

## Research Article

# Determination of Bolus Dielectric Constant for Optimum Coupling of Microwaves through Skin for Breast Cancer Imaging

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We find the complex dielectric constant of the bolus liquid with the best microwave impedance match to skin-covered fatty breast tissue. The real dielectric constant and conductivity of the ideal medium are determined by minimizing the magnitude of the reflection coefficient from the two-interface frequency-dependent system, using published measured skin and breast fat dielectric characteristics in the 400 MHz to 10 GHz frequency range. Error bounds are provided to indicate the coupling degradation when using real, nonideal media for the bolus liquid. Two frequency regimes are identified. Below 6.45 GHz, conventional liquid mixtures can approach the optimal coupling permittivity, but the reflection magnitudes for these best cases are high. Above 6.45 GHz, perfect coupling is possible, but only for impractically high values of bolus dielectric constant and loss.

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## 1. INTRODUCTION

In cases where breast tumors occur in fatty healthy breast tissue, microwaves can be used to take advantage of the large dielectric contrast difference between the tumor scatterer and its surrounding background to detect the cancer. Several breast cancer detection systems have been proposed [1–9], each indicating great potential for discriminating the high-water-content tumor volume from the low-water-content fatty surrounding tissue.

One open question is the choice of coupling fluid, often referred to as bolus liquid, which allows the greatest transfer of waves from the antennas through the skin and into the breast tissue. Published work investigating empirically enhancing the coupling with suitable mixtures of glycerin and water [10, 11], indicates that proper choice makes a significant difference in sensing performance.

Although the skin layer is thin (on average, 1.6 mm [12]), it is a high-water-content biological tissue and thus has a significant effect on the coupling into breast tissue. The problem is additionally complicated since all biological tissue is frequency dependent. This paper uses a simple one dimensional impedance model to determine the best bolus complex

dielectric constant necessary to minimize reflection from skin-covered fatty breast tissue and hence transfer the maximum percentage of microwave sensing signal into the breast.

In practice, it may not be possible to obtain a real bolus liquid with the desired dielectric properties across the intended frequency range. However, having the information about the desired complex permittivity characteristics across large frequency ranges provides potential guidance in the selection of bolus liquid.

## 2. ONE DIMENSIONAL IMPEDANCE MODEL

Since the skin layer covering the internal breast tissue is very thin compared to all other length scales of the problem geometry, it is reasonable to model the wave interaction as a three-layer one-dimensional model. A further simplification considers only normally incident plane waves. While in reality sensing waves are almost never uniform and normally incident on tissue boundaries, this assumption selects the most important of the great number of possible illumination cases.

Following the standard method [13], assume that the bolus with complex permittivity  $\epsilon = \epsilon_b \epsilon_0$  fills space to the left

