Research Article

MIMO Antenna Array Design with Polynomial Factorization

Wen-Qin Wang, Huaizong Shao, and Jingye Cai

*School of Communication and Information Engineering, University of Electronic Science and Technology of China (UESTC),
Chengdu 611731, China*

Correspondence should be addressed to Wen-Qin Wang; wqwang@uestc.edu.cn

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One of the main advantages of multiple-input multiple-output (MIMO) antenna is that the degrees-of-freedom can be significantly increased by the concept of virtual antenna array, and thus the MIMO antenna array should be carefully designed to fully utilize the virtual antenna array. In this paper, we design the MIMO antenna array with the polynomial factorization method. For a desired virtual antenna array, the polynomial factorization method can optimally design the specified MIMO transmitter and receiver. The array performance is examined by analyzing the degrees-of-freedom and statistical output signal-to-interference-plus-noise ratio (SINR) performance. Design examples and simulation results are provided.

1. Introduction

Multiple-input multiple-output (MIMO) antenna array has received much attention in communication, radar, and navigation systems [1–4]. MIMO idea is not new; its origin in control systems can be traced back to 1970s [5]. The early 1990s saw an emergence of MIMO ideas into the field of wireless communication systems [6–8]. More recently, the ideas of MIMO appear in sensor and radar systems [9–12]. Different from conventional phased-array systems [13], in MIMO antenna systems each antenna transmits a unique waveform, orthogonal to the waveforms transmitted by other antennas, and a matched filter bank is used to extract the orthogonal waveform components. When the orthogonal signals are transmitted from different antennas, the return of each orthogonal signal will carry independent information. The phase difference caused by different transmitting antennas along with the phase differences caused by different receiving antennas can form a new virtual antenna array steering vector. With optimally designed antenna array geometry, we can create a very long array steering vector with a small number of antennas [14, 15]. More importantly, this provides good flexibility and reconfigurability in array configuration, thus enabling a flexible and reconfigurable system.

Although MIMO antenna arrays are widely used in various applications, radar-related application is assumed in

this paper. Even so, the rich literature of research in this area appears in threefold: (a) the comparison between conventional phased-array and MIMO array, and the development of MIMO architecture [16–18]; (b) MIMO waveform diversity and transmitter design [19, 20]; and (c) the system performance and the receiver design for enhanced performance [21, 22]. In fact, one of the main advantages of MIMO antenna array is that the degrees-of-freedom can be significantly increased by the concept of virtual antenna array, and thus the MIMO antenna array should be carefully designed to fully utilize the virtual antenna array. This paper focuses on the MIMO sparse antenna array design to obtain a uniform virtual array for the system.

Many array design methods have been proposed for conventional phased arrays [23–26]. Specially, linear programming-based algorithms were proposed by [27, 28] for reduced sidelobe thinned array design. Genetic algorithm was proposed by [29] to optimally design thin large phased arrays. Simulated annealing-based technique was proposed by [30] to synthesize the position and weight coefficients of a linear array that minimizes peak sidelobes. A hybrid approach combining particle swarm optimization with combinational techniques was proposed by [31] to synthesize planar thinned arrays. Recently a general polynomial factorization-based design of sparse periodic linear arrays was proposed by [32] to design sparse periodic arrays. In

the method, transmit and receive aperture polynomials are selected such that their product results in a polynomial representing the desired combined transmit/receive effective aperture function.

In this paper, we use the polynomial factorization method proposed in [32] to design MIMO antenna array and comparatively analyze the designed array performance. To obtain the desired virtual MIMO antenna array, we use the polynomial factorization-based method to design the transmit and receive antenna arrays. Since there may be multiple solutions for the transmit and receive antenna arrays and different solutions may have different array performance, we use adaptive beamforming algorithm to comparatively analyze the statistical output signal-to-interference-plus-noise ratio (SINR) performance of the designed arrays. Extensive design examples and simulation results are also provided.

The remaining sections are organized as follows: after introducing the polynomial factorization in Section 2, we describe the polynomial factorization-based MIMO antenna array design in Section 3. Next, the output SINR performance of the designed MIMO antenna arrays is analyzed by adaptive beamforming on the receiver in Section 4, and design examples and simulation results are provided in Section 5. Finally, this paper is concluded in Section 6 with a short discussion of future work.

