Research Article

Finite-Time Formation of Unmanned Aerial Vehicles with Switching Topologies and Disturbances: An Average Dwell Time Approach

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This paper investigates the finite-time formation problem of unmanned aerial vehicles (UAVs) with switching topologies and external disturbances. The formation problem is first transformed into the finite-time stability and bounded problems of a switched system, respectively. In particular, the finite-time unachievable topology is concerned for unreliable information exchanges. By employing the average dwell time (ADT) method, sufficient criteria are established to deal with the switching topologies such that the desired time-varying formation can be achieved in finite time. Then, the topology-dependent controller can be designed in terms of matrix technique. Finally, an illustrative example is given to verify the effectiveness of our proposed formation strategy.

1. Introduction

During the past decade, the unmanned aerial vehicles (UAVs) have grown from academic researches into practical applications capable of various missions in both civilian and military areas [1–3]. Recently, with the substantial progresses in swarm intelligence, cooperation issues of multiple UAVs and robots have been receiving increasing attention for distinguishing advantages. Based on information exchanges according to the communication topology, a group of UAVs can work collectively to meet the need for high robustness, reliability, flexibility, and efficiency [4–10]. In particular, the formation problem is one of the most hot research topics for UAVs, which means that the UAVs can keep a certain flying configuration while accomplishing the tasks. Note that the communication among the UAVs can play a very significant role in formation since the information exchanges are conducted via the communication networks according to

the communication topology [11–15]. However, it is worth mentioning that the communication topology may change at time due to unreliable communication networks or security reason, and this may lead to unpredictable mission failure or performance degradation. As a consequence, the UAVs with switching topologies are of theoretical importance and practical background. So far, there are different remarkable approaches reported in the literature and the references therein [16–18].

On another research front line, the switched systems have been receiving increasing attention due to their novel ability in describing practical systems with mode switchings [19–22]. In particular, as one of the most effective analysis and synthesis methods for switched systems, the average dwell time (ADT) has been widely utilized. In comparison to the dwell time method, the ADT can further reduce the conservatism for each subsystem [23–25]. Therefore, a natural question arises from the prior discussions: can we introduce

the merit of ADT of switched systems to deal with the switching topologies? The answer to this question is challenging, but the efforts can lead to encouraging results. Furthermore, in most applications, the stability is required to be achieved over a finite-time interval, which gives rises to the researches on finite-time stability, and burgeoning investigations on finite-time consensus problems of multiagent systems with switching topologies have been made [26–28]. However, it is worth mentioning that the unachievable switching topologies are not concerned in most existing results. It is noted that there is also a need for the finite-time formation of UAVs by taking into account its realistic background. Until now, to the best of the authors' knowledge, there still exists a considerable gap in solving the formation problem of UAVs with switching topologies based on the switched system theory, let alone those with unachievable topologies. The aim of this article is to make one of the first attempts to shorten such a gap.

Inspired by the ADT method in switched systems, in this paper, the finite-time formation of UAVs with switching topologies and disturbances is studied in terms of the ADT approach. Compared with the existing results, the main contributions of our paper can be summarized as follows. (1) The finite-time formation problem of UAVs with unachievable switching topologies is investigated for the first time. More precisely, the switching topologies are categorized into two types: the achievable topologies and the unachievable topologies. The standard of category relies on whether the finite-time formation can be reached for a fixed topology. For example, it can be verified that the directed topology without a directed spanning tree is unachievable topology. This can further extend the applications of switching topologies and is more practical for UAVs. It can be found that the established results with unachievable topologies are more practical in the applications. (2) The disturbances of UAVs are also considered for formation robustness with the finite-time boundedness problem. On the basis of ADT with the multiple Lyapunov function (MLF) method, sufficient conditions are established for ensuring the finite-time formation and the desired topology-dependent controller gains are obtained accordingly.

The remainder of our paper is given as follows: Section 2 introduces the finite-time formation control problem of UAVs with some essential preliminaries. In Section 3, the main analysis and synthesis procedures are provided with the ADT method. Section 4 presents the numerical example to demonstrate the effectiveness of our proposed formation strategies. Finally, the paper is concluded in Section 5.

Notation 1. $\mathbb{R}^n$ and $\mathbb{R}^m \times n$ denote the n dimensional Euclidean space and the space of $m \times n$ real matrices, respectively. $A - B > 0$ ($A - B < 0$) denotes that $A - B$ is positive definite (negative definite). $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of matrix A , respectively. $\mathcal{L}_2[0, \infty)$ denotes the space of square-integrable vector functions over $[0, \infty)$. $A \otimes B$ stands for the Kronecker product. $*$ represents the ellipsis symmetry terms in symmetric block matrices. All matrices are compatible for algebraic operations.

2. Problem Formulation and Preliminaries

2.1. The UAV Dynamics. Consider a group of UAVs with the following kinematics and dynamics:

$$\begin{cases} \dot{x}_i = v_i \cos a_i \cos b_i, \\ \dot{y}_i = v_i \cos a_i \sin b_i, \\ \dot{z}_i = v_i \sin a_i, \end{cases} \quad (1)$$

$$\begin{cases} \dot{v}_i = \frac{T_i - D_i}{m_i} - g \sin a_i - d_{1i}, \\ \dot{a}_i = \frac{g}{v_i} (n_{gi} \cos \phi_i - \cos a_i) + d_{2i}, \\ \dot{b}_i = \frac{L_i \sin \phi_i}{m_i v_i \cos a_i} + d_{3i}, \end{cases} \quad (2)$$

where $[x_i, y_i, z_i]^T$ denotes the position of the i th UAV, v_i represents the ground speed, a_i stands for the flight-path angle, b_i is the heading angle, T_i denotes the engine thrust, D_i represents the drag, m_i denotes the mass, g stands for the acceleration due to gravity, L_i is the lift force, $n_{gi} = L_i/m_i g$ denotes the g -load value, ϕ_i represents the banking angle, and $d_{ki}, k = 1, 2, 3$ stands for the vortex disturbances.

