Research Article

Resonant Frequency Calculation and Optimal Design of Peano Fractal Antenna for Partial Discharge Detection

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Ultra-high-frequency (UHF) approaches have caught increasing attention recently and have been considered as a promising technology for online monitoring partial discharge (PD) signals. This paper presents a Peano fractal antenna for UHF PD online monitoring of transformer with small size and multiband. The approximate formula for calculating the first resonant frequency of the Peano fractal antenna is presented. The results show that the first resonant frequency of the Peano fractal antenna is smaller than the Hilbert fractal antenna when the outer dimensions are equivalent approximately. The optimal geometric parameters of the antenna were obtained through simulation. Actual PD experiments had been carried out for two typically artificial insulation defect models, while the proposed antenna and the existing Hilbert antenna were both used for the PD measurement. The experimental results show that Peano fractal antenna is qualified for PD online UHF monitoring and a little more suitable than the Hilbert fractal antenna for pattern recognition by analyzing the waveforms of detected UHF PD signals.

1. Introduction

Partial discharge (PD) online monitoring is an effective approach to inspect insulation defects and identify potential faults in power transformer [1]. Hence, it is important for monitoring PD signals online for power transformer. Compared with traditional detection methods, the ultra-high-frequency (UHF) technology has advantages such as high sensitivity and strong anti-interference, which make it more suitable for PD online monitoring [2]. By receiving the UHF electromagnetic waves of PD occurred in a power transformer, the UHF detection technology can measure the PD magnitudes and locate the PD source [3–7].

Antenna is the core component of an UHF PD online monitoring system. The performance of antenna will affect the extraction and postprocessing of PD signals. Currently, there are many types of UHF antennas used in PD detection for electrical plants. Literatures [8, 9] presented a two-wire Archimedean planar spiral antenna and its application in PD detection. A dipole antenna model and its waveform characteristics were introduced in [10], and a small loop

antenna was given in [11] to detect PD signals in transformer insulation oil. In addition to transformers, UHF antennas have been used for PD detection for other high voltage apparatuses. The horn antenna, biconical log-periodic antenna, loop antenna, and dipole antenna were used for PD detection for gas insulated switchgear (GIS) [12, 13].

Two criteria have to be considered for design of UHF antennas detecting PD in transformer [14]. On the one hand, the resonant frequencies of UHF PD antennas are required to fall into a lower range between 300 MHz and 1000 MHz with a wide bandwidth [5]. The lower first resonant frequency is important for the fractal antenna used in detecting UHF PD signals. Publication [15] presented the fundamental frequencies of Hilbert fractal antenna, while the calculated formula was presented in publication [16]. On the other hand, for the purpose of not affecting the safe operation of transformers and the convenience of installation, an antenna as small as possible is needed. The fractal antenna showed superior in these two respects, and publication [17] presented a compact Hilbert fractal antenna for UHF PD detection for power transformer. Literature [18] presented that the Peano fractal

antenna resonated at a lower fundamental frequency than the same order Hilbert antenna. It is expected that the outer dimension of Peano antenna is smaller than Hilbert antenna when their performances are both good.

This paper presents an approximate resonant frequency calculation formula and optimal design of UHF Peano fractal antenna for online monitoring PD of power transformers. The operation principle and the approximate resonant frequency calculation formula of the antenna are proposed. Besides, the antenna optimal design procedure is also addressed in the paper. The performances of the optimal antenna are given and discussed through simulation. To validate its performance, actual experiments were carried out on the proposed antenna and the existing Hilbert antenna for PD measurements of two typically artificial oil-paper defects in laboratory. The compared results show that the Peano fractal antenna is a little more suitable than the Hilbert fractal antenna for PD online UHF monitoring. The paper is organized as follows: Section 2 proposes the approximate resonant frequency calculation formula of the Peano fractal antenna. The actual optimal design procedure of antenna is given in Section 3. Section 4 presents the experiments and the experimental results. The conclusions are given in Section 5.