2. General Polynomial Factorization Method

The polynomial factorization method is based on factorizing a polynomial $P_M(x)$ of degree M in positive powers of x and with unity coefficients [33]

$$P_L(x) = \sum_{l=0}^L x^l. \quad (1)$$

Let the positive integer of the number of coefficients $L + 1$ be expressible as a product of $K + 1$ irreducible positive integers $L_k + 1$, $0 \leq k \leq K$:

$$L + 1 = \prod_{k=0}^K (L_k + 1). \quad (2)$$

Considering the product $P_{L_0}(x)P_{L_1}(x^{S_0})$, we have

$$\begin{aligned} P_{L_0}(x)P_{L_1}(x^{S_0}) &= \left(\sum_{l=0}^{L_0} x^l \right) \left(\sum_{l=0}^{L_1} x^{lS_0} \right) \\ &= \sum_{l=0}^{L_0} x^l + x^{S_0} \left(\sum_{l=0}^{L_0} x^l \right) \\ &\quad + x^{2S_0} \left(\sum_{l=0}^{L_0} x^l \right) + \cdots + x^{L_1 S_0} \left(\sum_{l=0}^{L_0} x^l \right) \\ &= \sum_{l=0}^{L_1 S_0 + L_0} x^l. \end{aligned} \quad (3)$$

Therefore, $P_L(x)$ can be expressed as [32, 33]

$$P_L(x) = \prod_{k=0}^K P_{L_k}(x^{S_k}), \quad (4)$$

with

$$S_k = \prod_{i=0}^k (L_{i-1} + 1), \quad (5)$$

where $L_{-1} = 0$.

The factorization of a specified $L + 1$ into a product of irreducible integers can be carried out using Euclid's algorithm [34]. When $L + 1$ can be expressed as a power of two, that is, $L + 1 = 3^v$, $P_L(x)$ can be factorized as

$$P_L(x) = (1 + x)(1 + x^2) \cdots (1 + x^{2^{v-1}}). \quad (6)$$

Note that the solutions to the cases when $L + 1$ cannot be expressed as a power of two can be found in [32]. The decomposition in (6) has been used in designing a sparse antenna array with a uniform aperture function [35] and a linearly tapered aperture function [36].

As an example, we suppose $L + 1 = 10$; using Euclid's algorithm we then have

$$L + 1 = 10 = 2 \times 5 = 5 \times 2. \quad (7)$$

Then, according to the above polynomial factorization method, we can get

$$\begin{aligned} P_L(x) &= (x^0 + x^1) \left[(x^2)^0 + (x^2)^1 + (x^2)^2 + (x^2)^3 + (x^2)^4 \right] \\ &= (x^0 + x^1)(x^0 + x^2 + x^4 + x^6 + x^8) \\ &= (1 + x)(1 + x^2 + x^4 + x^6 + x^8) \end{aligned} \quad (8)$$

or

$$\begin{aligned} P_L(x) &= (x^0 + x^1 + x^2 + x^3 + x^4) \left[(x^5)^0 + (x^5)^1 \right] \\ &= (x^0 + x^1 + x^2 + x^3 + x^4)(x^0 + x^5) \\ &= (1 + x + x^2 + x^3 + x^4)(1 + x^5). \end{aligned} \quad (9)$$

That is to say, there may be multiple possible factorizations for a given polynomial.

3. Polynomial Factorization-Based MIMO Array Design

Consider a MIMO radar system with a transmitter equipped with M colocated antennas and a receiver equipped with N colocated antennas. Suppose that both the transmit and receive arrays are close to each other in space (possibly the same array) so that they see targets at the same directions. The

signal received by each antenna is the weighted summation of all the transmitted waveform

$$x_n(t) = \sum_{m=1}^M \alpha_{n,m} \phi_m(t), \quad m = 1, 2, \dots, M; \quad n = 1, 2, \dots, N, \quad (10)$$

where $x_n(t)$ is the received signal at the n th antenna, $\phi_m(t)$ is the transmitted waveform at the m th antenna, and $\alpha_{n,m}$ is the channel coefficient with the m th antenna as input and the n th antenna as output. Suppose $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)]$ and $\boldsymbol{\phi}(t) = [\phi_1(t), \phi_2(t), \dots, \phi_N(t)]$, and the above equation can be rewritten in a matrix

$$\mathbf{x}(t) = \sigma_s [\mathbf{a}^T(\theta) \boldsymbol{\phi}(t)] \mathbf{b}(\theta), \quad (11)$$

where θ is the target direction, σ_s is the complex-valued reflection coefficient of the focal point θ_s , $(\cdot)^T$ is the transpose operator, and $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$ are the actual transmit and receive steering vectors associated with the direction θ .