By applying the feedback linearization method as

$$\begin{cases} \ddot{x}_i = u_{xi} + w_{xi}, \\ \ddot{y}_i = u_{yi} + w_{yi}, \\ \ddot{z}_i = u_{zi} + w_{zi}, \end{cases} \quad (3)$$

where $[u_{xi}, u_{yi}, u_{zi}]^T$ denotes the virtual acceleration control input and $[w_{xi}, w_{yi}, w_{zi}]^T$ represents the disturbances, the relationships between virtual control variables and the actual control variables are given by

$$\begin{cases} T_i = m_i [(u_{zi} + g) \sin a_i + u_{xyi} \cos a_i] + D_i, \\ n_{gi} = \frac{(u_{zi} + g) \cos a_i - u_{xyi} \sin a_i}{g \cos \phi_i}, \\ \phi_i = \arctan \left[\frac{u_{yi} \cos b_i - u_{xi} \sin b_i}{(u_{zi} + g) \cos a_i - u_{xyi} \sin a_i} \right], \end{cases} \quad (4)$$

with $u_{xyi} = u_{xi} \cos b_i + u_{yi} \sin b_i$, $a_i = \arcsin(\dot{z}_i/v_i)$, and $b_i = \arctan(\dot{y}_i/\dot{x}_i)$.

Consequently, by denoting

$$\begin{cases} \xi_i(t) = [x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i]^T, \\ u_i(t) = [u_{xi}, u_{yi}, u_{zi}]^T, \\ w_i(t) = [w_{xi}, w_{yi}, w_{zi}]^T, \end{cases} \quad (5)$$

the UAV dynamics can be obtained as follows:

$$\dot{\xi}_i(t) = A\xi_i(t) + Bu_i(t) + Bw_i(t), \quad (6)$$

where

$$A = \begin{bmatrix} 0 & I_3 \\ 0 & 0 \end{bmatrix}, \quad (7)$$

$$B = \begin{bmatrix} 0 \\ I_3 \end{bmatrix}.$$

2.2. Graph Theory. The directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ is adopted to describe the communication topology of the UAVs. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ represents the weighted adjacency matrix with

$$\begin{cases} a_{ij} > 0 (v_i, v_j) \in \mathcal{E}, \\ a_{ij} = 0, & \text{otherwise.} \end{cases} \quad (8)$$

$\mathcal{V}(\mathcal{G}) = \{v_1, \dots, v_N\}$ and $\mathcal{E}$ denote the sets of nodes and edges, respectively. The $L = (l_{ij})_{N \times N}$ denotes the Laplacian matrix with

$$\begin{cases} l_{ij} = -a_{ij}, & i \neq j, \\ l_{ii} = \sum_{j=1, j \neq i}^N a_{ij}. \end{cases} \quad (9)$$

If $\mathcal{G}$ has a directed spanning tree, then 0 is the eigenvalue of L . More details of the algebraic graph theory can be found in [29].

The switching topologies are studied in this paper. Let $\{\mathcal{G}_{\sigma(t)} : \sigma(t) \in \mathcal{S} = 1, 2, \dots, n\}$ denote all possible switching graphs, where $\sigma(t)$ represents the index set for all switching graphs. The switching signal $\sigma(t) : [0, \infty) \rightarrow p$, and its value is the index of the graph at time t . The switching sequence is $t_1, t_2, \dots$, where $[t_h, t_{h+1})$, $h = 1, 2, \dots$, is uniformly bounded nonoverlapping [30].

Moreover, for any possible $L_{\sigma(t)} \in \mathbb{R}^{N \times N}$ of graph $\mathcal{G}_{\sigma(t)}$ and a full row rank matrix $E \in \mathbb{R}^{(N-1) \times N}$ defined as

$$E = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \quad (10)$$

there exists a matrix $M_{\sigma(t)} \in \mathbb{R}^{N \times (N-1)}$ such that $L_{\sigma(t)} = M_{\sigma(t)}E$.

Remark 1. It is worth mentioning that when $\mathcal{G}_{\sigma(t)}$ is achievable ($L_{\sigma(t)}$ has a directed spanning tree), $M_{\sigma(t)}$ is of full column rank and the eigenvalues of $EM_{\sigma(t)}$ are the same as

that of $L_{\sigma(t)}$. When $\mathcal{G}_{\sigma(t)}$ is unachievable, the above conditions of $M_{\sigma(t)}$ will not be satisfied.

Without loss of generality, suppose that there are r achievable topologies with $\sigma(t) = 1, 2, \dots, r$ which means that the formation can be achieved with the $L_{\sigma(t)}$ and $n - r$ unachievable topologies with $r + 1$ and $r + 2, \dots, n$, otherwise. All the possible communication topologies can be divided into the achievable set denoted as $\mathcal{AT}$ and the unachievable set denoted as $\mathcal{UAT}$.

2.3. Finite-Time Formation. For the time-varying formation problem, denote $h(t) = [h_1(t), h_2(t), \dots, h_N(t)]^T \in \mathbb{R}^{6N}$, where $h_i(t) = [h_{xi}(t), h_{yi}(t), h_{zi}(t), h_{vxi}(t), h_{vyi}(t), h_{vzi}(t)]^T$ is a piecewise continuously differentiable, and $e(t) = [e_1(t), e_2(t), \dots, e_N(t)]^T \in \mathbb{R}^{6N}$, where $e_i(t) = \xi_i(t) - h_i(t)$.