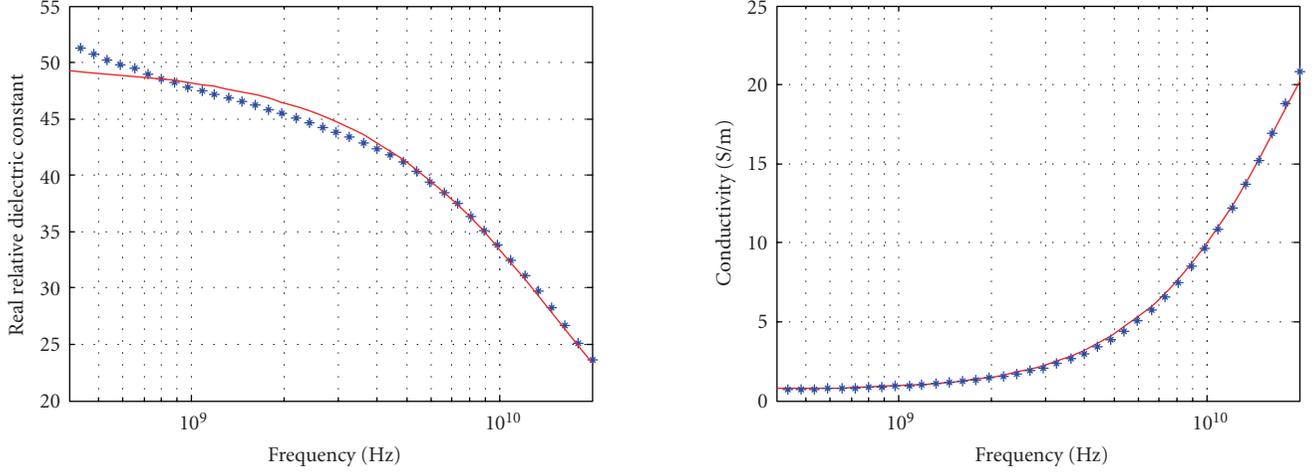


FIGURE 1: Measured (points) [14] and Cole-Cole model fitted (solid curve) dielectric values for wet skin in the microwave frequency range.

TABLE 1: Coefficients for Cole-Cole models of tissues for 400 MHz–20 GHz.

Tissue	$\epsilon_\infty$	$\Delta\epsilon_1$	$\tau_1$ (ps)	$\alpha_1$	$\sigma_s$ (S/m)
Breast fat	3.145	1.701	14.68	0.05826	0.03605
Wet skin	2.0	48	9.5	0.19	0.710

of the origin,  $z < 0$ , that fatty breast tissue with  $\epsilon = \epsilon_f \epsilon_0$  fills the space to the right of plane at  $z = \Delta z$ , and that skin with  $\epsilon = \epsilon_s \epsilon_0$  occupies the space of thickness  $\Delta z$  between the two. The incident and reflected, right-going and left-going, and transmitted waves in the three layers can be represented as

$$\begin{aligned} E_o e^{-jk_b z} + \Gamma E_o e^{+jk_b z}, \\ A E_o e^{-jk_s z} + B E_o e^{+jk_s z}, \\ T E_o e^{-jk_f z}, \end{aligned} \quad (1)$$

where the reflection and transmission coefficients are given by  $\Gamma$  and  $T$ ,  $A$  and  $B$  are unknown skin layer coefficients, and  $k_b = \omega \sqrt{\mu_0 \epsilon_b \epsilon_0} = k_0 \sqrt{\epsilon_b}$  is the complex wave number for the bolus layer, given in terms of the radian frequency and permeability and permittivity of free space and complex dielectric constant. Similar expressions hold for the wave numbers in the skin and fat layers. Continuity of transverse impedance at the two interfaces results in

$$\begin{aligned} \frac{1 + \Gamma}{1 - \Gamma} \frac{\omega \mu_0}{k_b} &= \frac{A + B}{A - B} \frac{\omega \mu_0}{k_s}, \\ \frac{A e^{-jk_s \Delta z} + B e^{+jk_s \Delta z}}{A e^{-jk_s \Delta z} - B e^{+jk_s \Delta z}} \frac{\omega \mu_0}{k_s} &= \frac{\omega \mu_0}{k_f}. \end{aligned} \quad (2)$$

Solving first for  $B$  in the latter equation, substituting it into the former, cancelling the common factor of  $A$ , and then solving for the three-layer reflection coefficient yield

$$\Gamma = \frac{e^{jk_s \Delta z} (k_b - k_s) (k_f + k_s) - e^{-jk_s \Delta z} (k_b + k_s) (k_f - k_s)}{e^{jk_s \Delta z} (k_b + k_s) (k_f + k_s) - e^{-jk_s \Delta z} (k_b - k_s) (k_f - k_s)}. \quad (3)$$

The matched case,  $\Gamma = 0$ , occurs when

$$k_b^{\text{opt}} = k_s \frac{(e^{jk_s \Delta z} - e^{-jk_s \Delta z}) k_s + (e^{jk_s \Delta z} + e^{-jk_s \Delta z}) k_f}{(e^{jk_s \Delta z} + e^{-jk_s \Delta z}) k_s + (e^{jk_s \Delta z} - e^{-jk_s \Delta z}) k_f}. \quad (4)$$

Equation (4) identifies the best bolus dielectric constant, as long as its imaginary part is negative. If it becomes positive, the coupling medium must have gain rather than conductive loss and hence is impractical.

