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Research Article

On the Second-Order Statistics of Correlated Cascaded Rayleigh Fading Channels

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The second-order statistics of two correlated cascaded (double) Rayleigh fading channels are analyzed, where different relevant second-order cross-correlation functions of in-phase and quadrature components of the cascaded Rayleigh channels are derived. The level crossing rate (LCR) and average fade duration (AFD) of the cascaded channels are evaluated, and a single-integral form of the LCR is derived. Numerical results of the LCR and AFD are presented, and the effect of the correlation is illustrated.

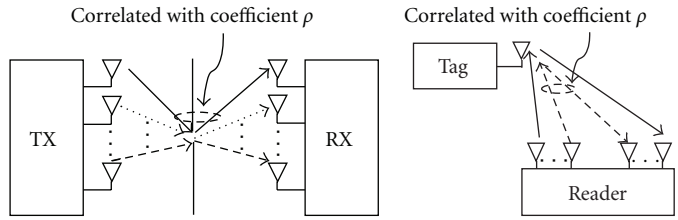
1. Introduction

Recently, the cascaded fading channels have attracted a lot of research interests [1–12]. The cascaded fading channels can be used to model different wireless communication scenarios, such as the keyhole fading channel [4], the mobile-to-mobile transmission channel [5], dual-hop fading channels [8], and radio frequency identification (RFID) pinhole channels [9–12].

In the published works of [1–9], the cascaded processes are assumed to be independent. For cascaded keyhole or pinhole fading channels, since the faded signals terminate and originate at the same keyhole or pinhole location, the correlation between cascaded fading processes may have some impacts on the performance of wireless transmissions. In [10–12], the bit error rate (BER) of RFID with multiple tags has been examined for correlated cascaded (forward-backscatter) channels. In fact, the completely dependent or correlated cascaded Rayleigh fading channels have been studied in earlier works [13], where the corresponding BERs of different modulations have been evaluated. The scenarios of keyhole multiple-input multiple-output (MIMO) channels and RFID pinhole channels with a correlation coefficient are illustrated in Figure 1.

In this paper, in contrast to the evaluation of first-order performance for correlated cascaded Rayleigh channels, such as the bit error rate given in [10–12], second-order statistics of correlated cascaded Rayleigh processes are analyzed and evaluated. The second-order statistics such as the LCR and AFD are useful for the design of practical wireless communication systems [14–19]. The LCR of a faded signal is defined as the average rate at which the signal envelope crosses a given level, and the AFD is the average duration that the signal envelope becomes lower than the given threshold [14, 15]. As addressed in [20], the second-order statistics, LCR and AFD, are also important for the design of real RFID systems. In [8], the LCR and AFD were analyzed for independent cascaded Rayleigh fading channels. In [16], the LCR and AFD were applied to the burst-error analysis for independent cascaded Rayleigh fading channels. In [17], the LCR and AFD of cooperative selection diversity were studied, where independent cascaded Rayleigh fading channels were assumed. In [18, 19], the LCR and AFD were evaluated for independent cascaded Nakagami-m fading channels.

In the context, two completely correlated Rayleigh processes [13] in cascaded are modelled, corresponding second-order correlation functions are derived, and the LCR and AFD of the correlated cascaded Rayleigh processes are evaluated. The result of the paper can be applied to the analysis of second-order statistics of correlated cascaded Rayleigh fading channels, such as a path of keyhole or pinhole fading channels shown in Figure 1.



(a) Keyhole MIMO channels (b) RFID pinhole channels

Figure 1: The scenarios of keyhole MIMO channels and RFID pinhole channels.

In Section 2, the correlated cascaded Rayleigh random processes and their in-phase and quadrature processes are modelled, where various second-order correlation functions are derived. In Section 3, the LCR and AFD of the correlated cascaded processes are evaluated. In Section 4, numerical results of the LCR and AFD are given and the effect of the correlation is illustrated. Conclusions are drawn in Section 5.