When the transmitted waveforms are orthogonal,

$$\int \phi_m(t) \phi_n^*(t) dt = \begin{cases} \delta(t), & m = m^* \\ 0, & m \neq m^*, \end{cases} \quad (12)$$

where $()^*$ denotes a conjugate operator. The radar return due to the m th transmitted waveform can be separated by matched-filtering the received signal with $\phi_m(t)$, $m = 1, 2, \dots, M$; that is,

$$\mathbf{s}_m \triangleq \int_{T_0} \mathbf{x}(t) \phi_m^*(t) dt, \quad m = 1, 2, \dots, M, \quad (13)$$

where T_0 is the radar pulse width. Therefore, there are a total of $M \times N$ independent signals, which can be written as a $MN \times 1$ virtual data vector

$$\mathbf{y} \triangleq [\mathbf{s}_1^T, \mathbf{s}_2^T, \dots, \mathbf{s}_M^T]^T = \sigma_s \mathbf{a}(\theta) \otimes \mathbf{b}(\theta), \quad (14)$$

where $\otimes$ is the Kronecker product; that is, the virtual antenna array is

$$\mathbf{v}(\theta) = \mathbf{a}(\theta) \otimes \mathbf{b}(\theta). \quad (15)$$

Suppose that the m th transmitting antenna is located at $x_{T,m} = (\lambda/2)u_m$ and the n th receiving antenna is located at $x_{R,n} = (\lambda/2)v_n$, where λ is the wavelength. Considering a far-field point target, the transmitter and receiver steering vectors can be represented, respectively, by

$$\begin{aligned} \mathbf{a}(\theta) &= [e^{ju_1\pi \sin \theta}, e^{ju_2\pi \sin \theta}, \dots, e^{ju_M\pi \sin \theta}]^T, \\ \mathbf{b}(\theta) &= [e^{jv_1\pi \sin \theta}, e^{jv_2\pi \sin \theta}, \dots, e^{jv_N\pi \sin \theta}]^T. \end{aligned} \quad (16)$$

Note that the first antenna of $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$, respectively, is the reference antenna. According to (15), we can get

$$\mathbf{v}(\theta) = \begin{bmatrix} e^{j(v_1+u_1)\pi \sin \theta} & e^{j(v_1+u_2)\pi \sin \theta} & \dots & e^{j(v_1+u_M)\pi \sin \theta} \\ e^{j(v_2+u_1)\pi \sin \theta} & e^{j(v_2+u_2)\pi \sin \theta} & \dots & e^{j(v_2+u_M)\pi \sin \theta} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(v_N+u_1)\pi \sin \theta} & e^{j(v_N+u_2)\pi \sin \theta} & \dots & e^{j(v_N+u_M)\pi \sin \theta} \end{bmatrix}. \quad (17)$$

The array response in the n th receiving antenna for the m th transmitted waveform can be expressed as

$$v_{m,n}(\theta) = e^{j(v_n+u_m)\pi \sin \theta}. \quad (18)$$

It can be noticed that the MIMO antenna array response is the same as the target response received by a receiving array with MN antennas located at

$$\{x_{T,m} + x_{R,n}\}, \quad m = 1, 2, \dots, M, \quad n = 1, 2, \dots, N. \quad (19)$$

This MN -element array is just the virtual MIMO array. An utmost of MN -element virtual array can be obtained by using only $M + N$ physical antenna elements. It is as if we have a receiving array of MN elements.