The next step in the modeling process is to specify the wave numbers for the skin and breast fat as functions of frequency. Accurate measurements of these types of tissues have been conducted [14, 15]. The Cole-Cole dispersive medium relation, given by

$$\frac{\epsilon_{f,s}(\omega)}{\epsilon_0} = \epsilon_\infty + \frac{\Delta\epsilon_1}{1 + (j\omega\tau_1)^{-\alpha_1}} + \frac{\sigma_s}{j\omega\epsilon_0}, \quad (5)$$

accurately and compactly matches both the real dielectric constant and the conductivity of a frequency-dependent medium. For breast tissue with 85–100% fat in the 400 MHz to 20 GHz, frequency range [15] has determined coefficients for (5), while for wet skin, one possible model is plotted in Figure 1. Table 1 gives the coefficients for these two models.

### 3. OPTIMAL DIELECTRIC CONSTANT RESULTS

Inserting the Cole-Cole dielectric models for skin and breast fat into the formulas for the wave numbers and then into (4) and choosing  $\Delta z = 1.6$  mm [12] yield optimal values for  $k_b^{\text{opt}}$ , which can be squared and divided by  $\omega^2 \mu_0 \epsilon_0$  to give the complex dielectric constant:

$$\epsilon_b^{\text{opt}}(\omega) = \epsilon'_b(\omega) - j \frac{\sigma(\omega)}{\omega \epsilon_0}. \quad (6)$$

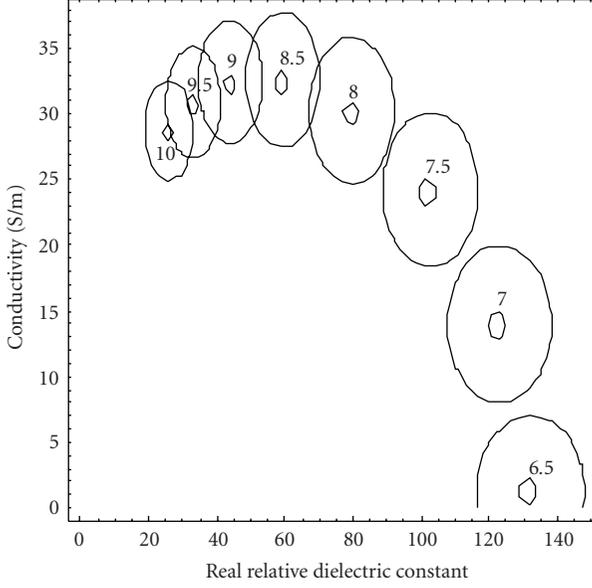


FIGURE 2: Exact complex dielectric values for bolus liquid with perfect match,  $|\Gamma| = 0$ , and contours of constant reflection magnitude  $|\Gamma| = 0.03$  for 6.5 to 10 GHz, in steps of 0.5 GHz.

The values for real relative dielectric constant  $\epsilon'_b$  and conductivity  $\sigma$ , as a function of frequency, are plotted in Figure 2. For 10, 9.5, 9, 8.5, 8, 7.5, 7, and 6.5 GHz, the points corresponding to the ordered pair  $(\epsilon'_b, \sigma)$  for zero reflection, as well as the elliptical contours corresponding to  $|\Gamma| = 0.03$  (arbitrarily chosen for graphical clarity) are indicated. The contours give an error bound showing the tolerance about the ideal complex values for liquids with close—but not exact—dielectric matching characteristics. Figure 2 also provides a sense of impedance variation of this three-layer geometry for wideband microwave systems. A bolus liquid which is perfect for one frequency may be considerably mismatched at another.

While the optimal complex permittivity values of Figure 2 provide perfect coupling, they are impractically high, both in real part and in conductivity. Even if such bolus fluids could be found, the propagation loss from antenna to skin would eliminate any coupling advantages. Extending the reflection magnitude contours to larger values and into the more physically realizable permittivity parameter space requires separate plots for each frequency. This set is shown in Figure 3, for reflection magnitudes of 0.1, 0.15, and 0.2, for 6.5 to 10 GHz. Also indicated in Figure 3 are the asymptotes that define the limits of reflection magnitude. For different frequencies, there are bolus fluids with  $\epsilon'_b$  and  $\sigma$  combinations that result in greater reflection, but for  $\epsilon'_b$  and  $\sigma$  combinations to the left and below a given asymptote, no frequency choice will couple with lower reflection than that asymptote value. Observing the plots of Figure 3 leads to simple rules for improving coupling to breast tissue.