2. Second-Order Correlation Functions of Correlated Cascaded Rayleigh Processes

2.1. Modeling. Let $\alpha_i(t)$ $(i = 1, 2)$ be a Rayleigh process with the probability density function (pdf) given by the following:

$$f_i(\alpha_i) = \frac{\alpha_i}{\sigma_i^2} e^{-\alpha_i^2/(2\sigma_i^2)}, \quad \alpha_i \geq 0, \tag{1}$$

where $2\sigma_i^2 = E[\alpha_i^2]$. Then, two correlated Rayleigh processes in cascade can be modelled by the following:

$$R(t, \tau) = \alpha_1(t)\alpha_2(t + \tau), \quad t, \tau \geq 0, \tag{2}$$

where τ is the time or phase offset between α_1 and α_2. The joint pdf of α_1 and α_2 is [21]

$$\begin{aligned} f_{12}(\alpha_1, \alpha_2) = {} & \frac{\alpha_1\alpha_2}{\sigma_1^2\sigma_2^2(1-\rho^2)} e^{-(\sigma_2^2\alpha_1^2+\sigma_1^2\alpha_2^2)/(2\sigma_1^2\sigma_2^2(1-\rho^2))} \\ & \times I_0\left(\frac{\alpha_1\alpha_2|\rho|}{\sigma_1\sigma_2(1-\rho^2)}\right), \quad \alpha_1, \alpha_2 \geq 0, \end{aligned} \tag{3}$$

where $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind, and the correlation coefficient is in the range $-1 < \rho < 1$. Notice that when $\rho = 1$, α_1 and α_2 have a linear relation, and R reduces to the well-known chi-square distribution. With $f_{12}(\alpha_1, \alpha_2)$ given by (3), it is straightforward to show that the pdf of R is

$$\begin{aligned} f_R(r) &= \int_0^\infty \frac{1}{\alpha_2} f_{12}\left(\frac{r}{\alpha_2}, \alpha_2\right) d\alpha_2 \\ &= \frac{r}{\sigma_1^2\sigma_2^2(1-\rho^2)} I_0\left(\frac{r|\rho|}{\sigma_1\sigma_2(1-\rho^2)}\right) K_0\left(\frac{r}{\sigma_1\sigma_2(1-\rho^2)}\right), \end{aligned} \tag{4}$$

where $K_0(\cdot)$ is the zeroth order modified Bessel function of the second kind, and [22, (3.478.4)] is used to evaluate the integral. In Figure 2, the pdf with different correlation coefficients is shown. From Figure 2, when ρ increases, the cascaded Rayleigh processes has a larger probability of yielding smaller values.

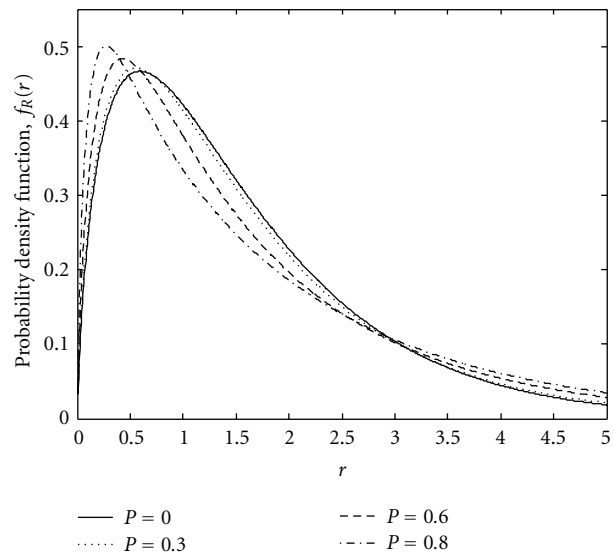


Figure 2: Probability density function of correlated cascade Rayleigh processes.

2.2. Second-Order Correlation Functions of In-Phase and Quadrature Processes. For $i = 1, 2$, let X_i and Y_i be the underlying in-phase and quadrature components of $\alpha_i(t)$, and

$$\alpha_i^2(t) = X_i^2(t) + Y_i^2(t). \tag{5}$$

The time derivatives of X_i, Y_i, and $\alpha_i(t)$ give the second-order information of cascaded Rayleigh processes.

