Research Article

Wideband Flat Radomes Using Inhomogeneous Planar Layers

Mohammad Khalaj-Amirhosseini

College of Electrical Engineering, Iran University of Science and Technology, Narmak, Tehran 16846-13114, Iran

Correspondence should be addressed to Mohammad Khalaj-Amirhosseini, khalaja@iust.ac.ir

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Inhomogeneous planar layers (IPLs) are optimally designed as flat radomes in a desired frequency range. First, the electric permittivity function of the IPL is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The performance of the proposed structure is verified using some examples.

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1. INTRODUCTION

Radomes are sheltering structures to protect antennas against severe weather such as high winds, rain, icing, and/or temperature extremes [1]. In addition, radomes must not interfere with normal operation of the antennas. Therefore, the input reflection of radomes must be negligible at the usable frequency band of the protected antennas. Inhomogeneous planar layers (IPLs) are widely used in microwave and antenna engineering [2–4]. In this paper, we propose utilizing IPLs as flat radomes [5, 6] in a desired frequency range. To optimally design IPLs, the electric permittivity function of them is expanded in a truncated Fourier series, first. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The identical procedure has been used to optimally design IPLs as impedance matchers between two different mediums previously [7]. Finally, the usefulness of the proposed structure is verified using some examples.

2. ANALYSIS OF IPLs

In this section, the frequency domain equations of the IPLs are reviewed. Figure 1 shows a typical IPL with thickness d , whose left and right mediums are the free space and whose electric permittivity function is $\epsilon_r(z)$. One way to fabricate the IPLs is to place several thin homogeneous dielectric layers beside each other. It is assumed that the incidence plane wave propagates obliquely toward positive x and z direction with an angle of incidence θ_i and electric field strength E^i . Also,

two different polarizations are possible, one is TM and the other is TE. Of course, we know that the wave radiated by an antenna can be decomposed to many plane waves with different angle of incidence.

The differential equations describing IPLs have nonconstant coefficients and so, except for a few special cases, no analytical solution exists for them. There are some methods to analyze the IPLs such as finite difference [8], Taylor's series expansion [9], Fourier series expansion [10], the equivalent sources method [11], and the method of moments [12]. Of course, the most straightforward method to analyze IPLs is subdividing them into K thin homogeneous layers with thickness

$$\Delta z = \frac{d}{K} \ll \lambda_{\min} \cong \frac{c}{f_{\max} \sqrt{\max(\epsilon_r(z))}} \quad (1)$$

in which c is the velocity of the light and $f_{\max}$ is the maximum analysis frequency. The $ABCD$ parameters [13] of the IPL are obtained from those of the layers as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \cdots \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}, \quad (2)$$

where the $ABCD$ parameters of the k th layer are as follows:

$$\begin{aligned} A_k &= D_k = \cos(\Delta\theta_k), \\ B_k &= Z_c^2((k-0.5)\Delta z, \theta_i) C_k = jZ_c((k-0.5)\Delta z, \theta_i) \sin(\Delta\theta_k). \end{aligned} \quad (3)$$

FIGURE 1: A typical IPL as a flat radome (a) TE polarization mode, (b) TM polarization mode.

FIGURE 2: The electric permittivity function $\varepsilon_r(z)$, considering $(\varepsilon_r)_{\min} = 1.05$.

FIGURE 4: The magnitude of $|\Gamma_{in}(f)|$ for TE polarization, considering $(\varepsilon_r)_{\min} = 1.05$.

FIGURE 3: The electric permittivity function $\varepsilon_r(z)$, considering $(\varepsilon_r)_{\min} = 1.10$.

FIGURE 5: The magnitude of $|\Gamma_{in}(f)|$ for TM polarization, considering $(\varepsilon_r)_{\min} = 1.05$.

In (3),

$$\Delta\theta_k = \frac{2\pi f}{c} \sqrt{\varepsilon_r((k-0.5)\Delta z) - \sin^2(\theta_i)\Delta z} \quad (4)$$

is the electrical length of the k th layer and $Z_c(z, \theta_i)$ is the characteristic impedance of the IPL, defined as the ratio of

FIGURE 6: The magnitude of $|\Gamma_{in}(f)|$ for TE polarization, considering $(\epsilon_r)_{\min} = 1.10$.