- (1) For frequencies above 9.5 GHz, increase  $\sigma$  and reduce  $\epsilon'_b$ .
- (2) Between about 8 and 9.5 GHz, just increase  $\sigma$ .

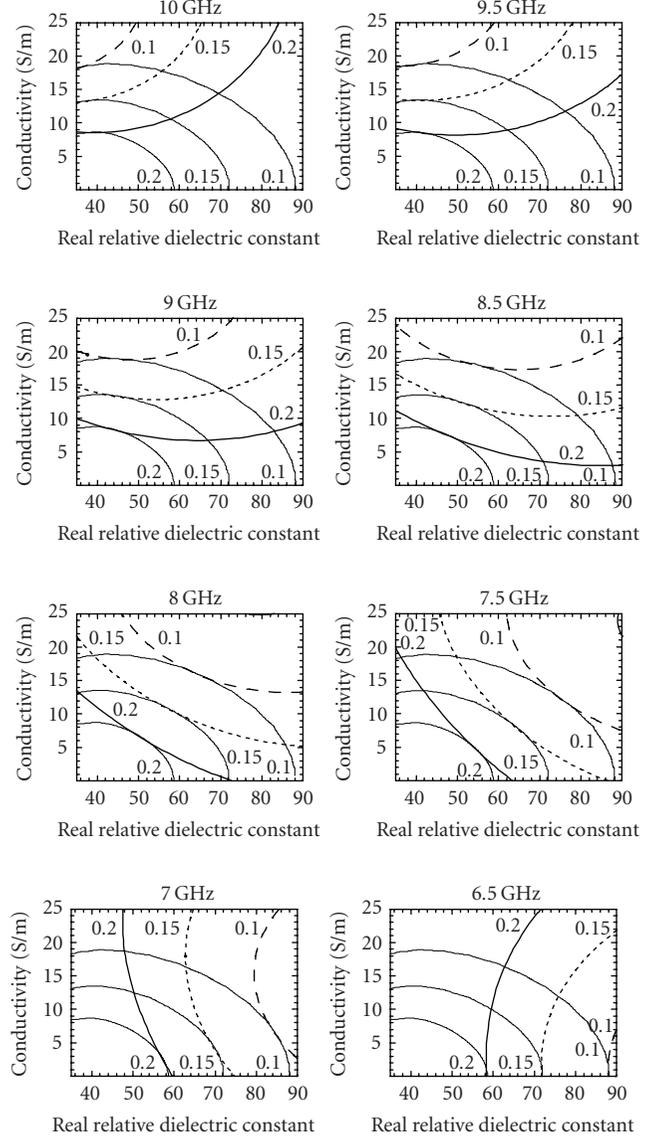


FIGURE 3: Contours of constant reflection magnitude  $|\Gamma| = 0.1$  (dashed curves), 0.15 (dotted curves), and 0.2 (solid curves) as indicated, for frequencies: 10.0, 9.5, 9.0, 8.5, 8.0, 7.5, 7.0, and 6.5 GHz, as well as the common asymptotes for  $|\Gamma| = 0.1, 0.15,$  and 0.2, for practical values of bolus complex permittivity. The curve contours are extended from the centers shown in Figure 2.

- (3) Between 7 and 8 GHz, increase  $\epsilon'_b$  and  $\sigma$ .
- (4) Between 6.5 and 7 GHz, increase  $\epsilon'_b$  and reduce  $\sigma$ .

Just below 6.5 GHz, the imaginary part of the optimal wave number (4) becomes positive, which makes the conductivity given by (6) negative. For these cases, the best bolus liquid is a pure dielectric with zero conductivity. Contours of constant reflection magnitude, given by (3) with  $k_b$  kept purely real, are plotted in Figure 4. The best dielectric constant for any particular frequency is found by picking the darkest contour (the lowest reflection magnitude level) crossed by the vertical line through that frequency and associating the dielectric constant at the crossing. The white

