Recovery Capabilities of Rateless Codes on Simulated Turbulent Terrestrial Free Space Optics Channel Model

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Free Space Optics (FSO) links are affected by several impairments: optical turbulence, scattering, absorption, and pointing. In particular, atmospheric optical turbulence generates optical power fluctuations at the receiver that can degrade communications with fading events, especially in high data rate links. Innovative solutions require an improvement of FSO link performances, together with testing models and appropriate channel codes. In this paper, we describe a high-resolution time-correlated channel model able to predict random temporal fluctuations of optical signal irradiance caused by optical turbulence. Concerning the same channel, we also report simulation results on the error mitigation performance of Luby Transform, Raptor, and RaptorQ codes.

1. Introduction

FSO is an optical wireless line-of-sight communication system able to offer good broadband performances, electromagnetic interference immunity, high security, license-free operation, low power consumption, ease of relocation, and straightforward installation [1]. It represents a modern technology significantly functional when it is impossible, expensive, or complex to use physical connections or radio links.

Thanks to these features, FSO is suitable for different broadband telecom applications as airborne, satellite scenarios, Next Generation Networks (NGN), and, finally, “Last Mile” communication links. In addition, FSO bandwidth performance can be further improved by using Wavelength Division Multiplexing (WDM) techniques reaching over 1.28 Tbps capacity [2, 3].

Unfortunately, as the transmission medium in a terrestrial FSO link is the air, these communications are strongly dependent on various atmospheric phenomena (as rain, snow, optical turbulence, and especially fog) that can cause losses and fading. Therefore, in worst-case conditions, it could be necessary to increase the optical transmission power although, at the same time, it is needed to comply with safety regulations. The commonly used wavelengths in outdoor FSO communications are 830, 1064, and 1550 nm, but, for the previously mentioned reasons, the highest is preferred for transmission [2].

Current laser technologies offer high-power sources at the most important wavelengths for communications. Novel technologies enable us to improve several applications previously limited by the fixed wavelength and power of other laser technologies. Today, we no longer have to work around the closest fit wavelength, but we can find the best wavelength to fit FSO communications [4, 5]. Moreover, new communication windows for FSO links could be explored using nonlinear optics techniques that are able to generate coherent emission at a wide range of wavelengths, from visible to THz [6–9].

The effects of already mentioned impairments are scattering (i.e., Rayleigh and Mie) losses, absorption, and scintillation. The first two can be described by proper attenuation coefficients [10] and increase if the atmospheric conditions get worse. Furthermore, hydrometeor scattering effects could introduce losses in FSO links [11], which can be estimated by proper models [12, 13]. At last, scintillation is a random phenomenon appreciable even under clear sky. It is due to atmospheric turbulences that originate local variations of the medium refractive index, thus generating optical irradiance fluctuations. Such impairments are predictable only using proper statistical models, able to describe the behavior of optical signals in free space propagation.
Therefore, due to the scintillation, in FSO links the irradiance fluctuates and could drop below a threshold under which the receiver is not able to detect the useful signal. In this case, communications suffer from cancellation errors, which cause link outages. This phenomenon becomes relevant at high distance, but it can also be observed in 500 m long FSO links. Moreover, the optical turbulence intensity can change by more than an order of magnitude during the course of a day: it reaches its maximum around midday (when the temperature is the highest) and, conversely, it is lower during the night [5].

In order to reduce or eliminate these impairments, different methods (hardware and software) were studied and reported in the literature. Hardware solutions focus on aperture averaging effects [14] to reduce irradiance fluctuations, in particular by using a bigger dimension detector or multidetector systems [15–22]. On the other hand, software techniques mostly focus on transmission codes [23–29].

Rateless codes are an innovative solution suitable for channels affected by erasure or burst errors. They add a redundant coding (also settable on the fly) to the source data, allowing the receiver to successfully recover the whole payload that, otherwise, would be corrupted or partially lost.

In order to test rateless codes recovery capabilities in FSO channels, we have to know detailed information about the occurring signal fading, in particular, its depth, temporal duration, and statistics. For this reason, we have implemented a time-correlated channel model able to generate an irradiance time series at the receiver side, at wide range of turbulence conditions (weak to strong).

Generated time series represents a prediction of temporal irradiance fluctuations caused by scintillation. By using the generated data, we were able to test the recovery capabilities of several types of rateless codes. In a previous work [23], information rates of Raptor codes and punctured LDPC codes with feedback were compared. In this paper, we instead present the FSO mitigation improvements obtained using coding schemes without any feedback.

In particular, we tested Luby Transform (LT) codes [30], Raptor codes (RCs) [31, 32], and newer RaptorQ (RQ) codes [33], for a 100 Mbps (Fast Ethernet speed) OOK modulation.

