Research Article

Can $\lambda/4$ Radiators Be Really Isotropic?

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In search for isotropic radiators with reasonable quality Factor (Q), bandwidth, and efficiency, one looks for practical radiators with a typical resonant length of $\lambda/4$. We present here a Green's function analysis in Fourier of a $\lambda/4$ microstrip element and a far-field integral method in configuration (real) space of single and dual U-shaped $\lambda/4$ elements. Both solutions analytically prove that the power radiation patterns are isotropic in nature (while the thickness and the width tend to zero), although the polarizations are not symmetrical in all cuts. It is also shown that the power isotropic U-shaped radiator, for which the surface current density is infinite, can be replaced by another finite-size radiator, having finite-surface current density, such that its far-field is exactly the same as the far-field of the U-shaped isotropic radiator.

1. Introduction

Accumulated experience, empirical and theoretical, with $\lambda/4$ microstrip elements proved their efficiency as elements of antenna arrays, as can be shown, for example, in [1]. The beamwidth of these elements is relatively broad, where the vertical part of the element contributes to the end-fire direction. The beamwidth becomes broader as the width and the height of the elements become narrower. This phenomenon has motivated us to reinvestigate the age-old problem of the unity gain antenna. Such an antenna would be considered as an achievement from the technological point of view: one is eager to have a multidirectional circularly polarized antenna for satellites and space vehicles in order to ensure communicability with Earth ground stations [2]. Indeed, [3] states that the need for isotropic antennas not academic in that a radar designed with such an antenna and so interconnected that the polarization is reversed on reception, would receive equal signals from most reflecting objects, no matter where they were placed on a sphere centered at the antenna. In [4] it is shown for the so-called " $\lambda/4$ transmission line antenna" that as the distance between the conductors becomes smaller, the pattern tends to be more isotropic. A relatively recent summary of the search for an isotropically radiating source is given in [5],

from which we learn that an infinite long source current distribution based on parallel to xy "turnstile" distributed uniformly over the z -axis radiated power isotropically. In this case the feed function is $K_0(kz)$, where $k = 2\pi/\lambda$ is the wavenumber and $K_0(\cdot)$ is the zero order of the modified Bessel function of the second kind. The pattern of one turnstile is shown, for example, in [6].

The peculiarity of the class of isotropic power radiators offered here is that the source has a finite size and that the far-field is analytically solvable. Moreover, it is shown [7, 8] that the origin of their phenomenon is the magnetoelectric symmetry of the radiator. These radiators are based on an infinite-surface current density, but this is not a serious impediment, because a finite-size, finite-surface current distribution radiator can be constructed such that the fields outside this radiator are the same as for the basic infinite-current isotropic power radiator, as will be shown in what follows.

The structure of the paper is as follows: in Section 2 we present the far-field calculation of an idealized $\lambda/4$ element and present its radiation cuts for various values of width and height above ground. It is shown that as the width and the height above ground of the element become smaller relative to the wavelength, the power pattern becomes more and more isotropic. A U-shaped power

FIGURE 1: The geometry of a $\lambda/4$ antenna element above an infinite ground plane. The height of the element above ground is H , its width is W , and its length is $\lambda/4$.

isotropic radiator is presented in Section 3. The proof of the isotropic behavior of the far-field radiation power density of this radiator is described, as well as its polarization patterns. In Section 4 we present a double U radiator, for which the polarization patterns are more symmetric. In Section 5 we show how to replace the U-shaped radiator, for which its surface current density is infinite, by a spherical surface having $\lambda/2$ radius and finite-surface current density, for which its far-field is exactly the same as the far-field of the U-shaped radiator. Conclusions are given in Section 6.

2. Patterns of an Idealized $\lambda/4$ Microstrip Element

Consider the geometry of a $\lambda/4$ element above an infinite ground plane as shown in Figure 1. For simplicity there is no dielectric substrate, although the general analysis takes such substrate of thickness H and relative dielectric constant ϵ_r into consideration. The surface current density on the radiator is composed of horizontal and vertical component as follows:

$$\mathbf{J}(x, y, z) = J_x(x, y)\hat{\mathbf{x}} + J_z(y, z)\hat{\mathbf{z}}, \quad (1)$$

where bold letters are used to designate vectors hereafter. A detailed Fourier analysis of the radiated fields [9–11] can be summarized as follows.