According to (18), the far-field radiation pattern at an angle θ away from the broadside of the virtual MIMO array with uniform weighting is expressed as

$$\begin{aligned} P_V(\theta) &= \sum_{m=1}^M \sum_{n=1}^N e^{j(v_n+u_m)\pi \sin \theta} \\ &= \sum_{m=1}^M e^{ju_m\pi \sin \theta} \sum_{n=1}^N e^{jv_n\pi \sin \theta} \\ &= P_T(\theta) P_R(\theta), \end{aligned} \quad (20)$$

where $P_T(\theta) = \sum_{m=1}^M e^{ju_m\pi \sin \theta}$ and $P_R(\theta) = \sum_{n=1}^N e^{jv_n\pi \sin \theta}$ denote, respectively, the far-field radiation pattern for the transmitter and receiver. For a more general format, by substituting $x = e^{j2\pi \sin \theta d/\lambda}$ with d being the interelement spacing, the virtual array, transmit array, and receive array radiation patterns can be expressed, respectively, as

$$P_V(x) = \sum_{i=0}^{M+N-1} w_V(i) x^i, \quad (21a)$$

$$P_T(x) = \sum_{i=0}^{M-1} w_T(i) x^i, \quad (21b)$$

$$P_R(x) = \sum_{i=0}^{N-1} w_R(i) x^i, \quad (21c)$$

where $w_V(i)$, $w_T(i)$, and $w_R(i)$ denote the element weighting functions for the virtual array, transmit array, and receive array, respectively. It follows also that [32]

$$P_V(x) = P_T(x) P_R(x). \quad (22)$$

Therefore, the far-field radiation pattern of the virtual array is equal to the product of the transmitter radiation pattern and receiver radiation pattern. This means that we can use different transmit and receive arrays with a combined virtual MIMO array that is a close equivalent to that of a single full array with no missing elements. To reach this aim, like the general polynomial factorization-based design of sparse periodic linear array discussed in [32], we can design the transmit and receive arrays based on factorizing the desired

FIGURE 1: Illustration of the MIMO antenna array design steps.

MIMO virtual array polynomial $P_V(x)$ as a product of two lower order polynomials with some zero-valued coefficients. In this paper, we aim to optimally design the transmit and receive arrays for a given virtual MIMO antenna array. The design steps are illustrated in Figure 1.

4. Array Performance Analysis by Adaptive Beamforming

The output of the matched filter of the MIMO array system can be written as

$$\mathbf{y} = \alpha_0 \mathbf{v}(\theta) + \mathbf{n}, \quad (23)$$

where $\mathbf{n}$ denotes the noise plus interference. The output SINR of the beamformer directly impacts the MIMO array performance. The minimum variance distortionless response (MVDR) [37] is a widely employed beamformer choice. The MDVR beamformer for a look angle θ is given by

$$\mathbf{w}(\theta) = \frac{\mathbf{R}^{-1} \mathbf{v}(\theta)}{\mathbf{v}^H(\theta) \mathbf{R}^{-1} \mathbf{v}(\theta)}, \quad (24)$$

where $(\cdot)^H$ is the conjugate transpose and $\mathbf{R}$ is the covariance matrix of the noise and interference signal. The beamformer output can then be expressed as

$$z = \mathbf{w}^H(\theta) \mathbf{y} = \alpha_0 + \mathbf{w}^H(\theta) \mathbf{n}. \quad (25)$$

Correspondingly, the output SINR can be expressed as

$$\eta \triangleq \frac{|\alpha_0|^2}{\mathbf{w}^H(\theta) \mathbf{R} \mathbf{w}(\theta)}. \quad (26)$$

FIGURE 2: The PDF $f_\eta(x)$ as a function of $x/\eta_{\max}$.

Since the output SINR is random due to the random covariance matrix $\mathbf{R}$, we can examine the SINR distribution under the standard assumption that $\mathbf{n}$ has a complex Gaussian distribution with zero mean and covariance matrix $\mathbf{R}$. In this case, the probability density function (PDF) of SINR can be represented by [38, 39]

$$f_\eta(x) = \frac{K!}{(N_{\text{dof}} - 2)! (K + 1 - N_{\text{dof}})! \eta_{\max}} \times \left(1 - \frac{x}{\eta_{\max}}\right)^{N_{\text{dof}}-2} \left(\frac{x}{\eta_{\max}}\right)^{K+1-N_{\text{dof}}}, \quad 0 \leq x \leq \eta_{\max}, \quad (27)$$

where K is the number of the training signals for obtaining the covariance matrix, N_{dof} is the number of the system degrees-of-freedom, and $\eta_{\max}$ is the maximum output SINR which is achieved when the covariance matrix is known exactly

$$\eta_{\max} = |\alpha_0|^2 \mathbf{v}^H(\theta) \mathbf{R}^{-1} \mathbf{v}(\theta). \quad (28)$$

Figure 2 shows the PDF $f_\eta(x)$ as a function of $x/\eta_{\max}$.