2. Channel Model

Several distribution models were developed in the literature to estimate optical turbulence effects (e.g., lognormal, negative exponential models, K-distribution) [34–38]. In detail, lognormal model is suitable for weak turbulence conditions, while negative exponential model is more appropriate for a very strong turbulence (i.e., saturate regime). Another model, which has provided excellent agreement with numerous experimental data, is the K-distribution. In particular, it is suitable under strong turbulence conditions although an extension of this model also permits us to cover weak fluctuation regimes (i.e., K-distribution).

Instead, a more versatile model is the Gamma-Gamma distribution, which is able to estimate optical turbulence from weak to moderate-strong fluctuation. This distribution is provided by two independent Gamma statistics which arise, respectively, from large-scale and small-scale atmospheric effects [37, 38].

For the above-mentioned properties, the Gamma-Gamma model is proper for our applications and it will be better described in Section 2.1.

2.1. Gamma-Gamma Model. The Gamma-Gamma statistics are able to describe the FSO scintillation phenomena in a broad range of turbulence conditions and, for this reason, it is suitable to design our correlated model. The Gamma-Gamma model provides the probability density function (PDF) of received optical irradiance (I) in the following form:

\[
p(I) = \frac{2(\alpha \beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha) \Gamma(\beta)} \Gamma\left[\frac{\alpha+\beta}{2}-1\right] K_{\alpha-\beta}\left(2\sqrt{\alpha \beta I}\right),
\]

where \( \Gamma(\cdot) \) is the Gamma function and \( K_n(\cdot) \) is the modified Bessel function of the second kind of order \( n \), while \( \alpha \) and \( \beta \) are two parameters expressed, for plane wave model at zero inner scale case [37], as follows:

\[
\begin{align*}
\alpha &= \left\{ \exp\left[\frac{0.49 \sigma_R^{12/5}}{\left(1 + 1.11 \sigma_R^{12/5}\right)^{7/6}}\right] - 1 \right\}^{-1}, \\
\beta &= \left\{ \exp\left[\frac{0.51 \sigma_R^{12/5}}{\left(1 + 0.69 \sigma_R^{12/5}\right)^{5/6}}\right] - 1 \right\}^{-1},
\end{align*}
\]

where \( \sigma_R^2 \) is the Rytov variance, related to the optical turbulence: it constitutes a measure of the strength of scintillation [37, 38] and, for plane wave, is defined by

\[
\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6},
\]

where \( C_n^2 \) is the refractive index structure parameter, \( k \) is the wavenumber, and \( L \) is the path length (distance from the transmitter to the receiver).

Starting from the PDF, we can generate a random irradiance time series, but it is mandatory to define a temporal correlation relationship between the samples. Assuming a plane-wave propagation, the spatial relationship linking the optical irradiance values is given by the following covariance function:

\[
B_1(\rho) = \exp\left[\left(\frac{0.49 \sigma_R^2}{\left(1 + 1.11 \sigma_R^{12/5}\right)^{7/6}}\right) \times \frac{7}{6} \times \frac{k(\rho)^2 \eta_x}{4L} + \left(\frac{0.50 \sigma_R^2}{\left(1 + 0.69 \sigma_R^{12/5}\right)^{5/6}}\right) \frac{k(\rho)^2 \eta_y}{L}^{5/12}\right] \\
\times K_{5/6}\left(\sqrt{\frac{k(\rho)^2 \eta_y}{L}}\right) - 1,
\]

\( (4) \)
where $\rho$ is the spatial variable, $\mathbf{1}_F(\cdot)$ is the confluent hyper-
gamma function of the first kind, and $\eta_x$ and $\eta_y$ are two parameters defined as follows [38]:

$$
\eta_x = \frac{2.61}{1 + 1.11\sigma_R^{12/5}}
$$

$$
\eta_y = 3 \left(1 + 0.69\sigma_R^{12/5}\right).
$$

In order to convert the spatial covariance into a time function, we applied Taylor’s frozen eddies hypothesis [37, 38]. By defining $V_T$ as the average transverse wind speed (orthogonal to the propagation direction), we can write the following:

$$
\rho = V_T t,
$$

where $t$ is the time.

Setting the $V_T$ in expression (6) and substituting it into (4), the covariance becomes a function of the only independent variable $t$. In order to simulate our channel, we chose $V_T = 1 \text{ ms}^{-1}$, thus obtaining irradiance correlation times close to those experimentally and theoretically reported in the literature [23, 39].