The Fourier transform of the tangential electric field on the plane which includes the patch is given by

$$\tilde{\mathbf{E}}_t(k_x, k_y) = \frac{\eta}{k_0} \left[\frac{k_0^2 (\tilde{\mathbf{J}} \cdot \hat{\mathbf{k}}_t^\perp) \hat{\mathbf{k}}_t^\perp}{j\gamma_2 \cot(\gamma_2 H) - \gamma_1} + \frac{\gamma_1 \gamma_2 (\tilde{\mathbf{J}} \cdot \hat{\mathbf{k}}_t^\parallel) \hat{\mathbf{k}}_t^\parallel}{j\epsilon_r \gamma_1 \cot(\gamma_2 H) - \gamma_2} \right], \quad (2)$$

where $\eta = 120\pi \Omega$ is the free space impedance, $k_0 = 2\pi/\lambda$ is the wavenumber, ϵ_r is the relative dielectric constant layer under the patch, and H is the thickness of the dielectric layer,

and $\tilde{\mathbf{J}}$ is the surface current density on the patch in Fourier space. The other variables are

$$\begin{aligned} \gamma_1 &= \sqrt{k_0^2 - k_x^2 - k_y^2}, \\ \gamma_2 &= \sqrt{\epsilon_r k_0^2 - k_x^2 - k_y^2}, \\ \mathbf{k}_t &= k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}}, \quad k_t = \sqrt{k_x^2 + k_y^2}, \\ \hat{\mathbf{k}}_t^\perp &= -\frac{k_y}{k_t} \hat{\mathbf{x}} + \frac{k_x}{k_t} \hat{\mathbf{y}}, \\ \hat{\mathbf{k}}_t^\parallel &= \frac{k_x}{k_t} \hat{\mathbf{x}} + \frac{k_y}{k_t} \hat{\mathbf{y}}. \end{aligned} \quad (3)$$

The electric field in spatial coordinates can be calculated by taking the inverse Fourier transform of $\tilde{\mathbf{E}}(k_x, k_y)$. However, this transform cannot be done analytically, so further results will be written in the Fourier domain.

The complex input power at the antenna terminals is

$$P_{\text{in}} = -\frac{1}{8\pi^2} \iint_{-\infty}^{\infty} \tilde{\mathbf{E}}_t(k_x, k_y) \cdot \tilde{\mathbf{J}}^*(k_x, k_y) dk_x dk_y. \quad (4)$$

This expression has real and imaginary parts. The contribution to the real part comes from the radiation into free space and from surface waves excitation effects. In the case of air as a dielectric substrate ($\epsilon_r = 1$) surface waves do not exist. The contribution to free space radiation comes from integration in the last equation over the range

$$k_x^2 + k_y^2 < k_0^2 \quad (5)$$

which is called the “visible range.”

Transforming the equation for P_{in} into spherical coordinates and restrict it to the visible range gives the following result:

$$k_x = k_0 \sin \theta \cos \varphi, \quad k_y = k_0 \sin \theta \sin \varphi, \quad (6)$$

$$P_r(\theta, \varphi) = \int_0^{\pi/2} \int_0^{2\pi} f(\theta, \varphi) \sin \theta d\theta d\varphi, \quad (7)$$

where $f(\theta, \varphi)$ is the power radiation of the antenna that is expressed in terms of the Fourier transforms of the surface current density components $\tilde{J}_x$ and $\tilde{J}_y$:

$$\begin{aligned} f(\theta, \varphi) &= \frac{15k_0^2}{\pi} \left[\frac{|\tilde{J}_x \sin \varphi + \tilde{J}_y \cos \varphi|^2 \cos^2 \theta}{(\epsilon_r - \sin^2 \theta) \cot^2(Hk_0 \sqrt{\epsilon_r - \sin^2 \theta}) + \cos^2 \theta} \right. \\ &\quad \left. + \frac{|\tilde{J}_x \cos \varphi + \tilde{J}_y \sin \varphi|^2 \cos^2 \theta (\epsilon_r - \sin^2 \theta)}{(\epsilon_r - \sin^2 \theta) + \epsilon_r^2 \cos^2 \theta \cot^2(Hk_0 \sqrt{\epsilon_r - \sin^2 \theta})} \right]. \end{aligned} \quad (8)$$