Le f_m be the maximal Doppler frequency of the cascaded fading channels and θ_k be the uniformly distributed phase on $[0, 2\pi]$. Based on the modeling used in [13, 14], for $i = 1, 2$, we may write (X_i, Y_i) in the following forms [23]

$$\begin{aligned} X_i(t) &= \lim_{n\to\infty} \sqrt{2\sigma_i^2} \sum_{k=1}^{n} a_k \cos(\omega_k t - \omega_{c,i} T_k), \\ Y_i(t) &= \lim_{n\to\infty} \sqrt{2\sigma_i^2} \sum_{k=1}^{n} a_k \sin(\omega_k t - \omega_{c,i} T_k), \end{aligned} \tag{6}$$

where $\omega_k = 2\pi f_m \cos\theta_k$, a_k is the corresponding fractional power for the kth component, $\omega_{c,i}$ is the frequency, and T_k is the time-delay [15]. In practice, T_k can be modelled by

an exponential distribution with mean $\overline{T}$ [15]. The time derivatives of (X_i, Y_i) are

$$\dot{X}_i(t) = -\lim_{n\to\infty}\sqrt{2\sigma_i^2}\sum_{k=1}^{n} a_k\omega_k\cos\theta_k\sin(\omega_k t - \omega_{c,i}T_k),$$
$$\dot{Y}_i(t) = \lim_{n\to\infty}\sqrt{2\sigma_i^2}\sum_{i=1}^{n} a_k\omega_k\cos\theta_k\cos(\omega_k t - \omega_{c,i}T_k). \tag{7}$$

Let $\langle\cdot\rangle$ denote the average operator. With (7), we can directly obtain that the second-order autocorrelation functions are

$$\langle X_i(t)\dot{X}_i(t+\tau)\rangle = \langle Y_i(t)\dot{Y}_i(t+\tau)\rangle = 0, \quad i = 1,2. \tag{8}$$

Similarly, the second-order intercross-correlation functions of the in-phase and quadrature processes can be obtained as

$$\begin{aligned}\langle X_i(t)\dot{Y}_i(t+\tau)\rangle &= \langle \dot{X}_i(t)Y_i(t+\tau)\rangle \\ &= \langle \dot{X}_i(t)\dot{Y}_i(t+\tau)\rangle = 0, \quad i = 1,2.\end{aligned} \tag{9}$$

Let $\Delta\omega = \omega_{c,2} - \omega_{c,1}$ and $\omega_m = 2\pi f_m$. By using (7) and some manipulations, the second-order cross-correlation function $\langle\dot{X}_1(t)\dot{X}_2(t+\tau)\rangle$ and $\langle\dot{Y}_1(t)\dot{Y}_2(t+\tau)\rangle$ can be derived as

$$\begin{aligned}\langle\dot{X}_1(t)\dot{X}_2(t+\tau)\rangle &= \langle\dot{Y}_1(t)\dot{Y}_2(t+\tau)\rangle \\ &= \sigma_1\sigma_2\int_0^{2\pi}\int_0^{\infty}\frac{e^{-T_k/\overline{T}}}{2\pi\overline{T}} \\ &\quad\times\cos(\omega_m\tau\cos\theta_k - \Delta\omega T_k)d\theta_k dT_k \\ &\quad+\sigma_1\sigma_2\omega_m^2\int_0^{2\pi}\int_0^{\infty}\frac{e^{-T_k/\overline{T}}}{4\pi\overline{T}} \\ &\quad\times\cos 2\theta_k\cos(\Delta\omega T_k) \\ &\quad\times\cos(\omega_m\tau\cos\theta_k)d\theta_k dT_k \\ &\quad+\sigma_1\sigma_2\omega_m^2\int_0^{2\pi}\int_0^{\infty}\frac{e^{-T_k/\overline{T}}}{4\pi\overline{T}} \\ &\quad\times\cos 2\theta_k\sin(\Delta\omega T_k) \\ &\quad\times\sin(\omega_m\tau\cos\theta_k)d\theta_k dT_k \\ &= \frac{\sigma_1\sigma_2\omega_m^2}{2\left[1+\left(\Delta\omega\overline{T}\right)^2\right]} \\ &\quad\times[J_0(\omega_m\tau) - J_2(\omega_m\tau)],\end{aligned} \tag{10}$$

where $J_i(\cdot)$ $(i = 0,1,2,\ldots)$ denotes the ith-order Bessel function of the first kind, and [22, (3.715), (8.339), (3.895), and (8.473.1)] are applied to evaluate the integral.