Consequently, the formation controller for each UAV can be designed as follows:

$$u_i(t) = K_{1\sigma(t)}(\xi_i(t) - h_i(t)) + K_{2\sigma(t)} \sum_{j=1}^N a_{ij\sigma(t)} ((\xi_i(t) - h_i(t)) - (\xi_j(t) - h_j(t))) + \mu_{ih}(t), \quad i = 1, 2, \dots, N, \quad (11)$$

where $\dot{h}_i(t) = Ah_i(t) + B\mu_{ih}(t)$ and $K_{1\sigma(t)}$ and $K_{2\sigma(t)}$ are topology-dependent controller gains to be determined, respectively.

As a result, the closed-loop dynamics of the UAVs can be obtained by

$$\begin{aligned} \dot{\xi}(t) &= (I \otimes A)\xi(t) + (I \otimes BK_{1\sigma(t)}) (\xi(t) - h(t)) \\ &\quad + (L_{\sigma(t)} \otimes BK_{2\sigma(t)}) \times (\xi(t) - h(t)) \\ &\quad + (I \otimes B)\mu_h(t) + (I \otimes B)w(t), \end{aligned} \quad (12)$$

where $\mu_h(t) = [\mu_{1h}^T(t), \mu_{2h}^T(t), \dots, \mu_{Nh}^T(t)]^T$.

In addition, note that $e(t) = \xi(t) - h(t)$; then, it follows that

$$\begin{aligned} \dot{e}(t) &= (I \otimes A)e(t) + (I \otimes BK_{1\sigma(t)}) e(t) \\ &\quad + (L_{\sigma(t)} \otimes BK_{2\sigma(t)}) e(t) + (I \otimes B)w(t). \end{aligned} \quad (13)$$

Premultiplying both sides of (13) by $(E \otimes I)$ yields that

$$\begin{aligned} \dot{\varepsilon}(t) &= (I \otimes A)\varepsilon(t) + (I \otimes BK_{1\sigma(t)}) \varepsilon(t) \\ &\quad + (EM_{\sigma(t)} \otimes BK_{2\sigma(t)}) \varepsilon(t) + (I \otimes B)w(t), \end{aligned} \quad (14)$$

where $\varepsilon(t) = (E \otimes I)e(t)$ denotes the disagreement of $e(t)$ and $w(t) = (E \otimes I)w(t)$, respectively.

To this end, the following definitions are given for later use.

Definition 1 (average dwell time, ADT) [31]. For any $T > t \geq 0$, denote $N_\sigma(T, t)$ as the number of switching over $[t, T]$. $\sigma(t)$ is said to have an average dwell time (ADT) $\tau_a > 0$, if there exist positive numbers N_0 such that

$$N_\sigma(T, t) \leq N_0 + \frac{T - t}{\tau_a}. \quad (15)$$

Definition 2 (finite-time formation, FTF) [32]. Given constants $c_1 > 0$, $c_2 > 0$, and $T > 0$, with $c_1 < c_2$, matrix $R > 0$, and a switching signal $\sigma(t) \in \mathcal{S}$, the finite-time formation is said to be achieved with $(c_1, c_2, T, R, \sigma(t))$, if it holds that

$$\varepsilon^T(0)R\varepsilon(0) \leq c_1 \Rightarrow \varepsilon^T(t)R\varepsilon(t) \leq c_2, \quad \forall t \in (0, T]. \quad (16)$$

Definition 3 (uniformly finite-time formation, UFTF) [32]. Given constants $c_1 > 0$, $c_2 > 0$, and $T > 0$, with $c_1 < c_2$ and matrix $R > 0$, the uniformly finite-time formation is said to be achieved with (c_1, c_2, T, R) , if it holds that

$$\varepsilon^T(0)R\varepsilon(0) \leq c_1 \Rightarrow \varepsilon^T(t)R\varepsilon(t) \leq c_2, \quad \forall t \in (0, T]. \quad (17)$$

Definition 4 (finite-time bounded formation, FTBF) [32]. Given constants $c_1 > 0$, $c_2 > 0$, $d_w \geq 0$, and $T > 0$, with $c_1 < c_2$, matrix $R > 0$, and a switching signal $\sigma(t) \in \mathcal{S}$, the finite-time bounded formation is said to be achieved with $(c_1, c_2, T, R, \sigma(t), d_w)$, if it holds that

$$\begin{aligned} \varepsilon^T(0)R\varepsilon(0) \leq c_1 \Rightarrow \varepsilon^T(t)R\varepsilon(t) \leq c_2, \quad \forall t \in (0, T], \\ \forall w(t): \int_0^T w^T(t)w(t)dt \leq d_w. \end{aligned} \quad (18)$$

Definition 5 (uniformly finite-time bounded formation, UFTBF) [32]. Given constants $c_1 > 0$, $c_2 > 0$, $d_w \geq 0$, and $T > 0$, with $c_1 < c_2$ and matrix $R > 0$, the uniformly finite-time bounded formation is said to be achieved with (c_1, c_2, T, R, d_w) , if for any switching signal $\sigma(t) \in \mathcal{S}$ it holds that

$$\begin{aligned} \varepsilon^T(0)R\varepsilon(0) \leq c_1 \Rightarrow \varepsilon^T(t)R\varepsilon(t) \leq c_2, \quad \forall t \in (0, T], \\ \forall w(t): \int_0^T w^T(t)w(t)dt \leq d_w. \end{aligned} \quad (19)$$

Remark 2. It should be pointed out that the concept of finite-time stability is different from Lyapunov asymptotic stability. A system can be finite-time stable but not Lyapunov asymptotically stable and vice versa.

3. Main Results

In this section, the FTF and the FTBF problems of UAVs are solved with details.

