{\tilde{\varphi}_2^2}{l_2} + \frac{\varphi_2^2}{l_2}, \\ \dot{W}_V \leq & -\left(\lambda_{V,1} - \frac{l_2\bar{\xi}_V^4}{4}\right)e_{V,1}^2 - \left(\lambda_{V,2} - \frac{\sqrt{\varphi_2}\bar{\xi}_V}{2} - \frac{\mu_{VM}}{2} \right. \\ & \left. - \frac{l_2\bar{\xi}_V^4}{4} - \frac{\varphi_2\bar{\xi}_V^2}{4}\right)e_{V,2}^2 + \frac{\sqrt{\varphi_2}\bar{\xi}_V}{2} + \frac{\mu_{VM}}{2} \\ & + \frac{\varphi_2\bar{\xi}_V^2}{4} + \frac{\varphi_2^2}{l_2} - \frac{\tilde{\varphi}_2^2}{2l_2}. \end{aligned} \quad (90)$$

Let $\lambda_{V,1} > (l_2\bar{\xi}_V^4/4)$, $\lambda_{V,2} > (\sqrt{\varphi_2}\bar{\xi}_V/2) + (\mu_{VM}/2) + (l_2\bar{\xi}_V^4/4) + (\varphi_2\bar{\xi}_V^2/4)$, $l_2 > 0$. And define the following compact sets:

$$\begin{aligned} \Omega_{e_{V,1}} & = \left\{ e_{V,1} \mid |e_{V,1}| \leq \sqrt{\left(\frac{\sqrt{\varphi_2}\bar{\xi}_V}{2} + \frac{\mu_{VM}}{2} + \frac{\varphi_2^2}{l_2} + \frac{\varphi_2\bar{\xi}_V^2}{4}\right) / \left(\lambda_{V,1} - \frac{l_2\bar{\xi}_V^4}{4}\right)} \right\}, \\ \Omega_{e_{V,2}} & = \left\{ e_{V,2} \mid |e_{V,2}| \leq \sqrt{\left(\frac{\sqrt{\varphi_2}\bar{\xi}_V}{2} + \frac{\mu_{VM}}{2} + \frac{\varphi_2^2}{l_2} + \frac{\varphi_2\bar{\xi}_V^2}{4}\right) / \left(\lambda_{V,2} - \frac{\sqrt{\varphi_2}\bar{\xi}_V}{2} - \frac{\mu_{VM}}{2} - \frac{l_2\bar{\xi}_V^4}{4} - \frac{\varphi_2\bar{\xi}_V^2}{4}\right)} \right\}, \\ \Omega_{\tilde{\varphi}_2} & = \left\{ \tilde{\varphi}_2 \mid |\tilde{\varphi}_2| \leq \sqrt{\left(\frac{\sqrt{\varphi_2}\bar{\xi}_V}{2} + \frac{\mu_{VM}}{2} + \frac{\varphi_2^2}{l_2} + \frac{\varphi_2\bar{\xi}_V^2}{4}\right) / \left(\frac{1}{2l_2}\right)} \right\}. \end{aligned} \quad (91)$$

TABLE 2: Membership functions of variables.

V	γ	θ	Q	Φ	δ_e
$\exp \left\{ -\left(\frac{V - i_V}{200} \right)^2 \right\}$	$\exp \left\{ -\left(\frac{\gamma - i_\gamma}{0.05} \right)^2 \right\}$	$\exp \left\{ -\left(\frac{\theta - i_\theta}{0.09} \right)^2 \right\}$	$\exp \left\{ -\left(\frac{Q - i_Q}{0.05} \right)^2 \right\}$	$\exp \left\{ -\left(\frac{\Phi - i_\Phi}{10} \right)^2 \right\}$	$\exp \left\{ -\left(\frac{\delta_e - i_{\delta_e}}{0.05} \right)^2 \right\}$

FIGURE 1: Altitude tracking performance.

FIGURE 2: Velocity tracking performance.

If $e_{V,1} \notin \Omega_{e_{V,1}}$ or $e_{V,2} \notin \Omega_{e_{V,2}}$ or $\tilde{\varphi}_2 \notin \Omega_{\tilde{\varphi}_2}$, $\dot{W}_V$ will be negative. Thus, $e_{V,1}$, $e_{V,2}$, and $\tilde{\varphi}_2$ are semiglobally uniformly ultimately bounded. This is the end of the proof.

3. Simulation Results

To verify the effectiveness of the new control strategy proposed in this article, the proposed controller is tested in the

simulation adopting the nonlinear models (1)-(7) implemented in MATLAB/SIMULINKs. The simulation step size is fixed, the step size is 0.01 s, and the simulation period is 100 s. The initial trim conditions of the FHV are shown in Table 1. The velocity step $\Delta V = 100$ m/s, altitude step $\Delta h = 200$. Both the velocity and altitude reference inputs are given by a second-order reference model with a damping ratio of 0.95 and a natural frequency of 0.03rad/s. All the aerodynamic coefficients and parameters are the same as [17, 23].

FIGURE 3: Responses of attitude angles.

FIGURE 4: Φ and δ_e . FIGURE 5: u_v and u_h .

FIGURE 6: Variation curves of flexible states.

FIGURE 7: Estimations of ϕ_1 and ϕ_2 .