FIGURE 7: The magnitude of $|\Gamma_{in}(f)|$ for TM polarization, considering $(\epsilon_r)_{\min} = 1.10$.

the transverse electric field to the transverse magnetic field, given by

$$Z_c(z, \theta_i) = \begin{cases} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\sqrt{\epsilon_r(z) - \sin^2(\theta_i)}}, & \text{TE,} \\ \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\epsilon_r(z)} \sqrt{\epsilon_r(z) - \sin^2(\theta_i)}, & \text{TM.} \end{cases} \quad (5)$$

Finally, the input impedance and reflection coefficient of the radome are determined as follows:

$$\begin{aligned} Z_{in}(f, \theta_i) &= \frac{AZ_L(\theta_i) + B}{CZ_L(\theta_i) + D}, \\ \Gamma_{in}(f, \theta_i) &= \frac{Z_{in}(f, \theta_i) - Z_S(\theta_i)}{Z_{in}(f, \theta_i) + Z_S(\theta_i)}, \end{aligned} \quad (6)$$

where

$$Z_L(\theta_i) = Z_S(\theta_i) = \begin{cases} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\cos(\theta_i)}, & \text{TE,} \\ \sqrt{\frac{\mu_0}{\epsilon_0}} \cos(\theta_i), & \text{TM} \end{cases} \quad (7)$$

are the equivalent load and source impedances, respectively.

3. SYNTHESIS OF RADOMES

In this section, a general method is proposed to optimally design the IPLs as radomes. First, we consider the following truncated Fourier series expansion for the electric permittivity function

$$\ln(\epsilon_r(z) - 1) = \sum_{n=0}^N C_n \cos\left(\frac{\pi n z}{d}\right). \quad (8)$$

The reason to use logarithm function at the left of (8) is to keep $\epsilon_r(z) \geq 1$. An optimum designed radome has to have the input reflection coefficient as small as possible in a desired frequency and incidence angle range. Therefore, the optimum values of the coefficients C_n in (8) can be obtained through minimizing the following error function corresponding to M frequencies, J incidence angles, and two possible polarizations TE and TM:

$$\text{Error} = \sqrt{\frac{1}{2MJ} \sum_{\text{POL}=\text{TE}}^{\text{TM}} \sum_{m=1}^M \sum_{j=1}^J |\Gamma_{in}(f_m, \theta_{i,j})|^2}. \quad (9)$$

The defined error function should be restricted by some constraints such as not having a significant reflection at all incidence angles $[0 - \theta_{i,\max}]$, and easy fabrication, respectively, as follows:

$$\begin{aligned} |\Gamma_{in}(f_m, \theta_{i,j})| &\leq \rho_{\max}, \quad \forall m = 1, 2, \dots, M, \quad \forall j = 1, 2, \dots, J, \\ \text{POL.} &= \text{TE and TM,} \end{aligned} \quad (10)$$

$$\epsilon_r(z) \leq (\epsilon_r)_{\max}, \quad (11)$$

$$\bar{\epsilon}_r = \frac{1}{d} \int_0^d \epsilon_r(z) dz \geq (\epsilon_r)_{\min}, \quad (12)$$

where $(\epsilon_r)_{\max}$ is the maximum value of $\epsilon_r(z)$, in the fabrication step. It is noticeable that the constraint (12) is necessary to avoid obtaining the wrong solution $\epsilon_r(z) = 1$ (the free space) in the optimization process. Also, to enforce the designed radomes to be symmetric, we have to use the following truncated Fourier series instead of (8) for the electric permittivity function

$$\ln(\epsilon_r(z) - 1) = \sum_{n=0}^N C'_n \cos\left(\frac{2\pi n z}{d}\right). \quad (13)$$

To solve the above constrained minimization problem, we can use the *fmincon*. *m* file in the MATLAB program. *fmincon* uses a sequential quadratic programming (SQP) method, in which a quadratic programming (QP) subproblem is solved at each of its iteration.

FIGURE 8: The electric permittivity function $\varepsilon_r(z)$, considering $(\varepsilon_r)_{\min} = 1.05$ (a) $d = 2.5$ cm, (b) $d = 5.0$ cm, (c) $d = 7.5$ cm, (d) $d = 10$ cm.

TABLE 1: The unknown coefficients of the truncated Fourier series for $d = 2.5$ cm.