When the two cascaded channels are using the same carrier frequency, we have $\Delta\omega = 0$. In Figure 3, this second-order cross-correlation of in-phase and quadrature processes with $\Delta\omega = 0$ is plotted for $\sigma_1^2 = \sigma_2^2 = 1$. From Figure 2, the vibration range of the cross-correlation may increase when f_m becomes higher.

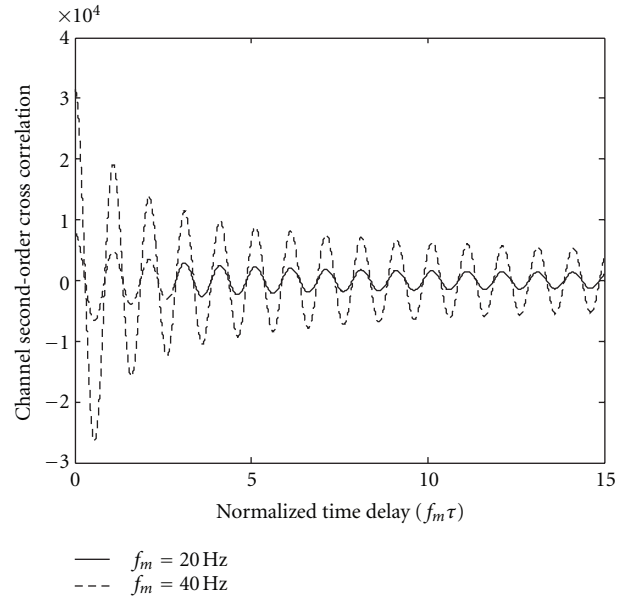


Figure 3: Cross-correlation function of in-phase and quadrature processes of the cascaded fading channels.

Other non-zero second-order correlation functions can be derived in a similar way. Following manipulations similar to those used for deriving (10), we can obtain the related second-order correlation functions as follows:

$$\begin{aligned}\langle\dot{X}_1(t)\dot{Y}_2(t+\tau)\rangle &= -\langle\dot{Y}_1(t)\dot{X}_2(t+\tau)\rangle \\ &= -\frac{\sigma_1\sigma_2\omega_m^2\Delta\omega\overline{T}}{2\left[1+\left(\Delta\omega\overline{T}\right)^2\right]} \\ &\quad\times[J_0(\omega_m\tau) - J_2(\omega_m\tau)], \\ \langle\dot{X}_1(t)X_2(t+\tau)\rangle &= -\langle X_1(t)\dot{X}_2(t+\tau)\rangle \\ &= -\langle\dot{Y}_1(t)Y_2(t+\tau)\rangle \\ &= -\langle Y_1(t)\dot{Y}_2(t+\tau)\rangle \\ &= -\frac{\sigma_1\sigma_2\omega_m J_1(\omega_m\tau)}{1+\left(\Delta\omega\overline{T}\right)^2}, \\ \langle X_1(t)\dot{Y}_2(t+\tau)\rangle &= \langle\dot{Y}_1(t)X_2(t+\tau)\rangle \\ &= -\langle\dot{X}_1(t)Y_2(t+\tau)\rangle \\ &= -\langle Y_1(t)\dot{X}_2(t+\tau)\rangle \\ &= \frac{\sigma_1\sigma_2\omega_m\Delta\omega\overline{T}J_1(\omega_m\tau)}{1+\left(\Delta\omega\overline{T}\right)^2}.\end{aligned} \tag{11}$$

The above second-order correlation functions can be applied to the derivation of the cross-correlation between $\dot{\alpha}_1(t)$ and $\dot{\alpha}_2(t+\tau)$ that will be useful for the LCR and AFD derivation in Section 3.