In our case $\epsilon_r = 1$, hence

$$f(\theta, \varphi) = \frac{15k_0^2}{\pi} \left[\frac{|\tilde{J}_x \sin \varphi + \tilde{J}_y \cos \varphi|^2}{\cot^2(Hk_0 \cos \theta) + 1} + \frac{|\tilde{J}_x \cos \varphi + \tilde{J}_y \sin \varphi|^2 \cos^2 \theta}{1 + \cot^2(Hk_0 \cos \theta)} \right] \quad (9)$$

$$= \frac{15k_0^2}{\pi} \sin^2(Hk_0 \cos \theta) \times \left(|\tilde{J}_x \sin \varphi + \tilde{J}_y \cos \varphi|^2 + |\tilde{J}_x \cos \varphi + \tilde{J}_y \sin \varphi|^2 \cos^2 \theta \right).$$

Return back to the $\lambda/4$ radiator, one can take an educated model for the horizontal current (near resonance) as

$$J_x(x, y) = \begin{cases} \frac{I}{W} \sin\left(\frac{\pi x}{2L}\right) & 0 \leq x \leq L, \\ -\frac{W}{2} \leq y \leq \frac{W}{2}, z = H \\ 0 & \text{elsewhere,} \end{cases} \quad (10)$$

and the vertical current

$$J_z(y, z) = \begin{cases} -\frac{I}{W} & x = L, -\frac{W}{2} \leq y \leq \frac{W}{2}, 0 \leq z \leq H, \\ 0 & \text{elsewhere.} \end{cases} \quad (11)$$

I is the current (in Amperes). Taking Fourier transform of $J_x(x, y)$

$$\begin{aligned} \tilde{J}_x(k_x, k_y) &= \iint J_x(x, y) \exp[j(k_x x + k_y y)] dx dy \\ &= \frac{I}{W} \int_0^L \sin\left(\frac{\pi x}{2L}\right) \exp(jk_x x) dx \\ &\quad \times \int_{-W/2}^{W/2} \exp(jk_y y) dy \\ &= I \frac{j \exp(j(\pi/2) \cdot (k_x/k_0)) k_x + k_0}{k_0^2 - k_x^2} \text{sinc}\left(k_y \frac{W}{2}\right). \end{aligned} \quad (12)$$

The vertical current can be replaced by two horizontal contributions [11]. This replacement is exact for the relevant fields on and above the plane $z = H$. The replacement is as follows:

$$\tilde{J}_h(k_x, k_y) = \tilde{J}_v(k_x, k_y) \frac{j\mathbf{k}_t}{\gamma^2}, \quad (13)$$

where $\tilde{J}_h(k_x, k_y) = \tilde{J}_{hx}(k_x, k_y)\hat{\mathbf{x}} + \tilde{J}_{hy}(k_x, k_y)\hat{\mathbf{y}}$ is the Fourier transform of the surface current density which replaces the vertical surface current density:

$$\tilde{J}_{hx}(k_x, k_y) = -I \frac{j \exp(j(\pi/2)(k_x/k_0)) k_x}{k_0^2 - k_x^2 - k_y^2} \text{sinc}\left(k_y \frac{W}{2}\right), \quad (14)$$

$$\tilde{J}_{hy}(k_x, k_y) = -I \frac{j \exp(j(\pi/2)(k_x/k_0)) k_y}{k_0^2 - k_x^2 - k_y^2} \text{sinc}\left(k_y \frac{W}{2}\right), \quad (15)$$

where $\text{sinc}(x) = \sin(x)/x$.

Substituting the surface current densities in the pattern formula, we can calculate the far-field pattern of the idealized $\lambda/4$ microstrip element as function of H and W . Figure 2(a) presents the E -plane power pattern cut and Figure 2(b) shows the H -plane power pattern cut. It is clearly seen that as W and H tend to zero, the pattern cuts become more and more isotropic.

3. The $\lambda/4$ U-Shaped Power Isotropic Radiator

We present here another current source which radiated its power isotropically. This time the calculations are done in configuration (or real) space. The geometry of the current source is shown in Figure 3, where the element is placed in a different position for convenience. The relevant current density is given by

$$\mathbf{J}(x, y, z) = \begin{cases} \delta(x)\delta(y)\delta\left(z + \frac{L}{2}\right)\hat{\mathbf{x}} \\ -\delta'(x)\delta(y) \cos\left(\frac{\pi}{4} + \frac{\pi z}{2L}\right)\hat{\mathbf{z}} & -\frac{L}{2} \leq z \leq \frac{L}{2}, \\ 0 & \text{elsewhere.} \end{cases} \quad (16)$$