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
$(\varepsilon_r)_{\min} = 1.05$	-29.046	24.689	-5.8701	-20.617	-20.551	29.999	8.6137	11.6949	-6.1217	7.6555	0.2282
$(\varepsilon_r)_{\min} = 1.10$	-30.000	-20.412	-11.794	-10.023	-0.0521	3.0247	8.9473	12.8205	10.4252	15.4492	23.7096

4. EXAMPLES AND RESULTS

We would like to design an IPL with thickness $d = 2.5$ cm (a practical and feasible chosen) as a radome in a frequency range DC to 8.0 GHz, considering $\theta_{i,\max} = 60^\circ$, $\rho_{\max} = 0.1 = -20$ dB, and $(\varepsilon_r)_{\max} = 10$. Using the proposed optimization approach, considering $N = 10$ spatial harmonics, $M = 80$ frequencies, $J = 2$ incidence angles (0° and 60°), and $(\varepsilon_r)_{\min} = 1.05$ or 1.10 , two radomes were synthesized. The unknown coefficients of the truncated Fourier series related to the synthesized radomes are written in Table 1. Figures 2 and 3 illustrate the obtained electric permittivity function $\varepsilon_r(z)$, respectively, considering $(\varepsilon_r)_{\min} = 1.05$ or 1.10 . Figures 4, 5, 6, and 7 illustrate the magnitude of

the input reflection coefficient $|\Gamma_{\text{in}}(f)|$ for TE and TM polarizations. It is observed that the designed radomes have a good performance in both desired frequency band and the incidence angle range. Meanwhile, the reflection coefficient degrades with increasing the angle of incidence. To show the effect of the thickness of IPLs, we increase d from 2.5 cm to 5, 7.5, and 10 cm. The unknown coefficients of the truncated Fourier series, the electric permittivity function $\varepsilon_r(z)$, and the magnitude of the input reflection coefficient $|\Gamma_{\text{in}}(f)|$ corresponding to $\theta_i = 60^\circ$ and considering $(\varepsilon_r)_{\min} = 1.05$ are shown in Table 2 and Figures 8, 9, and 10, respectively. It is observed that as the thickness of the IPL is chosen larger, the obtained electric permittivity function tends to a continuous function, whose property is matching between

TABLE 2: The unknown coefficients of the truncated Fourier series for $(\epsilon_r)_{\min} = 1.05$.

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
$d = 2.5$ cm	-29.046	24.689	-5.8701	-20.617	-20.551	29.999	8.6137	11.6949	-6.1217	7.6555	0.2282
$d = 5.0$ cm	-30.000	-3.4358	-19.617	-1.3242	-6.8844	0.9201	14.4735	1.9353	17.2640	2.1848	25.0426
$d = 7.5$ cm	-8.5288	-1.3020	-1.3469	-0.4938	-0.9210	0.1405	1.2922	0.5839	1.2605	1.2345	7.2452
$d = 10$ cm	-3.0284	-0.0009	-0.3182	0.0019	-0.0076	0.0026	0.1534	0.0023	0.0976	0.0021	0.0659

FIGURE 9: The magnitude of $|\Gamma_{in}(f)|$ for TE polarization with $\theta_i = 60^\circ$, considering $(\epsilon_r)_{\min} = 1.05$.

FIGURE 10: The magnitude of $|\Gamma_{in}(f)|$ for TM polarization with $\theta_i = 60^\circ$, considering $(\epsilon_r)_{\min} = 1.05$.

the air mediums and an intermediate medium. Also, it is seen that the efficiency for TM polarization is better than that for the TE polarization.

5. CONCLUSION

Inhomogeneous planar layers (IPLs) were optimally designed as flat radomes in a desired frequency. First, the electric permittivity function of the IPL is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The performance of the proposed structure is verified using some examples. It was observed that the designed radomes have a good performance in both desired frequency band and the incidence angle range, where the efficiency for TM polarization is better than that for the TE polarization. Also, as the thickness of the IPL is chosen larger, the obtained electric permittivity function tends to a continuous function, whose property is matching between the air mediums and an intermediate medium. The proposed method can be extended for IPLs, whose magnetic permeability is inhomogeneous solely or along with their electric permittivity. Also, we can consider IPLs for spherical wavefronts instead of planar ones in the future.

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