2. Resonant Frequency of Peano Fractal Antenna

Design of Peano fractal antennas is based on Peano fractal curves. Figure 1 shows a set of Peano fractal curves from the first to the third order. A Peano fractal curve is a continuous curve with a characteristic of strict self-similarity [19]. It is clear that the length of a Peano fractal curve is greater if the order of the curve is higher. If a Peano fractal curve has an infinite order, the curve will fill out all the space of the two-dimensional plane. For a Peano antenna with a side dimension L and an order of n , the length of each line segment d (shown in Figure 1) and the sum of all the line segments S are given by:

$$d = \frac{L}{3^n - 1}; \quad S = (3^{2n} - 1)d = (3^n + 1)L. \quad (1)$$

The resonant frequency calculated formula of the meander line antennas can be referred to [20]. Peano fractal wires are divided into parallel wire section, short circuit termination, and additional wire section, which are illustrated in Figure 2.

In a Peano fractal geometry of order n , there are m short circuited parallel wire sections, which can be expressed as follows:

$$m = \frac{3^{2n} - 1}{4}. \quad (2)$$

The length of the line segments s except the parallel wire sections is expressed as follows:

$$s = \frac{3^{2n} - 1}{2}d. \quad (3)$$

FIGURE 1: Peano curves from the first to the third order.

The characteristic impedance of a parallel wire transmission line consisting of wires with diameter b , spacing d is expressed as follows:

$$Z_0 = \frac{Z_c}{\pi} \log \frac{2d}{b}, \quad (4)$$

where Z_c is the intrinsic impedance of free space, $Z_c = 120 \pi \Omega$. Z_0 can be used to calculate the input impedance at the ends of the line, which is purely inductive;

$$L_{in,s} = \frac{Z_0}{\omega} \tan \beta d, \quad (5)$$

where ω is angular frequency, and $\omega = 2\pi f$, β is phase constant, and $\beta = 2\pi/\lambda$ and λ is the wavelength of the electromagnetic wave. The total input impedance of parallel wire transmission line of a Peano fractal antenna with n order can be expressed by

$$L_{in} = m \cdot \frac{Z_c}{\pi \omega} \cdot \log \frac{2d}{b} \cdot \tan \beta d. \quad (6)$$

When d is sufficiently small compared to the wavelength of the electromagnetic wave, $\tan(\beta d)$ can be expressed by the following Taylor formula [19]:

$$\tan \beta d = \beta d + \frac{1}{3}(\beta d)^3 + \frac{1}{5}(\beta d)^5 + \dots \quad (7)$$

The self-inductance due to a straight line of lengths as defined in (3) is

$$L_s = \frac{\mu_0}{2\pi} \cdot s \cdot \left(\log \frac{4s}{b} - 1 \right). \quad (8)$$

The total inductance of a Peano antenna with n orders is expressed as follows:

$$L_T = m \cdot \frac{Z_c}{\pi \omega} \cdot \log \frac{2d}{b} \cdot \tan \beta d + \frac{\mu_0}{2\pi} \cdot s \cdot \left(\log \frac{4s}{b} - 1 \right). \quad (9)$$

The total inductance of fractal antenna equals to inductance of the half-wave dipole antenna approximately referenced to publication [15]. And the inductance of the half-wave dipole antenna is expressed by

$$L_d = \frac{\mu_0}{\pi} \cdot \frac{\lambda}{4} \cdot \left(\log \frac{2\lambda}{b} - 1 \right), \quad (10)$$

FIGURE 2: Composition of Peano fractal wires for calculating the resonant frequency.

where μ_0 is the permeability of vacuum and equals to $4\pi \times 10^{-7} \text{ Hm}^{-1}$, for half-wave dipole antenna, $\lambda = 2L$. The resonant frequencies of the Peano fractal antenna with n order are calculated by the equation $L_T \approx L_d$. If the equivalent arm length of dipole antenna is changed, the multi-resonant frequencies can be obtained. All resonant frequencies of the Peano fractal antenna with n orders are obtained as follows:

$$\begin{aligned}
 m \cdot \frac{Z_c}{\pi\omega} \cdot \log \frac{2d}{b} \tan \beta d + \frac{\mu_0}{2\pi} \cdot s \cdot \left(\log \frac{4s}{b} - 1 \right) \\
 = \frac{\mu_0}{\pi} \cdot \frac{k\lambda}{4} \cdot \left(\log \frac{2k\lambda}{b} - 1 \right) \quad (11) \\
 f_r = \frac{c}{\lambda}.
 \end{aligned}$$

where c is velocity of light, $c = 3 \times 10^8 \text{ m/s}$, k is an odd number.