The detailed derivation of (27) can be found in the appendix of [38]. The corresponding cumulative distribution function is

$$F_\eta(x) = \frac{K!}{(N_{\text{dof}} - 2)! (K + 1 - N_{\text{dof}})!} \times \Gamma\left(\frac{x}{\eta_{\max}}; K + 2 - N_{\text{dof}}, N_{\text{dof}} - 1\right), \quad (29)$$

where $\Gamma(x; a, b)$ is the incomplete Beta function

$$\Gamma(x; a, b) \triangleq \int_0^x t^{a-1} (1-t)^{b-1} dt. \quad (30)$$

FIGURE 3: Comparative STD of the output SINR performance.

The expectation and variance of the output SINR can be derived, respectively, as

$$E(\eta) = \eta_{\max} \frac{K + 2 - N_{\text{dof}}}{K + 1}, \quad (31)$$

$$\text{var}(\eta) = \eta_{\max}^2 \frac{(N_{\text{dof}} - 1)(K + 2 - N_{\text{dof}})}{(K + 2)(K + 1)^2}. \quad (32)$$

It can be noticed from (31) that the expectation of the output SINR depends mainly on the ratio N_{dof}/K , not the number of the training data, K .

Equations (31) and (32) can be used to evaluate the designed MIMO array performance. Figure 3 gives the comparative standard deviation (STD) of the output SINR performance, where $K = 20$ is assumed. It can be noticed that, for a given number of the training signals, more degrees-of-freedom mean a better STD of the SINR performance.

Define

$$E(\eta) = \alpha \eta_{\max}. \quad (33)$$

The corresponding K is

$$K = \frac{1}{1 - \alpha} (N_{\text{dof}} - 2 + \alpha) \approx \frac{N_{\text{dof}}}{1 - \alpha}. \quad (34)$$

To ensure that the mean output SINR of the adaptive beamformer is within 3 dB from its maximum achievable value, the training signals should be satisfactory with [38]

$$K \geq 2N_{\text{dof}} - 3. \quad (35)$$

If we want to achieve $\alpha = 0.95$, the K should be

$$K \geq 3N_{\text{dof}}. \quad (36)$$

5. Design Examples and Performance Analysis

We consider two design examples with given virtual MIMO antenna arrays.

Example 1. $L = 9$ and $P_V(x) = \sum_{i=0}^9 x^i$.

According to (2), we factorize $L + 1 = 10$ into two prime integers 2 and 5, that is, $10 = 2 \times 5$. Then, according to (4), the virtual array radiation pattern $P_V(x)$ can then be factorized as

$$P_V(x) = (1 + x)(1 + x^2 + x^4 + x^6 + x^8). \quad (37)$$

We can use the first factor for the transmit array and the second factor for the receive array or the first factor for the receive array and the second factor for the transmit array; that is,

$$P_T(x) = 1 + x, \quad (38a)$$

$$P_R(x) = 1 + x^2 + x^4 + x^6 + x^8 \quad (38b)$$

or

$$P_T(x) = 1 + x^2 + x^4 + x^6 + x^8, \quad (39a)$$

$$P_R(x) = 1 + x. \quad (39b)$$

Suppose that the number of the training signals is $K = 25$; Figure 4 shows the possible two transmit-receive arrays. It can be noticed that 7 antennas are required for the MIMO antenna arrays and the number of the system degrees-of-freedom is 10. Figure 5 gives the statistical mean and STD of the output SINR performance. Note that, although the two arrays have the same statistical SINR performance (because they have the same number of degrees-of-freedom), they should be seen as two different MIMO antenna arrays in the design because they have different transmit array gain and receive array gain (MIMO antenna array has no transmit array gain whereas phased array has both transmit array gain, and receive array gain).