3.1. Finite-Time Formation without Disturbances

Theorem 1. Consider the group of UAVs (1) with switching topologies, for giving constants $\alpha_i \geq 0$, $\beta_i \geq 0$, $i \in \mathcal{S}$, $\mu > 1$, and the topology-dependent controller gains K_{1i} and K_{2i} , if there exist matrices $P_i > 0$, $\forall (i, j) \in \mathcal{S} \times \mathcal{S}$, $i \neq j$, such that

$$\prod_i -a_i(I \otimes P_i) < 0, \quad i \in \mathcal{AT}, \quad (20)$$

$$\prod_i \beta_i(I \otimes P_i) < 0, \quad i \in \mathcal{UAT}, \quad (21)$$

$$P_i \leq \mu P_j, \quad i \neq j, \quad (22)$$

where

$$\prod_i := \text{sym} \{ (I \otimes P_i A) + (I \otimes P_i B K_{1i}) + (E M_i \otimes P_i B K_{2i}) \}, \quad (23)$$

and the total dwell time of the unachievable topologies T^+ satisfies

$$T^+(0, t) < \frac{\ln c_2 \lambda_{\min}(P_i) \lambda_{\min}(R) - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(P_i) - N \ln \mu - aT}{\beta - a}. \quad (24)$$

Then, the FTF can be achieved with respect to $(c_1, c_2, T, R, \sigma(t))$.

Furthermore, when T^+ can be fixed, the switching signal has an ADT satisfying

$$\tau_a \geq \tau_a^* = \frac{T \ln \mu}{\ln c_2 \lambda_{\min}(P_i) \lambda_{\min}(R) - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(P_i) - aT - (\beta - a)T^+(0, t)}. \quad (25)$$

Proof 1. Choose the multiple Lyapunov-like functions as follows:

$$V_{\sigma(t)}(t) = \varepsilon^T(t) \left(I \otimes P_{\sigma(t)} \right) \varepsilon(t). \quad (26)$$

Then, the LMIs (20) and (21) mean that

$$\begin{aligned} \dot{V}_i(t) &< \alpha_i V_i(t), \quad i \in \mathcal{AT}, \\ \dot{V}_i(t) &< \beta_i V_i(t), \quad i \in \mathcal{UAT}. \end{aligned} \quad (27)$$

For $t \in [t_k, t_k + 1)$, it can be obtained by (27) that

$$\begin{aligned} V(t) &< e^{a_{\sigma(t_k)}(t-t_k)} V_{\sigma(t)}(t_k), \quad \sigma(t_k) \in \mathcal{AT}, \\ V(t) &< e^{\beta_{\sigma(t_k)}(t-t_k)} V_{\sigma(t)}(t_k), \quad \sigma(t_k) \in \mathcal{UAT}. \end{aligned} \quad (28)$$

Without loss of generality, denote $\sigma(t_k) = i$ and $\sigma(t_k^-) = j$ with $N_0 = 0$. Then, for any $i \neq j$, if (22) holds then

$$V_{\sigma(t_k)}(t_k) \leq \mu V_{\sigma(t_k^-)}(t_k^-). \quad (29)$$

Thus, for $t \in [0, T)$, letting $a = \max_{i \in \mathcal{S}}(a_i)$ and $\beta = \max_{i \in \mathcal{S}}(\beta_i)$ yields that

$$\begin{aligned} V(t) &< e^{aT^-(0,t) + \beta T^+(0,t)} V_{\sigma(t_0)}(0) \\ &= \mu^N e^{a(T-T^+(0,t)) + \beta T^+(0,t)} V_{\sigma(0)}(0), \end{aligned} \quad (30)$$

where N is the switching number of $\sigma(t)$ over $[0, T)$ and $T^+(0, t)$ is the total dwell time of $\mathcal{UAT}$.

On the other hand, it holds that

$$\begin{aligned} \lambda_{\min}(P_{\sigma(t)}) \varepsilon^T(t) \varepsilon(t) &\leq V_{\sigma(t)}(t) \\ &= \varepsilon^T(t) P_{\sigma(t)} \varepsilon(t) \leq \lambda_{\max}(P_{\sigma(t)}) \varepsilon^T(t) \varepsilon(t), \end{aligned} \quad (31)$$

which implies that when the FTF in Definition 2 can be achieved it can be verified that

$$T^+(0, t) < \frac{\ln c_2 \lambda_{\min}(\tilde{P}_i^{-1}) \lambda_{\min}(R) - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(\tilde{P}_i^{-1}) - N \ln \mu - aT}{\beta - a}. \quad (35)$$

Then, the FTF can be achieved with respect to $(c_1, c_2, T, R, \sigma(t))$.

$$\begin{aligned} \varepsilon^T(t) R \varepsilon(t) &\leq \lambda_{\max}(R) \varepsilon^T(t) \varepsilon(t) \\ &\leq \lambda_{\max}(R) \frac{V_{\sigma(t)}(t)}{\lambda_{\min}(P_{\sigma(t)})} \\ &< \frac{\lambda_{\max}(R)}{\lambda_{\min}(P_{\sigma(t)})} \mu^N e^{a(T-T^+(0,t)) + \beta T^+(0,t)} V_{\sigma(t_0)}(0) \\ &\leq \frac{\lambda_{\max}(R) \lambda_{\max}(P_{\sigma(t)})}{\lambda_{\min}(P_{\sigma(t)})} \mu^N e^{aT} e^{(\beta-a)T^+(0,t)} \varepsilon^T(0) \varepsilon(0) \\ &\leq c_1 \mu^N \frac{\lambda_{\max}(R) \lambda_{\max}(P_{\sigma(t)})}{\lambda_{\min}(P_{\sigma(t)}) \lambda_{\min}(R)} e^{aT} e^{(\beta-a)T^+(0,t)} < c_2. \end{aligned} \quad (32)$$

Moreover, the ADT τ_a can be obtained for a fixed $T^+(0, t)$ according to Definition 1, which completes the proof.