This paper focuses on the calculation for the first resonant frequency of the Peano fractal antenna. With (11), the first resonant frequency of the Peano fractal antenna with n order can be calculated by (12) as follows:

$$\begin{aligned}
 m \cdot \frac{Z_c}{\pi\omega} \cdot \log \frac{2d}{b} \cdot \beta d + \frac{\mu_0}{2\pi} \cdot s \cdot \frac{\mu_0}{2\pi} \cdot s \cdot \left(\log \frac{4s}{b} - 1 \right) \\
 = \frac{\mu_0}{\pi} \cdot \frac{\lambda}{4} \cdot \left(\log \frac{2\lambda}{b} - 1 \right) \\
 f_r = \frac{c}{\lambda}. \quad (12)
 \end{aligned}$$

It is clear that the first resonant frequency of the fractal antenna is mainly related to the order and side dimension of the antenna and width of conductor. Table 1 shows the first resonant frequencies of the Peano and Hilbert fractal antennas with different parameters calculated by (12), respectively.

FIGURE 3: Feed points selected of Peano fractal antenna for simulation.

FIGURE 4: The third Peano fractal antenna: (a) front face of antenna, (b) back of antenna.

FIGURE 5: VWSR curve of the Peano fractal antenna.

FIGURE 6: Input impedance of the Peano fractal antenna.

FIGURE 7: 3 D Radiation patterns at the select frequency.

The results show that the first resonant frequencies of fractal antennas become lower with the order increasing, which are in accord with the conclusions presented in publication [18]. Furthermore, the outer dimension of the third order Peano antenna is smaller than the fourth order Hilbert antenna when they resonate at the similar fundamental frequency. Since the lowest frequency of UHF PD signals is about

(a) 370 MHz

(b) 700 MHz

FIGURE 8: 2 D Radiation patterns at the select frequency.

300 MHz, it is then necessary to have a third order Peano fractal antenna to detect PDs in power transformers.

3. Optimal Design of Peano Fractal Antenna

Previous research results [14] show that the performance of a fractal antenna is affected by many factors such as the side dimension (L), thickness (k) of the print circuit board (PCB), width of conductor (b), feed point, and dielectric constant of the PCB. To obtain a Peano antenna with desired performance, the above factors need to be included and optimized in the design procedure.

A Peano fractal antenna with desirable performance and size for detecting PDs in transformers can be designed synthetically through simulation studies. The simulation model

FIGURE 9: Two types of artificial defect models: (a) corona-in-oil model, (b) surface discharging-in-oil model.

FIGURE 10: The PD experiment setup in laboratory.

TABLE 1: Resonant frequencies of Peano and Hilbert fractal antennas with different geometry parameters.

Antennas	L (mm)	n	b (mm)	f_r (MHz)
Peano	90	2	2	247.32
		3	2	102.89
	70	2	2	323.73
		3	2	135.50
Hilbert	100	2	2	395.51
		3	2	242.92
		4	2	141.37
		2	2	441.73
	90	3	2	271.78
		4	2	158.41
		2	2	575.45
		3	2	355.63
		4	2	208.09

TABLE 2: Different widths of conductor and thicknesses of PCB for antennas with different side dimension.