### 2.2. Non-Gaussian-Correlated Sequence

We can simulate predictions of irradiance fluctuations that make use of discrete irradiance time series—following Gamma-Gamma distribution—in which the samples are temporally spaced by one correlation time. The latter can be defined as the time in which the amplitude of normalized covariance function is equal to 0.5 and it represents a time distance beyond which we can consider the samples uncorrelated. The method that we have just described is known as Block Fading Model (BFM) [40] and it does not permit time resolution less than one correlation time. In order to reach better resolutions we have to introduce a time correlation between the irradiance samples. Unfortunately, it is not trivial to correlate a non-Gaussian distribution of samples without turning it into a Gaussian one. We can solve this problem by using a new suitable algorithm [41] defined by a correlation filter and a nonlinear memoryless block function. This algorithm is able to generate random processes with a defined marginal probability distribution and a power spectral density function (i.e., generating coloured, non-Gaussian signals). It is quite simple to be implemented as it only involves the Fourier transform and a sorting routine.

The correlation filter employed is created using the Fourier amplitudes associated with the target power spectral density. The block function is memoryless, and this ensures that the spectral properties of the generated signal are not altered during the execution of the algorithm [41].

In our case, the algorithm input parameters are the double side Fourier transform of the temporal irradiance covariance—obtained through the FFT of $B_i(t)$—and a random irradiance samples sequence that follows a Gamma-Gamma distribution. The temporal spacing between two adjacent samples is the reciprocal of the FFT sampling frequency ($f_s$). This means that we can choose the temporal resolution of the sequence according to our needs.

In Figure 1 we show an example of irradiance fluctuations (normalized with respect to irradiance mean value) simulation for $\sigma_R^2 = 0.7$, at a 1550 nm wavelength, $L = 500$ m, 0.2 s time interval, and 10 $\mu$s time spacing.

Figure 2 depicts the comparison between the covariance function of the simulated irradiance fluctuations displayed in Figure 1 (dashed red line) and the input temporal covariance function (solid blue line). It is worth noting that the two above-mentioned functions are very similar: so the algorithm we used is able to properly correlate irradiance samples. We also analyze the difference between the simulated irradiance fluctuations PDF and the Gamma-Gamma distribution. Figure 3 shows that both PDFs are comparable.
Normalized irradiance

0 0.2 0.4 0.6 0.8 1

0

2

4

6

8

Time (s)

(a)

30

20

10

0

0

1

2

3

4

5

6

Frequency

(b)

Figure 4: (a) Samples distribution histogram and (b) simulated temporal optical irradiance fluctuations in the case of BFM.

It demonstrates that the irradiance time series exhibits the correct probability density distribution.

In the BFM the spacing between the irradiance samples equals the correlation time. We generated an irradiance time series according to the BFM, Figure 4(a), whose Gamma-Gamma distribution histogram is shown in Figure 4(b). We used \( \sigma_R^2 = 1 \) (value corresponding to a strong turbulence condition), a time interval of 1 s, and the correlation time value extracted from the irradiance covariance function.

In our correlated channel model, for the same values of Rytov variance and time interval, considering a temporal spacing of 10 \( \mu \)s, we obtained the results depicted in Figure 5. Even though the two simulations are apparently similar (as expected, since they are based on the same irradiance PDF), the correlated model has a much larger temporal resolution if compared to the BFM. For this reason, we can conclude that our model is suitable for high data rate communication tests.

3. Simulation Analysis

At the receiver, the irradiance fluctuations cause communication failures and outages, when irradiance values drop below a fading depth (threshold under which the receiver is not able to detect data). In other words, when the optical signal drops below the above-mentioned fading depth threshold, we interpret it as an erasure error occurring in the FSO communication link.

It is also worth noting that at relatively strong turbulence conditions, power fluctuations become larger and, hence, the average value of the signal power decreases. For these reasons, in our work, we referred to the normalized average value of the signal power and, in particular, to the normalized average value of the irradiance at the receiver.

Using the channel model described in the previous section, we investigated the outage statistics and the performance of rateless codes at fast data rate. In detail, we tested the LT codes and RCs capabilities in order to mitigate erasure errors, which can be detected during a 500 m single terrestrial free space link. In our simulations, we used a 1550 nm wavelength. In addition, we did not consider the noise due to the photodetector, because it can be neglected if compared to the irradiance fluctuations caused by scintillation phenomena in the turbulence conditions taken into account.

In detail, we considered that the photodetector has a Noise Equivalent Power (NEP), that is, the minimum detectable input power, of 6 dB and 9 dB lower (fading depth) than the mean value of the irradiance at the receiver. Consequently, the latter is not constant but varies with the turbulence conditions.

3.1. Outage Statistics. In Figure 6 outage statistics histograms are depicted. They correspond to a ten-hour FSO communications simulation related to two different values of \( \sigma_R^2 \) and to a \( -9 \) dB fading depth. We can actually see that as \( \sigma_R^2 \) increases, the average outage duration time grows. At different fading depths, we have found a similar behaviour. Therefore, we wondered if we were able to reduce or to eliminate communications errors by using rateless codes and, if so, what configuration parameters should be set. In order to answer these questions, we chose to employ modern code typologies belonging to rateless codes family. In particular, we used LT codes, RCs, and RQ codes.