The relation between the wavenumber k and the length is given by

$$L = \frac{\lambda}{4} = \frac{\pi}{2k}. \quad (17)$$

Following Elliott's procedure [12], the electric far-field is given by

$$\mathbf{E} = -j\omega\mathbf{A}_t, \quad (18)$$

$$\mathbf{A}_t(r, \theta, \varphi, t) = \frac{\exp[j(\omega t - kr)]}{4\pi\mu_0^{-1}r} \mathcal{A}(\theta, \varphi), \quad (19)$$

FIGURE 2: (a) E -plane radiation pattern ($\phi = 0^\circ$ cut) of the ideal $\lambda/4$ antenna element for $H = W$, measured in wavelengths. The pattern becomes more isotropic as W and H become smaller. (b) H -plane radiation pattern ($\phi = 90^\circ$ cut) of the ideal $\lambda/4$ antenna element for $H = W$, measured in wavelengths. The pattern becomes more isotropic as W and H become smaller.

where $\mathcal{A}(\theta, \varphi)$ is the directional weighting function given by

$$\mathcal{A}(\theta, \varphi) = \int \mathbf{J}(x', y', z') \times \exp[jk(x' \sin \theta \cos \varphi + y' \sin \theta \sin \varphi + z' \cos \theta)] dx' dy' dz'. \quad (20)$$

The $\mathcal{A}_\theta$ and $\mathcal{A}_\varphi$ components are given in our case by

$$\mathcal{A}_\theta(\theta, \varphi) = \cos \theta \cos \varphi \mathcal{A}_x(\theta, \varphi) - \sin \theta \mathcal{A}_z(\theta, \varphi), \quad (21)$$

$$\mathcal{A}_\varphi(\theta, \varphi) = -\sin \varphi \mathcal{A}_x(\theta, \varphi), \quad (22)$$

$$\begin{aligned} \mathcal{A}_x(\theta, \varphi) &= \int \delta(x') \delta(y') \delta\left(\frac{\pi}{4k} + z'\right) \\ &\times \exp[jk(x' \sin \theta \cos \varphi + y' \sin \theta \sin \varphi + z' \cos \theta)] dx' dy' dz' \\ &= \exp\left(-j\frac{\pi}{4} \cos \theta\right), \end{aligned} \quad (23)$$

$$\begin{aligned} \mathcal{A}_z(\theta, \varphi) &= \int \cos\left(\frac{\pi}{4} + z'k\right) \delta'(x') \delta(y') \\ &\times \exp[jk(x' \sin \theta \cos \varphi + y' \sin \theta \sin \varphi + z' \cos \theta)] dx' dy' dz' \\ &= j \frac{\cos \varphi}{\sin \theta} \exp\left(-j\frac{\pi}{4} \cos \theta\right) \\ &\times \left[\exp\left(j\frac{\pi}{2} \cos \theta\right) - j \cos \theta \right], \end{aligned} \quad (24)$$

FIGURE 3: The geometry of a current source which radiates power isotropically. We put $L = \lambda/4$ and take the limit $H \rightarrow 0$ and $W \rightarrow 0$, keeping $I \cdot H = 1$, where I is the maximum value of the current.

hence, we have for the polar components $\mathcal{A}_\theta$, $\mathcal{A}_\varphi$

$$\mathcal{A}_\theta(\theta, \varphi) = -j \cos \varphi \exp\left[j\frac{\pi}{4} \cos \theta\right] \sim E_\theta(\theta, \varphi), \quad (25)$$

$$\mathcal{A}_\varphi(\theta, \varphi) = -\sin \varphi \exp\left[-j\frac{\pi}{4} \cos \theta\right] \sim E_\varphi(\theta, \varphi) \quad (26)$$

and finally

$$|\mathcal{A}_\theta(\theta, \varphi)|^2 + |\mathcal{A}_\varphi(\theta, \varphi)|^2 = 1. \quad (27)$$

Hence, the U-shaped $\lambda/4$ radiator whose thickness H and width W tend to zero radiates its power isotropically in the far-field.