L (mm)	k (mm)			b (mm)		
	Min	Step	Max	Min	Step	Max
60	1.0	—	1.0	1.0	0.5	3.0
70	1.0	0.1	1.5	1.0	0.5	3.0
80	1.0	0.1	1.5	1.0	0.5	3.0
90	1.0	0.1	2.0	1.0	0.5	3.0
100	1.0	0.1	2.0	1.0	0.5	3.0

in Ansoft contains 3 layers. The upper layer is filled with Peano curves (see Figure 1) constituted by copper; the middle layer is a board of insulating material, which is FR4 epoxy board with dielectric constant of 4.4. The down layer is a copper grounding shield.

The optimal UHF PD antenna should be with small size and wide frequency bandwidth, which was depicted in Section 1. The optimal process of a Peano fractal antenna is shown as follows. Firstly, five different side dimensions of Peano antenna were selected for simulation, $L = 60$ mm, 70 mm, 80 mm, 90 mm, and 100 mm. For each side dimension, different widths of conductor were explored. Other factors such as thickness of PCB feed points were also simulated for the voltage standing wave ratio (VSWR), gain, and radiation pattern. The parameters used for simulation are given in Table 2. Because the Peano curve is symmetrical, 25 feed points on half of the curve are obtained as the simulation condition, which are shown in Figure 3. Parameter r is used to describe the relative locations of these feed points. r is defined as the ratio of the distance along the conductor from a feed point and its closest end to the total conduct length of the antenna. By the simulations, the optimal antenna was selected with the smallest size and the widest frequency bandwidth. The parameters of the optimal antenna are determined as $L = 90$ mm, $k = 2$ mm, $b = 2$ mm, and $r = 0.059$ (i.e., point 3 in Figure 3).

Figure 4 shows the prototype of the designed third order Peano fractal antenna. Performance curves (e.g., voltage

FIGURE 11: Waveforms of UHF PD signals from the two defects: (a) signals from corona and surface models detected by Hilbert antenna; (b) signals from corona and surface models detected by Peano antenna.

TABLE 3: PD experiment conditions.

Defect model	Inception voltage (kV)	Breakdown voltage (kV)	Test voltage (kV)	Sample numbers
Corona discharge	5.7	12.5	7.0	50
			8.0	50
			9.0	50
Surface discharge	8.4	13.2	9.0	50
			10.0	50
			11.0	50

standing wave ratio (VSWR), input impedance, and radiation patterns) of the antenna are given from Figures 5 to 8. Figure 5 shows that between 0.3 GHz and 1 GHz the multiband antenna has 2 resonant frequencies (370 MHz, 700 MHz), where $VSWR < 5$. The pass frequency bands of the antenna are approximate 340 MHz~580 MHz, 650 MHz~740 MHz, and 920 MHz~1000 MHz. Figure 6 shows the input impedance of the antenna. It is noted that the absolute

value of real part is about 50 ohms, and the absolute value of imaginary part is less than 10 ohm when frequencies are within the bandwidth of the antenna. The results show that the antenna can match with a 50 ohms coaxial cable as needed. The three-dimensional radiation patterns and two-dimensional radiation patterns ($\phi = 0$ and 90°) at different frequencies, namely, 370 MHz and 700 MHz, are shown in Figures 7 and 8. Its patterns at the two frequencies are all nearly a hemisphere, and the gain variations at the two frequencies are relatively stable. The simulated results show that the optimal Peano fractal antenna has desirable performance with nearly wide frequency bandwidth but smaller size in comparison with the Hilbert fractal antenna reported in [14].

Figure 8 shows the minimum gain of the antenna is about -18 dBi. Besides, the detected UHF PD signals will be transferred to the processing center by the coaxial cable with the length of tens of meters. It is motivated to develop a signal processing circuit with an amplifier and a filter for the wideband detection in the frequency range between 300 MHz and 1 GHz. The gain of the amplifier is about 40 dB

FIGURE 12: Normalized power frequency spectra of UHF PD signals from the two models: (a) signals from corona and surface models detected by Hilbert antenna; (b) signals from corona and surface models detected by Peano antenna.

between 300 MHz and 1 GHz, and the gain of the whole UHF PD system is about 20 dBi.