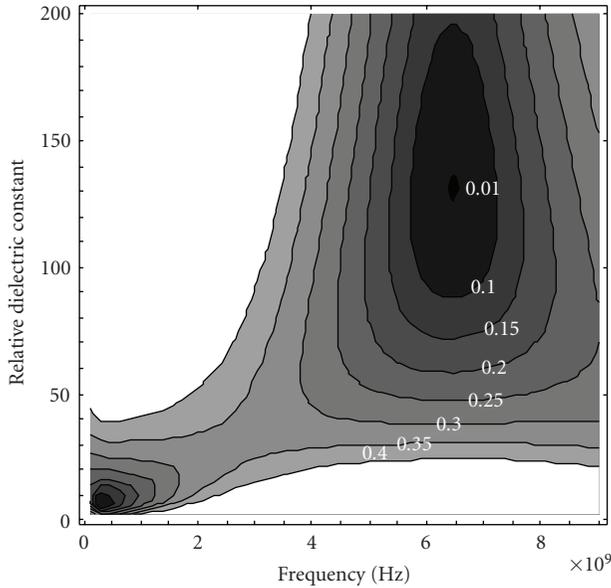


FIGURE 4: Contours of constant reflection magnitude for lossless bolus matching to skin-covered fatty breast tissue. A perfect match occurs at 6.45 GHz for a pure dielectric bolus with dielectric constant of 135. The worst coupling occurs at 2.67 GHz, with reflection coefficient of 0.34 for the best choice dielectric constant  $\epsilon' = 30.4$ .

regions of Figure 4 correspond to  $|\Gamma| > 0.4$ . Both the minimum reflection magnitudes and their variation with dielectric constant are much greater than those for frequencies above the critical value, found numerically to be 6.45 GHz.

For frequencies below 1 GHz or above 5 GHz, the optimal reflection magnitude is below 0.2 for bolus dielectric constants of about 10 and 100, respectively. However, in the diagnostically useful frequency range of 2–4.5 GHz, the reflection magnitude cannot be reduced below 0.3 for *any* choice of matching fluid.

For real liquids with nonzero conductivity, Figure 5 indicates the error bounds about the ideal dielectric choice. Each contour indicates the region of possible combinations of real dielectric constant and conductivity which would result in reflection magnitudes not more than 30% above the ideal value for that frequency. The horizontal maximum widths of the contours correspond to the vertical elongation of the contours of Figure 4. Note that while there is considerable tolerance for dielectric characteristics for 5.0 GHz, this is due to the reflection magnitude contour of Figure 4 being quite long in the vertical direction.

The lowest frequency range is examined in detail in Figure 6, which shows several common reflection magnitude contours for 400 MHz (solid curves), 900 MHz (dotted curves), and 1.3 GHz (dashed curves) reflection magnitude contours of 0.25, 0.4, and 0.55. Labeled points indicate particular bolus fluids: W1: water at 300 MHz, W2: water at 3 GHz, S1: 0.1 molal saline at 300 MHz, S2: 0.1 molal saline at 3 GHz, S3: 0.3 molal saline at 300 MHz [16], S4: 0.15 molal (0.9%) saline at 900 MHz, G1: 10 : 30 glycerin/water at 900 MHz, G2: 80 : 20 glycerin/water at 900 MHz, G3: 87 : 13 glycerin/water at 900 MHz, G4: 79 : 21 glycerin/water at 1.3 GHz, and G5: 87 : 13 glycerin/water at 1.3 GHz [11].