2.3. Cross-Correlation Function of $\dot{\alpha}_1(t)$ and $\dot{\alpha}_2(t)$. For $i = 1, 2$ the time derivative of $\alpha_i(t)$ has the form

$$\dot{\alpha}_i(t) = \frac{X_i(t)\dot{X}_i(t) + Y_i(t)\dot{Y}_i(t)}{\sqrt{X_i^2(t) + Y_i^2(t)}} = \frac{X_i(t)\dot{X}_i(t) + Y_i(t)\dot{Y}_i(t)}{\alpha_i(t)}. \tag{12}$$

The correlation function between $\dot{\alpha}_1(t)$ and $\dot{\alpha}_2(t+\tau)$ under fixed $\alpha_1(t)$ and $\alpha_2(t+\tau)$ can be written as follows:

$$\langle \dot{\alpha}_1(t)\dot{\alpha}_2(t+\tau)\rangle = \frac{\langle X_1\dot{X}_1X_2\dot{X}_2\rangle + \langle Y_1\dot{Y}_1Y_2\dot{Y}_2\rangle + \langle X_1\dot{X}_1Y_2\dot{Y}_2\rangle + \langle X_2\dot{X}_2Y_1\dot{Y}_1\rangle}{\alpha_1(t)\alpha_2(t+\tau)}, \tag{13}$$

where the variables t in (X_1, Y_1) and $(t+\tau)$ in (X_2, Y_2) are omitted for simplification. To derive $\langle \dot{\alpha}_1(t)\dot{\alpha}_2(t+\tau)\rangle$, we need to obtain the four expectations in the numerator of (13). Based on the model of central limit theorem considered in [14, 15], [24, (8.111)], all $X_i, \dot{X}_i, Y_i$, and $\dot{Y}_i$ have a zero-mean Gaussian distribution. On the other hand, it is well known that if the random variables (Z_1, Z_2, Z_3, Z_4) have a jointly zero-mean Gaussian distribution, $\langle Z_1Z_2Z_3Z_4\rangle$ can be evaluated by [19, (8.61)] the following:

$$\begin{aligned}\langle Z_1Z_2Z_3Z_4\rangle = {} & \langle Z_1Z_2\rangle\langle Z_3Z_4\rangle \\ & + \langle Z_1Z_3\rangle\langle Z_2Z_4\rangle \\ & + \langle Z_1Z_4\rangle\langle Z_2Z_3\rangle.\end{aligned} \tag{14}$$

Thus, we can use the property given by (14) to analyze the numerator of (13). The four expectations in the numerator of (13) can be derived by using (14) and the expectations of the forms $\langle Z_iZ_j\rangle$ and $\langle Z_i\dot{Z}_j\rangle$ $(i, j = 1, 2)$. For $Z = X$ or Y in (14), we have

$$\begin{aligned}&\langle X_i(t)Y_i(t+\tau)\rangle = 0, \\ &\langle X_1(t)X_2(t+\tau)\rangle = \langle Y_1(t)Y_2(t+\tau)\rangle = \frac{\sigma_1\sigma_2J_0(\omega_m\tau)}{1+\left(\Delta\omega\overline{T}\right)^2}, \\ &\langle X_1(t)Y_2(t+\tau)\rangle = -\langle X_2(t)Y_1(t+\tau)\rangle = \frac{-\Delta\omega\overline{T}\sigma_1\sigma_2J_0(\omega_m\tau)}{1+\left(\Delta\omega\overline{T}\right)^2}.\end{aligned} \tag{15}$$

Using the result obtained in Section 2.2, and (13)–(15), we can simplify (13) into the form

$$\begin{aligned}\langle \dot{\alpha}_1(t)\dot{\alpha}_2(t+\tau)\rangle &= \frac{\sigma_1^2\sigma_2^2\omega_m^2}{\alpha_1(t)\alpha_2(t+\tau)\left[1+\left(\Delta\omega\overline{T}\right)^2\right]} \\ &\quad\times\left[J_0^2(\omega_m\tau) - 2J_1^2(\omega_m\tau) - J_0(\omega_m\tau)J_2(\omega_m\tau)\right].\end{aligned} \tag{16}$$

To evaluate (16), we employ the following relation derived in [15, (1.5.20)] and [23, (4.31b)] for ρ and $\Delta\omega\overline{T}$

$$\rho^2(\omega_m\tau) = \frac{J_0^2(\omega_m\tau)}{1+\left(\Delta\omega\overline{T}\right)^2}. \tag{17}$$