4. Experiments and Results

To validate the performance of the designed UHF Peano fractal antenna, actual PD experiments with two typical transformer insulation defects were carried out in laboratory. The Peano and Hilbert antennas were both used to detect PD signals, as presented as follows. The performance of the existing Hilbert antenna is referred to [14].

4.1. Defect Models Experiments. There are two types of defect models built in experiment to generate UHF PD signals. Figure 9(a) shows the corona discharge model, which basically is a needle-to-plate electrode system. Figure 9(b) shows an experiment model of a cylinder-to-board electrode for surface discharge defects in oil. The thickness of the pressboard of each model is 0.5 mm. The experiment setup of UHF PD detection is shown in Figure 10. The artificial defect models were put into a container filled with transformer oil,

and the experiments were carried out in an electromagnetic shielded laboratory. The UHF antenna was placed beside the testing models. A digital oscilloscope was used to observe and record the UHF PD signals. The sampling frequency of the oscilloscope for recording the UHF PD signals was 5 GHz.

Table 3 shows the inception voltages, breakdown voltages, test voltages, and sample numbers of the two defect models in experiments. The Peano fractal antenna and the existing Hilbert antenna detected the PD signals at the same time. The dimension of the existing Hilbert antenna is 100 mm, and the pass frequency bands are about 450 MHz~610 MHz and 750 MHz~1000 MHz. When the test voltages were higher than the inception voltages, the transient UHF PD signals were detected by the antennas. The number of the PD samples was 50 for each model. One UHF PD signal was obtained at each voltage for every sample.

4.2. Analysis of UHF PD Waveforms. The differences in frequency spectra of UHF PD signals generated from the same defected model are significantly smaller than those generated from different types of defected models. Thus Figure 11

shows the examples of detected UHF PD signals of the two defect models by the two antennas. The UHF PD signals look similar but differ in details. The examples of normalized power frequency spectra of the measured UHF PD signals, generated by the two defect models, detected by the two antennas, are shown in Figure 12. The results show that the Peano fractal antenna with smaller dimension is also qualified for UHF PD detection. Besides, the spectra of the UHF PD signals detected by the proposed antenna even are a little wider than that detected by the Hilbert antenna, especially for the UHF PD signal generated by the corona discharge model. This implies that the Peano fractal antenna is a little more suitable than the Hilbert fractal antenna for pattern recognition by analyzing the waveforms of detected UHF PD signals.

5. Conclusions

This paper presents a compact multiband UHF Peano fractal antenna for PD online monitoring of high voltage power transformers. The approximate formula for calculating the first resonant frequency of the Peano fractal antenna was presented. The actual antenna was developed based on the optimal design procedure. The actual PD experiments were carried out to verify the performance of the antenna. The results of the work are concluded as follows.

- (a) In comparison with the first resonant frequency of the Hilbert fractal antenna calculated by the formula, the outer dimension of the third order Peano antenna is smaller than the fourth order Hilbert antenna when they resonate at the similar fundamental frequency. This implies that the outer dimension of the Peano fractal antenna is smaller than the Hilbert fractal antenna when their performances are similar.
- (b) The frequency passband of the developed Peano fractal antenna is hundreds of MHz. The radiation patterns show that the antenna can receive electromagnetic waves from the front of the antenna. The actual PD experiments including two typically artificial oil-paper defects were carried out to verify the performance of the antenna. In comparison with the existing Hilbert fractal antenna, the experimental results show that the proposed antenna with smaller dimension is also effectively applied for PD online monitoring of transformers.
- (c) The spectra of the UHF PD signals detected by the two antennas show that the PD signals measured by the UHF Peano fractal antenna are a little wider than that detected by the Hilbert antenna, especially for the corona discharge. It implies that the Peano fractal antenna is a little more suitable for pattern recognition by analyzing the waveforms of detected UHF PD signals.

In the future, there is still scope for improvement in manufacturing a compact fractal antenna with higher gain. The modeling of the fractal antenna including the dielectric loading effect will be investigated. Further studies are also

needed to establish protocols for recognition of UHF PD signals.

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