Based on the established conditions in Theorem 1, Theorem 2 is given to calculate the topology-dependent controller gains.

Theorem 2. Consider the group of UAVs (1) with switching topologies, for giving constants $\alpha_i \geq 0$, $\beta_i \geq 0$, $i \in \mathcal{S}$, and $\mu > 1$, if there exist matrices $P_i > 0$ and W_{2i} , $\forall (i, j) \in \mathcal{S} \times \mathcal{S}$, $i \neq j$, such that

$$\begin{aligned} \Xi_i - \alpha_i (I \otimes \tilde{P}_i) &< 0, \quad i \in \mathcal{AT}, \\ \Xi_i - \beta_i (I \otimes \tilde{P}_i) &< 0, \quad i \in \mathcal{UAT}, \\ \tilde{P}_i &\leq \mu \tilde{P}_j, \quad i \neq j, \end{aligned} \quad (33)$$

where

$$\Xi_i := \text{sym} \{ (I \otimes A \tilde{P}_i) + (I \otimes B K_{1i} \tilde{P}_i) + (E M_i \otimes B W_{2i}) \}, \quad (34)$$

and the total dwell time of the unachievable topologies T^+ satisfies

Furthermore, when T^+ can be fixed, the switching signal has an ADT satisfying

$$\tau_a \geq \tau_a^* = \frac{T \ln \mu}{\ln c_2 \lambda_{\min}(\tilde{P}_i^{-1}) \lambda_{\min}(R) - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(\tilde{P}_i^{-1}) - aT - (\beta - a)T^+(0, t)}. \quad (36)$$

In addition, when the topology-dependent controller gain K_{1i} is configured, K_{2i} can be obtained by

$$K_{2i} = W_{2i} \tilde{P}_i^{-1}. \quad (37)$$

Proof 2. By letting $\tilde{P}_i = P_i^{-1}$ and $W_{2i} = K_{2i} \tilde{P}_i$ and performing matrix congruent transformation, the conditions then follow directly from Theorem 1.

Remark 3. It is noted that K_{1i} is related with the motion modes of the formation center, such that K_{1i} is generally separately designed in advance of K_{2i} . When K_{1i} is configured appropriately, then the desired K_{2i} can be obtained by solving the conditions in Theorem 2 accordingly. For the large-scale UAVs, since the formation configuration $h(t)$ is prescribed, the controllers of UAVs can be designed according to $u_i(t)$,

where the information exchanges among the UAVs are also considered.

Note that if a common Lyapunov-like function can be found for (6), Corollary 1 can be derived.

Corollary 1. Consider the group of UAVs (1) with switching topologies, for giving constants $\alpha_i \geq 0$ and $\beta_i \geq 0$, $i \in \mathcal{S}$, if there exist matrices $P_i > 0$ and W_{2i} , $\forall (i, j) \in \mathcal{S} \times \mathcal{S}$, $i \neq j$, such that

$$\begin{aligned} \Xi_i - \alpha(I \otimes \tilde{P}_i) &< 0, \quad i \in \mathcal{AT}, \\ \Xi_i - \beta(I \otimes \tilde{P}_i), \quad i &\in \mathcal{UAT}, \end{aligned} \quad (38)$$

and the total dwell time of the unachievable topologies T^+ satisfies

$$T^+(0, t) < \frac{\ln c_2 \lambda_{\min}(\tilde{P}_i^{-1}) \lambda_{\min}(R) - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(\tilde{P}_i^{-1}) - aT}{\beta - a}. \quad (39)$$

Then, the UFTF can be achieved with respect to (c_1, c_2, T, R) .

In addition, when the topology-dependent controller gain K_{1i} is configured, K_{2i} can be obtained by

$$K_{2i} = W_{2i} \tilde{P}_i^{-1}. \quad (40)$$

3.2. Finite-Time Formation with Disturbances. When the external disturbances of UAVs are considered, the theorems of FTBF and UFTBF are presented.

Theorem 3. Consider the group of UAVs (1) with switching topologies and disturbances, for giving constants $\alpha_i \geq 0$, $\beta_i \geq 0$, $i \in \mathcal{S}$, $\mu > 1$, $\kappa_{1i} > 0$, $\kappa_{2i} > 0$, and the topology-dependent controller gains K_{1i} and K_{2i} , if there exist matrices $P_i > 0$, $\forall (i, j) \in \mathcal{S} \times \mathcal{S}$, $i \neq j$, such that

$$\begin{bmatrix} \Psi_{1i} & (I \otimes P_i B) \\ * & -\kappa_{1i} I \end{bmatrix} < 0, \quad i \in \mathcal{AT}, \quad (41)$$

$$\begin{bmatrix} \Psi_{2i} & (I \otimes P_i B) \\ * & -\kappa_{2i} I \end{bmatrix} < 0, \quad i \in \mathcal{UAT}, \quad (42)$$

$$P_i \preceq \mu_i P_j, \quad (43)$$

where

$$\begin{aligned} \Psi_{1i} &:= \text{sym} \{ (I \otimes P_i A) + (I \otimes P_i B K_{1i}) + (E M_i \otimes P_i B K_{2i}) \} \\ &\quad - \alpha_i (I \otimes P_i), \\ \Psi_{2i} &:= \text{sym} \{ (I \otimes P_i A) + (I \otimes P_i B K_{1i}) + (E M_i \otimes P_i B K_{2i}) \} \\ &\quad - \beta_i (I \otimes P_i), \end{aligned} \quad (44)$$

and the total dwell time of the unachievable topologies T^+ satisfies

$$T^+(0, t) < \frac{\ln (c_2 \lambda_{\min}(P_i) \lambda_{\min}(R) - \mu^N \lambda_{\max}(R) \lambda_{\max}(P_i) \lambda_{\min}(R) \kappa e^{\beta T} d_w)}{\beta - a} - \frac{N \ln \mu - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(P_i) - aT}{\beta - a}. \quad (45)$$

Then, the FTBF can be achieved with respect to $(c_1, c_2, T, R, \sigma(t), d_w)$.