From (17), for $\Delta\omega = 0$, the correlation coefficient is a function of the maximum Doppler frequency and the phase offset between the cascaded processes. Substituting (17) with $\Delta\omega = 0$, we can simplify (16) into the form

$$\begin{aligned}\langle \dot{\alpha}_1(t)\dot{\alpha}_2(t+\tau)\rangle &= \frac{-\sigma_1^2\sigma_2^2}{\alpha_1(t)\alpha_2(t+\tau)}\frac{d^2}{d\tau^2}J_0^2(\omega_m\tau) \\ &= \frac{-\sigma_1^2\sigma_2^2\omega_m^2(\rho^2)''}{\alpha_1(t)\alpha_2(t+\tau)} \\ &= \frac{-\sigma_1^2\sigma_2^2\omega_m^2(\rho^2)''}{R(t,\tau)},\end{aligned} \tag{18}$$

where we use the recursive relations of Bessel functions given by [22, (8.473.1), (8.473.4)], and $(\rho^2)''$ is defined by

$$(\rho^2)'' = \left[\frac{d^2\rho^2(x)}{dx^2}\right]_{x=\omega_m\tau}. \tag{19}$$

For the derivation of the LCR and AFD of the correlated cascaded Rayleigh channels, in the next section, without loss of generality, $\Delta\omega = 0$ is considered.

3. Derivation of LCR and AFD

3.1. LCR. The average LCR of a random process R at $R = r$ can be evaluated by [14, 15]

$$N_R(r) = \int_0^\infty \dot{r} f_{R,\dot{R}}(r,\dot{r})d\dot{r} = f_R(r)\int_0^\infty \dot{r} f_{\dot{R}|R}(\dot{r} \mid r)d\dot{r}, \tag{20}$$

where $\dot{r}$ denotes the time derivative of r, and $f_{\dot{R}|R}(\dot{r} \mid r)$ is the pdf of the time derivative $\dot{R}$ conditioned on $R = r$.

To evaluate the LCR given by (20), we first characterize the probability distribution of $\dot{R}$ below, where

$$\dot{R}(t,\tau) = \alpha_2(t+\tau)\dot{\alpha}_1(t) + \alpha_1(t)\dot{\alpha}_2(t+\tau). \tag{21}$$

Based on [15], $\dot{\alpha}_i$ is Gaussian distributed with zero mean and variance $\sigma_{\dot{\alpha}_i}^2 = 2\pi^2 f_m^2\sigma_i^2$. Consequently, conditioned on fixed (α_1, α_2), $\dot{R}$ is Gaussian distributed with zero mean and the variance

$$\dot{\sigma}_R^2 = \alpha_2^2\sigma_{\dot{\alpha}_1}^2 + \alpha_1^2\sigma_{\dot{\alpha}_2}^2 + 2\alpha_1\alpha_2\,\mathrm{cov}(\dot{\alpha}_1, \dot{\alpha}_2), \tag{22}$$

where $\mathrm{cov}(\dot{\alpha}_1, \dot{\alpha}_2)$ denotes the covariance of $\dot{\alpha}_1(t)$ and $\dot{\alpha}_2(t+\tau)$ under fixed (α_1, α_2), and is given by (18) for $\Delta\omega = 0$. Let $f_{\dot{R}|r,\alpha}(\dot{r} \mid \alpha_1, \alpha_2)$ denote the conditional pdf of $\dot{R}$ under fixed (α_1, α_2). Thus,

$$f_{\dot{R}|r,\alpha}(\dot{r} \mid \alpha_1, \alpha_2) \sim \mathrm{Gaussian}(0, \dot{\sigma}_R^2). \tag{23}$$