FIGURE 4: (a) Schematic view of an observer, of the far-field polarization of the U-shaped power isotropic radiator at the angles $\theta = 0, \pi/4, \pi/2$, and at multiples of $\pi/8$ of the angle φ . The observer looks into the direction of the negative Z-axis. The polarizations have been drawn by rotating them so that they appear perpendicular to the line of sight. The green drawing represents the direction of the current of the radiator at a given time. The red ellipses represent the electric field, while the blue ellipses the magnetic field. The tip of the arrow (in black) indicates the phase for a given time, and the arrow points in the direction of the rotation of the fields. (b) Similarly as in Figure 4(a), but here the observer looks into the direction of the Y-axis, and the polarizations are drawn for $\theta = 0, \pi/4, \pi/2, 3\pi/4, \pi$, and φ is between $-\pi/8$ and $-\pi/8$ in multiples of $\pi/8$. (c) Similarly as in Figure 4(a), but here the observer looks into the direction of the negative X-axis, $\theta = 0, \pi/4, \pi/2, 3\pi/4, \pi$ and $\varphi = 0, \pm\pi/8, \pm\pi/4, \pm3\pi/8$.

The polarization patterns of the radiator are presented is what. Figures 4(a), 4(b), and 4(c) present a view from directions Z, $-Y$, and X, respectively. The radiator is drawn at the center of the plots. It is shown that the polarization of the radiator strongly depends on the direction.

4. The $\lambda/4$ Double U-Shaped Power Isotropic Radiator

We have found the electric far-fields E_θ, E_φ of the U-shaped $\lambda/4$ radiator and have proved that the total power

FIGURE 5: The double U-shaped radiator. This radiator is composed of two U-shaped radiators, one radiator is rotated by $\pi/2$ radians in the azimuth relative to the second U-shaped radiator, and the phase difference between the currents of the two radiators is $\pi/2$.

radiated is isotropic. However, this element shows different polarizations (linear, elliptical, and circular) at different cuts. In case that much more “regular” polarizations are required, we propose to use a double U-shaped $\lambda/4$ radiator whose geometry is shown in Figure 5. The current density of this radiator is given by

$$\mathbf{J} = \delta(x)\delta(y)\delta\left(z + \frac{\lambda}{8}\right)(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) - [\delta'(x)\delta(y) + j\delta(x)\delta'(y)]\cos\left(\frac{\pi}{4} + \frac{2\pi}{\lambda}z\right)\hat{\mathbf{z}}. \quad (28)$$

The electric far-field components are given by

$$E_\theta = -(\sin\varphi + j\cos\varphi)\exp\left(j\frac{\pi}{4}\cos\theta\right), \quad (29)$$

$$E_\varphi = -(\sin\varphi + j\cos\varphi)\exp\left(-j\frac{\pi}{4}\cos\theta\right). \quad (30)$$

Figures 6(a) and 6(b) describe the view from directions Z and X, respectively. It is shown clearly that the polarization patterns are more symmetric than in the case of the U-shaped radiator. It can be shown that the symmetry of the radiation patterns can be described by known symmetry groups [7].

5. A Finite-Size Finite-Current Density Source Which Radiates Power Isotropically

We have seen that the U-shaped and the double U-shaped power isotropic radiators have infinite-surface current density. In order to remove this infinity, it is possible to replace the infinite-current density source by a finite-equivalent-current density source. A way to do this is to calculate

FIGURE 6: (a) Schematic view of an observer, of the far-field polarization of the double U-shaped power isotropic radiator at the angles $\theta = 0, \pi/4, \pi/2$, and at multiples of $\pi/8$ of the angle φ . The observer looks into the direction of the negative Z-axis. The polarizations have been drawn by rotating them so that they appear perpendicular to the line of sight. The green drawing represents the direction of the current of the radiator at a given time. The red ellipses represent the electric field, while the blue ellipses the magnetic field. The tip of the arrow (in black) indicates the phase for a given time, and the arrow points in the direction of the rotation of the fields. (b) Similarly as in Figure 6(a), but here the observer looks into the direction of the Y-axis, and the polarizations are drawn for $\theta = 0, \pi/4, \pi/2, 3\pi/4, \pi$ and $\varphi = 0, \pm\pi/8, \pm\pi/4, \pm3\pi/8$.