Furthermore, when T^+ can be fixed, the switching signal has an ADT satisfying

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln c_2 \lambda_{\min}(P_i) \lambda_{\min}(R) - \ln(\lambda_{\max}(R) \lambda_{\max}(P_i) (e^{a(T-T^+(0,t)) + \beta T^+(0,t)} c_1 + \lambda_{\min}(R) \kappa e^{\beta T} d_w))}. \quad (46)$$

Proof 3. Select the multiple Lyapunov-like function as follows:

$$V_i(t) = \varepsilon^T(t) (I \otimes P_i) \varepsilon(t). \quad (47)$$

Then, the LMIs (41) and (42) mean that

$$\begin{aligned} \dot{V}_i(t) &< \alpha_i V_i(t) + \kappa_{1i} \omega^T(t) \omega(t), \quad i \in \mathcal{AT}, \\ \dot{V}_i(t) &< \beta_i V_i(t) + \kappa_{2i} \omega^T(t) \omega(t), \quad i \in \mathcal{UAT}. \end{aligned} \quad (48)$$

Following same lines in the proof of Theorem 1, for $t \in [t_k, t_{k+1})$, it holds that

$$\begin{aligned} V(t) &< e^{a_{\sigma(t_k)}(t-t_k)} \mu V_{\sigma(t_k)}(t_k^-) \\ &\quad + \int_{t_k}^t e^{a_{\sigma(t_k)}(t-s)} \kappa_{1\sigma(t_k)} \omega^T(s) \omega(s) ds, \quad \sigma(t_k) \in \mathcal{AT}, \\ V(t) &< e^{\beta_{\sigma(t)}(t-t_k)} \mu V_{\sigma(t_k)}(t_k^-) \\ &\quad + \int_{t_k}^t e^{\beta_{\sigma(t)}(t-s)} \kappa_{2\sigma(t_k)} \omega^T(s) \omega(s) ds, \quad \sigma(t_k) \in \mathcal{UAT}. \end{aligned} \quad (49)$$

By letting $\pi_{\sigma(t)} = a_{\sigma(t)}$, $\kappa_{\sigma(t)} = \kappa_{1\sigma(t)}$, and $\sigma(t) \in \mathcal{AT}$ and $\pi_{\sigma(t)} = \beta_{\sigma(t)}$, $\kappa_{\sigma(t)} = \kappa_{2\sigma(t)}$, and $\sigma(t) \in \mathcal{UAT}$, one has

$$\begin{aligned} V(t) &\leq e^{\pi_{\sigma(t_k)}(t-t_k)} \mu V_{\sigma(t_k)}(t_k^-) + \int_{t_k}^t e^{\pi_{\sigma(t_k)}(t-s)} \kappa_{\sigma(t_k)} \omega^T(s) \omega(s) ds \\ &\leq e^{\pi_{\sigma(t_k)}(t-t_k)} \mu \left(e^{\pi_{\sigma(t_{k-1})}(t_k-t_{k-1})} \mu V_{\sigma(t_{k-1})}(t_{k-1}^-) \right. \end{aligned}$$

$$\begin{aligned} &\quad \left. + \int_{t_{k-1}}^t e^{\pi_{\sigma(t_{k-1})}(t_k-s)} \kappa_{\sigma(t_{k-1})} \omega^T(s) \omega(s) ds \right) \\ &\quad + \int_{t_k}^t e^{\pi_{\sigma(t_k)}(t-s)} \kappa_{\sigma(t_k)} \omega^T(s) \omega(s) ds \\ &\leq \dots \leq \mu^N e^{aT^-(0,t) + \beta T^+(0,t)} V_{\sigma(t_0)}(0) \\ &\quad + \int_0^t e^{\pi_{\sigma(t)}(t-s)} \mu^N \kappa_{\sigma(t)} \omega^T(s) \omega(s) ds \\ &\leq \mu^N \left(e^{a(T-T^+(0,t)) + \beta T^+(0,t)} V_{\sigma(t_0)}(0) + \kappa e^{\beta T} d_w \right), \end{aligned} \quad (50)$$

where $a = \max_{i \in \mathcal{S}}(a_i)$, $\beta = \max_{i \in \mathcal{S}}(\beta_i)$, and $\kappa = \max_{i \in \mathcal{S}}(\kappa_i)$. Consequently, it can be obtained that

$$\begin{aligned} \varepsilon^T(t) R \varepsilon(t) &\leq \lambda_{\max}(R) \varepsilon^T(t) \varepsilon(t) \leq \lambda_{\max}(R) \frac{V_{\sigma(t)}(t)}{\lambda_{\min}(P_{\sigma(t)})} \\ &< \frac{\lambda_{\max}(R)}{\lambda_{\min}(P_{\sigma(t)})} \mu^N \left(e^{a(T-T^+(0,t)) + \beta T^+(0,t)} V_{\sigma(t_0)}(0) \right. \\ &\quad \left. + \kappa e^{\beta T} d_w \right) \leq \frac{\lambda_{\max}(R) \lambda_{\max}(P_{\sigma(t)})}{\lambda_{\min}(P_{\sigma(t)})} \\ &\quad \cdot \mu^N \left(e^{a(T-T^+(0,t)) + \beta T^+(0,t)} \varepsilon^T(0) \varepsilon(0) + \kappa e^{\beta T} d_w \right) \\ &\leq \frac{\lambda_{\max}(R) \lambda_{\max}(P_{\sigma(t)})}{\lambda_{\min}(P_{\sigma(t)}) \lambda_{\min}(R)} \mu^N \left(e^{a(T-T^+(0,t)) + \beta T^+(0,t)} c_1 \right. \\ &\quad \left. + \lambda_{\min}(R) \kappa e^{\beta T} d_w \right) < c_2, \end{aligned} \quad (51)$$

which means that

$$T^+(0, t) < \frac{\ln(c_2 \lambda_{\min}(P_i) \lambda_{\min}(R) - \mu^N \lambda_{\max}(R) \lambda_{\max}(P_i) \lambda_{\min}(R) \kappa e^{\beta T} d_w)}{\beta - a} - \frac{N \ln \mu - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(P_i) - aT}{\beta - a}. \quad (52)$$

In addition, the ADT τ_a can be obtained for a fixed $T^+(0, t)$ according to Definition 1, which completes the proof.