Let $f_{R|\alpha}(r \mid \alpha_1, \alpha_2)$ denote the conditional pdf of R under fixed (α_1, α_2). Then, for the correlated cascaded processes, the LCR given by (20) can be rewritten in the form

$$\begin{aligned}N_R(r) &= \int\int\int_0^\infty \delta(r - \alpha_1\alpha_2)\dot{r} f_{\dot{R}|r,\alpha}(\dot{r} \mid r, \alpha_1, \alpha_2)d\dot{r} \\ &\quad\times f_{R|\alpha}(r \mid \alpha_1, \alpha_2)f_{12}(\alpha_1, \alpha_2)d\alpha_1 d\alpha_2 \\ &= \int_0^\infty\left(\int_0^\infty \dot{r} f_{\dot{R}|\alpha}\left(\dot{r} \mid \frac{r}{\alpha_2}, \alpha_2\right)d\dot{r}\right)\frac{1}{\alpha_2}f_{12}\left(\frac{r}{\alpha_2}, \alpha_2\right)d\alpha_2,\end{aligned} \tag{24}$$

where $\delta(\cdot)$ is the Dirac delta function, $f_{12}(r/\alpha_2, \alpha_2)$ has the form given by (3) with α_1 replaced by r/α_2, and $f_{\dot{R}|\alpha}(\dot{r} \mid r/\alpha_2, \alpha_2)$ is given by (23) also with α_1 replaced by r/α_2. Substituting (18) into (22) and (3) into (24), we can express the normalized LCR in the single-integral form

$$\begin{aligned}\frac{N_R(r)}{f_m} &= \frac{\sqrt{\pi} r}{\sigma_1^2\sigma_2^2(1-\rho^2)} I_0\left(\frac{r|\rho|}{\sigma_1\sigma_2(1-\rho^2)}\right) \\ &\quad \times \int_0^\infty \sqrt{\sigma_1^2 - \frac{4\sigma_1^2\sigma_2^2(\rho^2)''}{\alpha_2^2} + \frac{r^2\sigma_2^2}{\alpha_2^4}} \\ &\quad \times e^{-((\sigma_1^2\alpha_2^2+\sigma_2^2 r^2/\alpha_2^2)/2\sigma_1^2\sigma_2^2(1-\rho^2))} d\alpha_2\end{aligned} \tag{25}$$

that can be easily evaluated with a computing software, such as MATLAB. If $\rho = 0$, it can be easily check that (25) will reduce to the form given in [8, 17, 18] for independent cascaded Rayleigh channels.

3.2. AFD. Let $P(R \le r)$ be the cumulative distribution function (cdf) of R. With the normalized LCR given by (25), the normalized AFD can be evaluated by [14]

$$T_R(r) = f_m \cdot \frac{P(R \le r)}{N_R(r)}. \tag{26}$$

By using the pdf of R given by (4), the cdf in the above AFD can be evaluated as

$$\begin{aligned}P(R \le r) &= \int_0^r f_R(y)\,dy \\ &= \frac{1}{\sigma_1^2\sigma_2^2(1-\rho^2)} \int_0^r y I_0\left(\frac{y|\rho|}{\sigma_1\sigma_2(1-\rho^2)}\right) \\ &\quad \times K_0\left(\frac{y}{\sigma_1\sigma_2(1-\rho^2)}\right) dy \\ &= 1 - \frac{r|\rho|}{\sigma_1\sigma_2(1-\rho^2)} I_1\left(\frac{r|\rho|}{\sigma_1\sigma_2(1-\rho^2)}\right) \\ &\quad \times K_0\left(\frac{r}{\sigma_1\sigma_2(1-\rho^2)}\right) \\ &\quad - \frac{r}{\sigma_1\sigma_2(1-\rho^2)} I_0\left(\frac{r|\rho|}{\sigma_1\sigma_2(1-\rho^2)}\right) \\ &\quad \times K_1\left(\frac{r}{\sigma_1\sigma_2(1-\rho^2)}\right),\end{aligned} \tag{27}$$

where $I_1(\cdot)$ is the first-order modified Bessel function of the first kind, and a formula modified from [22, (6.521.4)] with $I_i(z) = j^{-i} J_i(jz)$ ($j = \sqrt{-1}$) is used to write the integral into a closed-form.

4. Numerical Results of LCR and AFD

From (17), due to the property of Bessel functions, ρ^2 is not a monotone function of τ. However, as τ increases, the vibration range of $J_0^2(\omega_m\tau)$ tends to be smaller, and ρ^2 becomes smaller.