Example 2. $L = 11$ and $P_V(x) = \sum_{i=0}^{11} x^i$.

According to (2), we can factorize $L + 1 = 12$ into the following three ways.

Factorization 1. We can factorize 12 into three prime integers 2, 2, and 3, that is, $12 = 2 \times 2 \times 3$. Then, according to (4), the virtual array radiation pattern $P_V(x)$ can then be factorized as

$$P_V(x) = (1 + x)(1 + x^2)(1 + x^4 + x^8). \quad (40)$$

Factorization 2. We can factorize 12 into three prime integers 2, 3, and 2, that is, $12 = 2 \times 3 \times 2$. Then, according to (4), the

FIGURE 4: Illustration of the designed MIMO antenna array positions. The x -axis stands for the MIMO antenna position. $y = 1$ denotes an antenna and $y = 0$ means no antenna.

FIGURE 5: Statistical output SINR performance.

virtual array radiation pattern $P_V(x)$ can then be factorized as

$$P_V(x) = (1+x)(1+x^2+x^4)(1+x^6). \quad (41)$$

Factorization 3. We can factorize 12 into three prime integers 3, 2, and 2, that is, $12 = 3 \times 2 \times 2$. Then, according to (4), the virtual array radiation pattern $P_V(x)$ can then be factorized as

$$P_V(x) = (1+x+x^2)(1+x^3)(1+x^6). \quad (42)$$

Then there are nine possible transmit-receive array designs:

$$P_T(x) = (1+x), \quad P_R(x) = (1+x^2)(1+x^4+x^8), \quad (43a)$$

$$P_T(x) = (1+x^2), \quad P_R(x) = (1+x)(1+x^4+x^8), \quad (43b)$$

$$P_T(x) = (1+x^4+x^8), \quad P_R(x) = (1+x^2)(1+x), \quad (43c)$$

$$P_T(x) = (1+x), \quad P_R(x) = (1+x^2+x^4)(1+x^6), \quad (43d)$$

$$P_T(x) = (1+x^2+x^4), \quad P_R(x) = (1+x)(1+x^6), \quad (43e)$$

$$P_T(x) = (1+x^6), \quad P_R(x) = (1+x)(1+x^2+x^4), \quad (43f)$$

$$P_T(x) = (1+x+x^2), \quad P_R(x) = (1+x^3)(1+x^6), \quad (43g)$$

$$P_T(x) = (1+x^3), \quad P_R(x) = (1+x+x^2)(1+x^6), \quad (43h)$$

$$P_T(x) = (1+x^6), \quad P_R(x) = (1+x+x^2)(1+x^3). \quad (43i)$$

Suppose that the number of the training signals is $K = 64$; Figure 6 shows the possible nine transmitter-receiver arrays. It can be noticed that 8 antennas are required for the transmit-receive pairs (a), (b), (d), (f), (h), and (i), and 7 antennas are required for the transmit-receive pairs (c), (e), and (g). All of the nine transmitter-receiver arrays have 10 degrees-of-freedom. Figure 7 gives the statistical mean and STD of the

FIGURE 6: Illustration of the nine possible transmit-receive array positions for the MIMO antenna array. The x axis stands for the MIMO antenna position. $y = 1$ denotes an antenna and $y = 0$ means no antenna.

output SINR performance. Since they have the same number of degrees-of-freedom, the nine transmit-receive arrays have the same statistical SINR performance.

6. Conclusion

In MIMO antenna array systems, the transmitter and receiver should be carefully designed to fully utilize the virtual MIMO antenna array. In this paper, we designed the MIMO antenna array with the polynomial factorization method and

comparatively analyzed the designed array performance. For a desired virtual antenna array, the polynomial factorization method can optimally design the specified transmitter and receiver antenna arrays. Design examples are also provided, which are verified by simulation results. Note that two MIMO antenna arrays may have the same statistical SINR performance; they often should be seen as two different MIMO antenna arrays in the design because they have different transmit array gain and receive array gain. In this paper, uniform virtual MIMO antenna array elements are assumed.

FIGURE 7: Statistical output SINR performance.

We are aware that nonuniform virtual MIMO antenna array elements may be desired in actual MIMO antenna arrays. This topic will be further investigated in our subsequent work.

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