Theorem 4. Consider the group of UAVs (1) with switching topologies and disturbances, for giving constants $\alpha_i \geq 0$, $\beta_i \geq 0$, $i \in \mathcal{S}$, $\mu > 1$, $\kappa_{1i} > 0$, and $\kappa_{2i} > 0$, if there exist matrices $\tilde{P}_i > 0$ and W_{2i} , $\forall (i, j) \in \mathcal{S} \times \mathcal{S}$, $i \neq j$, such that

$$\begin{aligned} \begin{bmatrix} \Phi_{1i} & (I \otimes B) \\ * & -\kappa_{1i} I \end{bmatrix} &< 0, \quad i \in \mathcal{AT}, \\ \begin{bmatrix} \Phi_{2i} & (I \otimes B) \\ * & -\kappa_{2i} I \end{bmatrix} &< 0, \quad i \in \mathcal{UAT}, \end{aligned} \quad (53)$$

$$\tilde{P}_i \leq \mu_i \tilde{P}_j,$$

Input: $A, B, L_{\sigma(t)}$
Output: $K_{2\sigma(t)}$
 1: Given constants $\alpha_{\sigma(t)} > 0, \beta_{\sigma(t)} > 0, \mu > 1, \sigma(t) \in \mathcal{S}, \kappa_{1i} > 0, \kappa_{2i} > 0$;
 2: Configure the formation centre by $A + BK_{1\sigma(t)}$ with desired eigenvalues;
 3: Obtaining $M_{\sigma(t)}$ by $L\sigma(t) = M\sigma(t)E$;
 4: Solving the LMIs in Theorem 4;
 5: Obtaining feasible solutions of $\tilde{P}_{\sigma(t)} > 0$ and $W_{2\sigma(t)}$;
 6: Obtaining the total dwell time of the unachievable topologies T^+ ;
 7: **return** $K_{2\sigma(t)} = W_{2\sigma(t)}\tilde{P}_{\sigma(t)}^{-1}$.

ALGORITHM 1: FTBF algorithm.

where

$$\begin{aligned} \Phi_{2i} &:= \text{sym} \left\{ (I \otimes A\tilde{P}_i) + (I \otimes BK_{1i}\tilde{P}_i) + (EM_i \otimes BW_{2i}) \right\} \\ &\quad - \beta_i (I \otimes A\tilde{P}_i), \\ \Phi_{1i} &:= \text{sym} \left\{ (I \otimes A\tilde{P}_i) + (I \otimes BK_{1i}\tilde{P}_i) + (EM_i \otimes BW_{2i}) \right\} \\ &\quad - \alpha_i (I \otimes A\tilde{P}_i), \end{aligned} \quad (54)$$

and the total dwell time of the unachievable topologies T^+ satisfies

$$T^+(0, t) < \frac{\ln \left(c_2 \lambda_{\min}(\tilde{P}_i^{-1}) \lambda_{\min}(R) - \mu^N \lambda_{\max}(R) \lambda_{\max}(\tilde{P}_i^{-1}) \lambda_{\min}(R) \kappa e^{\beta T} d_w \right)}{\beta - a} - \frac{N \ln \mu - \ln c_1 \lambda_{\max}(R) \lambda_{\max}(\tilde{P}_i^{-1}) - aT}{\beta - a}. \quad (55)$$

Then, the FTBF can be achieved with respect to $(c_1, c_2, T, R, \sigma(t), d_w)$.

Furthermore, when T^+ can be fixed, the switching signal has an ADT satisfying

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln c_2 \lambda_{\min}(\tilde{P}_i^{-1}) \lambda_{\min}(R) - \ln \left(\lambda_{\max}(R) \lambda_{\max}(\tilde{P}_i^{-1}) (e^{a(T-T^+(0,t)) + \beta T^+(0,t)} c_1 + \lambda_{\min}(R) \kappa e^{\beta T} d_w) \right)}. \quad (56)$$

In addition, when the topology-dependent controller gain K_{1i} is configured, K_{2i} can be obtained by

$$K_{2i} = W_{2i} \tilde{P}_i^{-1}. \quad (57)$$

Proof 4. By applying the same matrix congruent transformation in Theorem 2, the results can be directly obtained.

Based on the above results, Algorithm 1 can be given as follows.

Corollary 2. Consider the group of UAVs (1) with switching topologies and disturbances, for giving constants $\alpha_i \geq 0$ and

$\beta_i \geq 0, i \in \mathcal{S}$, if there exist matrices $\tilde{P}_i > 0$ and $W_{2i}, \forall (i, j) \in \mathcal{S} \times \mathcal{S}, i \neq j$, such that

$$\begin{aligned} \begin{bmatrix} \Phi_{1i} & (I \otimes B) \\ * & -\kappa_{1i} I \end{bmatrix} &< 0, \quad i \in \mathcal{AT}, \\ \begin{bmatrix} \Phi_{2i} & (I \otimes B) \\ * & -\kappa_{2i} I \end{bmatrix} &< 0, \quad i \in \mathcal{UAT}, \end{aligned} \quad (58)$$

and the total dwell time of the unachievable topologies T^+ satisfies

$$T^+(0, t) < \frac{\ln \left(c_2 \lambda_{\min}(\tilde{P}_i^{-1}) \lambda_{\min}(R) - \lambda_{\max}(R) \lambda_{\max}(\tilde{P}_i^{-1}) \lambda_{\min}(R) \kappa e^{\beta T} d_w \right)}{\beta - a} - \frac{\ln c_1 \lambda_{\max}(\tilde{P}_i^{-1}) - aT}{\beta - a}. \quad (59)$$

FIGURE 1: The switching communication topologies of UAVs.