FIGURE 8: Altitude tracking performance.

FIGURE 9: Velocity tracking performance.

FIGURE 10: Responses of attitude angles.

Design parameters are chosen as $k_{h,1} = 2$, $k_{h,2} = 0.8$, $\tau_h = 75$, $\lambda_{h,1} = 45$, $\lambda_{h,2} = 45$, $\lambda_{h,3} = 53$, $\lambda_{h,4} = 53$, $l_1 = 0.05$, $\tau_V = 15$, $\lambda_{V,1} = 10$, $\lambda_{V,2} = 2$, and $l_2 = 0.05$. In order to verify the robustness of the system, it is assumed that the aerodynamic coefficients of the FHV models have a perturbation of $\pm 40\%$.

$$C = \begin{cases} C_0, & 0 \leq t < 50 \text{ s}, \\ C_0[1 + 0.4 \sin(0.1\pi t)], & 50 \leq t < 100 \text{ s}, \end{cases} \quad (92)$$

where C is the value of uncertain coefficient; C_0 denotes the nominal value of C .

The fuzzy function approximators choose the Gauss basis functions as the membership functions, set the fuzzy set of each variable to 100. The input vectors are $\Psi_h = [\psi_{h,1}, \psi_{h,2}, \psi_{h,3}, \psi_{h,4}]^T$, $\Psi_V = [\psi_{V,1}, \psi_{V,2}]^T$. Where the fuzzy centers $i_V, i_\gamma, i_\theta, i_Q, i_\phi$ and i_{δ_e} of each fuzzy set are uniformly distributed on $[2500 \text{ m}, 2800 \text{ m}]$, $[-1.1^\circ, +1.1^\circ]$, $[0^\circ, 11.5^\circ]$, $[-5.7^\circ/\text{s}, +5.7^\circ/\text{s}]$, $[0, 2]$, and $[-23^\circ, +23^\circ]$, respectively. The membership functions of the above variables are shown

FIGURE 11: Φ and δ_e . FIGURE 12: u_V and u_h .

in Table 2. To show the superiority of the proposed method, we do the simulation study compared with recent research methods of others. The following two different cases are considered.

Case 1. The method proposed in this paper (controller 1) is compared with the novel back-stepping control method (controller 2) in [33] without adding the saturation auxiliary system.

The obtained simulation results are shown in Figures 1–7. As can be seen from Figures 1 and 2, both of the two methods can achieve the desired tracking effects, but the tracking accuracy of controller 1 is higher than that of con-

troller 2 in both altitude tracking effects and velocity tracking effects. According to Figure 3, the attitude angle changes of the two methods are roughly the same, but the change amplitude of controller 1 is less than that of controller 2. Figure 4 shows that the trend of Φ and δ_e are similar, the lines are relatively smooth, there is no high-frequency chattering phenomenon, and the amplitude of controller 1 is small. Figure 5 shows that u_V and u_h are bounded. Finally, variation curves of flexible states and the estimations of φ_1 and φ_2 are presented in Figures 6 and 7.

Case 2. Assuming that the actuators are subject to theoretical constraints: $\Phi \in [0.05, 1.2]$, $\delta_e \in [-18^\circ, 18^\circ]$. The closed-

FIGURE 13: Variation curves of flexible states.

FIGURE 14: Estimations of φ_1 and φ_2 .

loop system simulation of FHV is carried out by using the method presented in this paper (controller 1) and the method of adaptive neural control with nonaffine dynamics (controller 2) in [34].

When the actuators are constrained, the simulation results using the two methods are shown in Figures 8–14. From Figures 8 and 9, it can be seen that both two methods can achieve stable tracking of reference altitude and velocity, but the altitude tracking error and velocity tracking error in controller 2 are relatively large, the maximum altitude tracking error is more than 4 m, and the maximum velocity tracking error is close to 2 m/s. At the same time, it can be seen from Figure 10 that the attitude angle changes of controller 1 are obviously smoother than that of controller 2. In Figure 11, although controller 2 can maintain the robustness of the system, the actuators are saturated for a long time. When controller 1 is saturated, it can also rely on the auxil-

iary system to maintain the stability of the system. In Figure 12, u_v and u_h are bounded. Variation curves of flexible states and the estimations of φ_1 and φ_2 are presented in Figures 13 and 14. To sum up, in both cases, the superiority of the proposed method can be demonstrated.

4. Conclusions

- (1) In this paper, a fuzzy-approximation-based novel back-stepping control method is proposed. The method does not need virtual control laws, but only needs a final actual control law, and the control algorithm is more simplified
- (2) For each subsystem of FHV, only a fuzzy approximator is needed to approximate the total uncertainties in the subsystem. Meanwhile, the norm estimation

approach is introduced to directly estimate the norm of the weight vector in the fuzzy approximator, instead of the element itself, which reduces the computational complexity of the control algorithm and guarantees the real-time performance of the algorithm

- (3) The control method in this paper is based on nonaffine models and uses low-pass filters to deal with nonaffine dynamics, avoiding the loss of dynamic characteristics in affine models
- (4) The saturation problem of actuators in nonaffine models is discussed. By designing an auxiliary system, the original control problem with input constraints is transformed into a new control problem without input constraints
- (5) In two cases, the simulation results are compared with the recent methods to verify the advantages of the proposed method

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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