3.2. Rateless Codes for Error Mitigation. Fountain codes (FCs) are rateless and also suitable for q-ary erasure channels; FSO channels can be described similarly. FCs do not need
feedback [42]; in fact, they add to the source data a redundant coding (also settable on the fly) that allows the receiver to recover the whole payload, despite erasure errors. More in detail, random linear FCs for a group of $K$ packets produce a new set of $N$ encoded packets ($N > K$). In particular, FCs perform a linear combination (bitwise sum, modulo-2) of the $K$ source packets by means of a binary pseudorandom $G$ matrix ($K \times N$). Each generated encoded packet will be linked to one or more source packets and the number of such links is termed “degree” ($\delta$). The $G$ matrix depends on the degrees distribution and, for this reason, its definition is crucial for the code implementation. LT codes are more efficient than random linear FCs. In order to complete the decoding process, LT codes require, at least, the following two conditions: receiving a number of coded packets $O > K$ and having, for each decoding step, a coded packet with degree equal to 1. With regards to the degrees, LT codes use a robust soliton distribution [42], which guarantees, at the same overhead, a higher probability to complete the decoding process if compared to the ideal soliton distribution.

The computational costs of LT codes depend on $K$, but it is possible to improve their computational performance thanks to a proper management of sparse graphs. Nevertheless, the degrees distribution does not always guarantee a decoding sparse graph and, consequently, a good decoding speed. This issue is overcome with RCs. They are, substantially, LT codes in which a precoding step has been added to reduce the expected degree. In details, RCs show an expected degree equal to three. In this way, the decoding graph is always sparse and decoding computational costs are reduced.

Moreover, RCs are systematic codes (i.e., codes in which the first $K$ encoding packets are the same as the $K$ source packets and the last $N-K$ repair packets are the result of encoding [43]) and they work on Galois Field 2 (GF(2)). The number of source symbols is limited to 8192 [44].

RQ codes are an evolution of RCs. They are also systematic codes but they work on a much larger alphabet, in particular on GF(256). It can be demonstrated that with a larger alphabet, the failure probability is reduced at a certain overhead [45]. Thus, RQ codes show better recovering capability. They also permit us to easily find good systematic indices [44] hence supporting a number of source symbols much larger than RCs.

### 3.3. Results

We tested the performance of LT codes and RCs by simulating an OOK modulated transmission of 100 Mbps (1518 bytes frame size) over a distance of 500 m, at different fading depths and $\sigma_R^2$ values. More in detail, for each couple of the latter parameters, we evaluated the failure probabilities (for 1000 points) for an overhead ranging from 5% to 50% and for different values of $K$. Figure 7 shows the results concerning the case of $-6$ dB fading depth and $\sigma_R^2 = 1.0$. As we expected, as the overhead becomes larger, the decoding performance of both codes improves. Nevertheless, while LT codes are not always able to recover all the source data (i.e., the failure probability is always above $10^{-2}$), RCs are able to recover all the source data starting from $K = 500$ and for a 50% overhead. Therefore, using the above-mentioned configurations, we were able to demonstrate that RCs failure probability can reach values below $10^{-3}$.

However, the best performance is given for $K = 1000$ and starting from a 35% overhead. In Figure 8, we show the failure probability versus source packets number, for LT codes and RCs, at different overhead values. In this case, with
increasing $K$, LT codes performance improves more than RCs one. This is due to the fact that the performance of RCs is already good even for low values of $K$. We carried out further tests in order to evaluate the performance of RQ codes. The same configurations we have used for LT codes and RCs have been exploited herein. The failure probability versus overhead percentage related to RQ codes is depicted in Figure 9.

We can see how RQ codes are able to recover all the source data from $K = 500$ and for a 35% overhead. For higher values of $K$ the performance improves, so that an overhead less or equal to 25% is sufficient enough to recover data for $K > 2000$.

4. Conclusion

We produced a high-resolution FSO channel model that takes into account the temporal covariance of irradiance and hence is able to simulate the temporal irradiance fluctuations at the receiver, with a high resolution. Moreover, it also permits us to set the temporal spacing among the irradiance samples via a proper sampling frequency for the FFT of the temporal irradiance covariance. Our correlated model shows a much larger temporal resolution if compared to the BFM and, for this reason, it is appropriate for communication testing at high data rates.

We also tested, in our channel model, the performance of LT codes and RCs able to mitigate erasure errors caused by the scintillation phenomena. Our simulations illustrate that LT codes, with K values lower than 1000, are not able to cancel erasure errors, even with a 50% overhead. On the other hand, RCs can eliminate all the erasure errors, with a 50% overhead, already starting from $K = 500$. Nonetheless, RQ codes provide the best recovering performances. They work slightly better than RCs and are the best choice especially when high values of $K$ are required.

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