Then, the UFTBF can be achieved with respect to (c_1, c_2, T, R, d_w) .

In addition, when the topology-dependent controller gain K_{1i} is configured, K_{2i} can be obtained by

$$K_{2i} = W_{2i} \tilde{P}_i^{-1}. \quad (60)$$

4. Illustrative Example

In the following, a numerical example is given to verify the effectiveness of our proposed formation design.

Consider a group of four UAVs with the following switching communication topologies depicted in Figure 1.

The corresponding Laplacian matrices can be given as follows:

$$\begin{aligned} L_1 &= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ L_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \\ L_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \end{aligned} \quad (61)$$

It can be verified that the formation cannot be achieved since topology 1 and topology 2 do not have directed spanning trees, which implies that they are unachievable.

Accordingly, it can be obtained that

$$\begin{aligned} M_1 &= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ M_2 &= \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \\ M_3 &= \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \end{aligned} \quad (62)$$

The time-varying formation configuration is given as $h_i(t) = [3 \sin t, 0.6 \cos t, q_i \cos t, 3 \cos t, -0.6 \sin t, -\sin t]^T$, $q_1 = 1$, $q_2 = 0.8$, $q_3 = 0.6$, and $q_4 = 0.4$, and the external disturbances are set by $0.005 \sin t$.

The design parameters are chosen as $\alpha = 0.1$, $\beta = 0.5$, $\mu = 1.3$, $\kappa_{13} = 0.06$, $\kappa_{21} = 0.02$, and $\kappa_{22} = 0.03$. For the FTBF, the parameter c_1 is chosen as 0.5, the parameter c_2 is set as 50, T is given by 10 s, and d_w is supposed to be 0.0025.

The eigenvalues of $I \otimes (A + BK)$ are configured by $(-1 \pm i, -1 \pm i, \text{ and } -1 \pm i)$, one has

$$K_{1i} = \begin{bmatrix} -2 & 0 & 0 & -2 & 0 & 0 \\ 0 & -2 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & 0 & 0 & -2 \end{bmatrix}. \quad (63)$$

Based on the established results of Theorem 4, it can be obtained by solving the LMIs that

FIGURE 2: State trajectories of desired time-varying formation.

FIGURE 4: State trajectories of the closed-loop FTBF trajectory.

FIGURE 3: State trajectories of the open-loop FTBF trajectory.

FIGURE 5: State trajectories of formation errors.

$$\begin{aligned}
 K_{21} &= \begin{bmatrix} 0.3424 & 0 & 0 & 0.5493 & 0 & 0 \\ 0 & 0.3424 & 0 & 0 & 0.5493 & 0 \\ 0 & 0 & 0.3424 & 0 & 0 & 0.5493 \end{bmatrix}, \\
 K_{22} &= \begin{bmatrix} 0.4996 & 0 & 0 & 0.7677 & 0 & 0 \\ 0 & 0.4996 & 0 & 0 & 0.7677 & 0 \\ 0 & 0 & 0.4996 & 0 & 0 & 0.7677 \end{bmatrix}, \\
 K_{23} &= \begin{bmatrix} 0.4310 & 0 & 0 & 0.5133 & 0 & 0 \\ 0 & 0.4310 & 0 & 0 & 0.5133 & 0 \\ 0 & 0 & 0.4310 & 0 & 0 & 0.5133 \end{bmatrix},
 \end{aligned} \tag{64}$$

and the total dwell time of the unachievable topologies T^+ is calculated as 4.04 s.

In the simulation, the initial conditions of the UAVs are given as $[0.1; 0.2; 0.3; 0; 0; 0; -0.1; -0.2; 0.3; 0; 0; 0; 0.1; -0.3; 0.2; 0; 0; 0; 0.1; -0.2; 0.1; 0; 0; 0; 0]^T$ and T^+ is set as 2.5 s. Figures 2–4 show the desired time-varying formation trajectory, the open-loop FTBF trajectory, and the closed-loop FTBF trajectory, respectively. It can be seen that the designed formation controllers for each UAV can achieve

FIGURE 6: State trajectories of $\epsilon^T(t)Re(t)$.

the prescribed formation with unachievable topologies and external disturbances. Figure 5 depicts the formation errors of the UAVs with the finite-time interval, and Figure 6 indicates the corresponding FTBF conditions. It can be found that the formation errors are finite-time bounded with switching topologies according to Definition 4 with $\epsilon^T(0)Re(0) = 0.32$ and $\epsilon^T(10)Re(10) = 0.002$, which supports our theoretical analysis.

5. Conclusion

In this paper, the formation problem of UAVs with switching topologies and disturbance is concerned with an ADT approach. In particular, the concept of FTBF is introduced to deal with the finite-time stability problem and disturbances. Moreover, both achievable and unachievable topologies are taken into account for a more practical background of communications among the UAVs. Based on model transformation, sufficient conditions are established for the FTBF and the desired topology-dependent controller can be designed accordingly. In the end, a numerical example is given to demonstrate the effectiveness of our proposed control method. Our future study will focus on the cases with model uncertainties and communication delays.

Data Availability

All data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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