FIGURE 7: A finite-size spherical radiator carrying a finite-surface current density, for which its fields outside the sphere are exactly the same as the fields of the U-shaped $\lambda/4$ power isotropic radiator. The radius of the sphere is $R = \lambda/4$.

an equivalent spherical surface current density, where the radius of the sphere is $R > \lambda/4$ (see Figure 7), where the fields outside the sphere are the same as for the infinite-current density radiator. For convenience, this calculation is performed using Gaussian units. Following [13], we apply the multipole procedure. Let I be the region inside the spherical surface of radius R , and II be the region outside the spherical surface, the fields in region II are given by

$$\mathbf{B}^{\text{II}} = \sum_{l,m} \left[a_{E\text{II}}(l,m) h_l^{(1)}(kr) \mathbf{X}_{lm} - \frac{j}{k} a_{M\text{II}}(l,m) \nabla \times h_l^{(1)}(kr) \mathbf{X}_{lm} \right], \quad r > R, \quad (31)$$

$$\mathbf{E}^{\text{II}} = \sum_{l,m} \left[\frac{j}{k} a_{E\text{II}}(l,m) \nabla \times h_l^{(1)}(kr) \mathbf{X}_{lm} + a_{M\text{II}}(l,m) h_l^{(1)}(kr) \mathbf{X}_{lm} \right], \quad r > R, \quad (32)$$

where

$$h_l^{(1)} = j_l(kr) + j n_l(kr), \quad (33)$$

$j_l(kr)$, $n_l(kr)$ are the spherical Bessel and Neumann functions [14]. The coefficients $a_{E\text{II}}(l,m)$, $a_{M\text{II}}(l,m)$ can be calculated for a given source current by

$$a_{E\text{II}}(l,m) = \frac{4\pi k^2}{j\sqrt{l(l+1)}} \times \int Y_{lm}^* \left\{ \frac{\nabla \cdot \mathbf{J}}{j\omega} \frac{\partial}{\partial r} [r j_l(kr)] + \frac{jk}{c} (\mathbf{r} \cdot \mathbf{J}) j_l(kr) \right\} d^3x, \quad (34)$$

$$a_{M\text{II}}(l,m) = \frac{4\pi k^2}{jc\sqrt{l(l+1)}} \int Y_{lm}^* \nabla \cdot (\mathbf{r} \times \mathbf{J}) j_l(kr) d^3x. \quad (35)$$

Inserting the current density of the U-shaped power isotropic radiator

$$\mathbf{J}(x,y,z) = \begin{cases} \delta(x)\delta(y)\delta(z+\lambda)\hat{\mathbf{x}} \\ -\delta'(x)\delta(y)\cos\left(\frac{\pi}{4} + \frac{2\pi z}{\lambda}\right)\hat{\mathbf{z}} & -\frac{\lambda}{8} \leq z \leq \frac{\lambda}{8}, \\ 0, & \text{elsewhere,} \end{cases} \quad (36)$$

we have the expressions for the coefficients

$$a_{E\text{II}}(l,m) = \begin{cases} -\frac{\sqrt{2\pi(2l+1)}}{c} k^2 B(m) \\ \times \left[I_c(l) + \frac{\sqrt{2}}{2} j_l\left(\frac{\pi}{4}\right) \right], & l = \text{odd}, \\ -\frac{\sqrt{2\pi(2l+1)}}{c} k^2 B(m) \\ \times \left[I_s(l) + \frac{\sqrt{2}}{2} j_l\left(\frac{\pi}{4}\right) \right], & l = \text{even}, \end{cases} \quad (37)$$

$$a_{M\text{II}}(l,m = \pm 1) = \begin{cases} \frac{\sqrt{2\pi(2l+1)}}{c} k^2 \left[I_c(l) + \frac{\sqrt{2}}{2} j_l\left(\frac{\pi}{4}\right) \right], & l = \text{odd}, \\ -\frac{\sqrt{2\pi(2l+1)}}{c} k^2 \left[I_s(l) + \frac{\sqrt{2}}{2} j_l\left(\frac{\pi}{4}\right) \right], & l = \text{even}, \end{cases} \quad (38)$$

and $a_{M\text{II}}(l,m \neq \pm 1) = 0$, where

$$B(m) = \begin{cases} 1, & m = 1, \\ -1, & m = -1, \\ 0, & \text{else,} \end{cases} \quad (39)$$

$$I_c(l) = \frac{\sqrt{2}}{2l} \left\{ \frac{\pi/4}{l+1} \left[j_l\left(\frac{\pi}{4}\right) - j_{l+1}\left(\frac{\pi}{4}\right) \right] + j_l\left(\frac{\pi}{4}\right) \right\}, \quad (40)$$

$$I_s(l) = \frac{\sqrt{2}}{2(l+1)} \times \left\{ \frac{\pi/4}{l} \left[\left(\frac{4}{\pi} - 1\right) j_l\left(\frac{\pi}{4}\right) - j_{l+1}\left(\frac{\pi}{4}\right) \right] + j_l\left(\frac{\pi}{4}\right) \right\}. \quad (41)$$