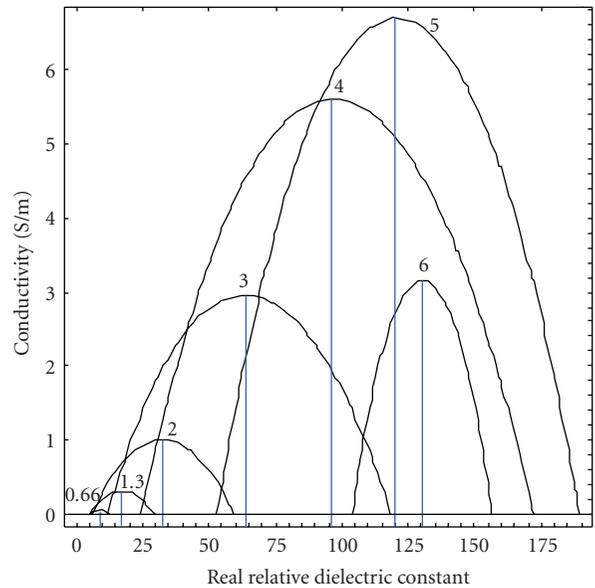


FIGURE 5: Contours of constant reflection magnitude at levels 30% above the best level determined for pure dielectric with no loss, for frequencies: 0.667, 1.33, 2.0, 3.0, 4.0, 5.0, and 6.0 GHz, as indicated on each curve on the figure. The 30% increased reflection contours have values: 0.17, 0.34, 0.42, 0.435, 0.37, 0.246, and 0.08, respectively. The optimal match dielectric constant for each frequency is indicated by the vertical line from the contour peak to the horizontal axis.

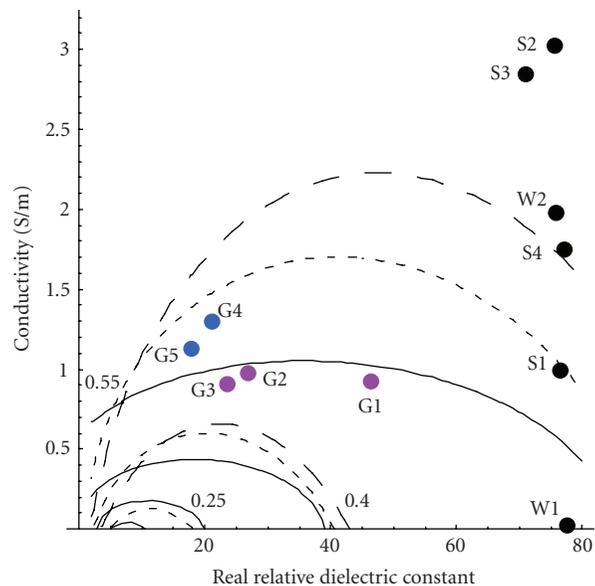


FIGURE 6: Low-frequency detail of Figure 5, for 400 MHz (solid curves), 900 MHz (dotted curves), and 1.3 GHz (dashed curves) reflection magnitude contours of 0.25, 0.4, and 0.55. Labeled points indicate particular bolus fluids: W1: water at 300 MHz, W2: water at 3 GHz, S1: 0.1 molal saline at 300 MHz, S2: 0.1 molal saline at 3 GHz, S3: 0.3 molal saline at 300 MHz [16], S4: 0.15 molal (0.9%) saline at 900 MHz, G1: 10 : 30 glycerin/water at 900 MHz, G2: 80 : 20 glycerin/water at 900 MHz, G3: 87 : 13 glycerin/water at 900 MHz, G4: 79 : 21 glycerin/water at 1.3 GHz, and G5: 87 : 13 glycerin/water at 1.3 GHz [11].

#### 4. CONCLUSION

This analysis has determined the best dielectric constant and conductivity characteristics for a bolus liquid to match to skin-covered fatty breast tissue for microwave in the 400 MHz to 10 GHz frequency range. The results are based on a simple 1D plane-wave model, using accurate frequency dependent measurements of dielectric characteristic of skin and breast fat.

Above 6.45 GHz, specific bolus dielectric values can match exactly to the skin/breast layers, although their dielectric constants and conductivities must be very high, in the 5–30 S/m range. About 10% variation in real dielectric constant and 15% in conductivity result in reflection magnitudes of only 0.03. Although the optimal values are impractical for typical liquids, they indicate the directions in complex permittivity space that bolus fluids should approach to increase coupling efficiency. The contours for larger reflection magnitudes of 0.1, 0.15, and 0.2 extend into permittivity values typical of water-mixture bolus fluids.

Below 6.45 GHz, the best bolus dielectric constants are purely real, and the lowest possible reflections magnitudes are as high as 0.34 for 2.67 GHz. For practical liquids with moderate conductivity, the reflection magnitude increases slowly, with only 30% reflection increase for liquids with 6.5 S/m at 5 GHz, or 0.17 S/m at 1.33 GHz. Thus, there is much greater tolerance for variation of real dielectric constant and conductivity from the optimal values for bolus fluids below 6.45 GHz, but the reflection magnitudes are considerably higher than for frequencies above 6.45 GHz.

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