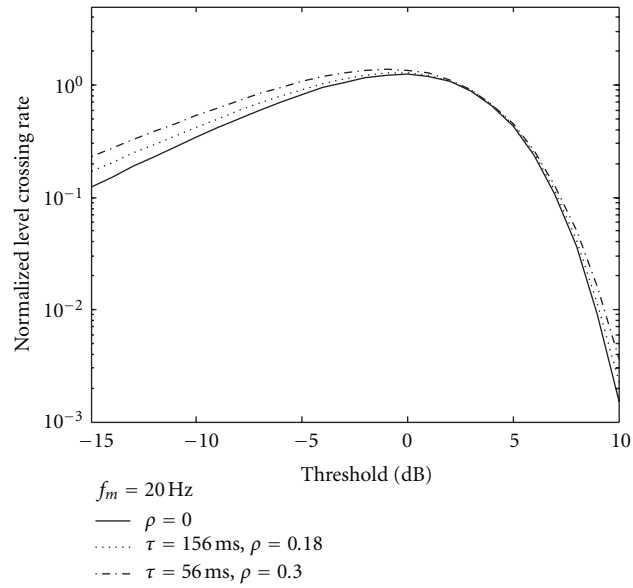


Figure 4: Normalized LCR for $\sigma_1^2 = \sigma_2^2 = 1$.

To demonstrate the impact of the correlation on the LCR and AFD, the normalized LCR of the correlated cascaded Rayleigh channels for different values of τ is plotted in Figure 4. The result in Figure 4 implies that when correlation increases, the cascaded envelope R has more chances to enter a deep fading area. For example, as shown in Figure 4, the correlated cascaded channels may have a 5% ∼ 10% more crossing rate than independent cascaded channels to go into deep fade below −10 dB.

In Figure 5, the numerical AFD is plotted. According to Figure 5, as the correlation increases, the average duration when R temporarily stays in the deep fading area can be shorter although R enters the area more frequently.

Thus, the correlation between cascaded channels not only affects the BER as shown in [10–13] but also changes the second-order statistics which are more serious in a deep fading area.

5. Conclusions

Different second-order correlation functions of completely correlated cascaded fading channels have been derived and the corresponding LCR and AFD are analyzed, which can facilitate the design of wireless communication or RFID systems on keyhole or pinhole channels. Under the consideration of the channel correlation, the LCR and AFD will depart from those obtained for independent cascaded channels. Numerical results show that the correlation has a higher impact on the second-order statistics in the deep fading region. The evaluation of LCR and AFD will also be helpful for the design of interleaving transmission and encoding schemes over correlated cascaded fading channels.

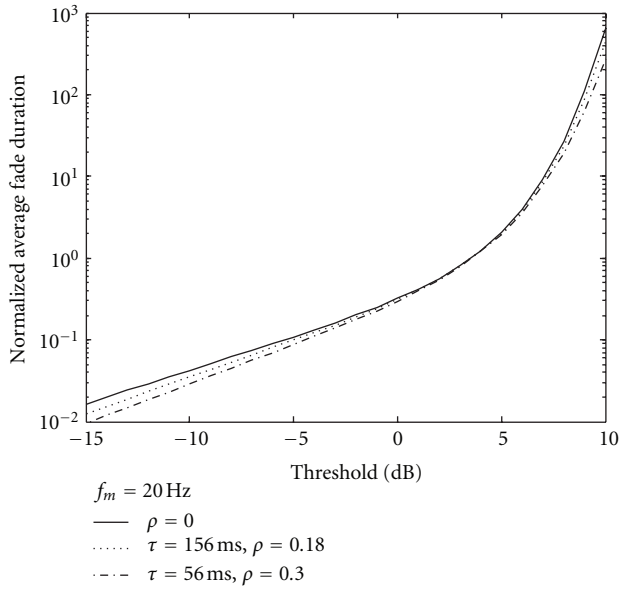


Figure 5: Normalized AFD for $\sigma_1^2 = \sigma_2^2 = 1$.

Further research on related topics may consider the second-order statistics of diversity schemes over correlated cascaded channels.

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