The details of the calculations are given in [7]. In order to find the surface current density on the surface of the sphere of radius R such that the fields outside the sphere will be the same as the fields of the isotropic radiator, we expand the fields inside the sphere:

$$\mathbf{B}^{\text{I}} = \sum_{l,m} \left[a_{E\text{I}}(l,m) j_l(kr) \mathbf{X}_{lm} - \frac{j}{k} a_{M\text{I}}(l,m) \nabla \times j_l(kr) \mathbf{X}_{lm} \right], \quad r < R, \quad (42)$$

FIGURE 8: Constant amplitude curves of the surface current density on the surface of the spherical radiator drawn in Figure 7.

FIGURE 9: Convergence of the multipole expansion of the far-field power for the isotropic spherical radiator. It is shown in two scales: in a coarse scale (a) and in a fine scale (b), for $0 < \theta < \pi$ and $\varphi = \pi/4$. It is shown that 6 multipole terms are enough for excellent convergence of the far-field power.

$$\mathbf{E}^I = \sum_{l,m} \left[\frac{j}{k} a_{E_l}(l, m) \nabla \times j_l(kr) \mathbf{X}_{lm} + a_{M_l}(l, m) j_l(kr) \mathbf{X}_{lm} \right],$$

$$r < R.$$
(43)

We can find the coefficients $a_{E_l}(l, m)$, $a_{M_l}(l, m)$ from the equation

$$\mathbf{E}_t^I = \mathbf{E}_t^{II}, \quad r = R, \quad (44)$$

then $\mathbf{B}^I$ is known, and the surface current density on the sphere is given by

$$\begin{aligned}
 \mathbf{J}(R, \theta, \varphi) &= \hat{\mathbf{n}} \times (\mathbf{B}_t^{\text{II}} - \mathbf{B}_t^I) \\
 &= \sum_{l,m} \left\{ a_{E_{ll}}(l, m) \right. \\
 &\quad \times \left[h_l^{(1)}(kr) - \frac{(l+1)h_l^{(1)}(kr) - kRh_{l+1}^{(1)}(kr)}{(l+1)j_l^{(1)}(kr) - kRj_{l+1}(kr)} \right] \hat{\mathbf{n}} \times \mathbf{X}_{lm} \\
 &\quad + \frac{j}{kR} a_{M_{ll}}(l, m) \\
 &\quad \times \left[(l+1)h_l^{(1)}(kr) - kRh_{l+1}^{(1)}(kr) - \frac{h_l^{(1)}(kr)}{j_l(kR)} \right. \\
 &\quad \left. \left. \times [(l+1)j_l(kr) - kRj_{l+1}(kr)] \right] \mathbf{X}_{lm} \right\}, \quad (45)
 \end{aligned}$$

where the coefficient in region I has been expressed in terms of the coefficients in region II [7].

Figure 8 describes constant surface current density curves on the sphere surface.

The far-field can be expressed in terms of the multipole expansion by

$$\begin{aligned}
 \frac{dP}{d\Omega} &= \frac{c}{4\pi k^2} \\
 &\times \left| \sum_{l,m} (-j)^{l+1} [a_{E_{ll}}(l, m) \mathbf{X}_{lm} \times \hat{\mathbf{n}} + a_{M_{ll}}(l, m) \mathbf{X}_{lm}] \right|^2. \quad (46)
 \end{aligned}$$

Figure 9 shows the first 6 terms in the former expansion are enough for an excellent convergence. In the graphs we see the far-field power density as function of θ for $\varphi = \pi/4$ as an example.

6. Conclusions

The $\lambda/4$ microstrip element has been investigated in the limit case where its width and its height above ground tend to zero. We have shown that in this case the far-field power density of the radiator tends to be isotropic in half space. Adding the image of this element, we have the U-shaped power isotropic radiator. The polarization pattern of the U-shaped radiator has low symmetry; hence, we have presented the double U-shaped power isotropic radiator, for which the polarization pattern is better. In order to remove the infinity in the current density of the radiator, we change the U-shaped radiator by a power isotropic spherical surface, having finite-surface current. However, we have to learn how to control, in practice, the details of the current density of the antenna.

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