Performance of a circular patch microstrip antenna is highly affected by the effective dielectric constant of a used substrate material. When the circular patch is conformed on a cylindrical body, the effective dielectric constant is changing with curvature due to the changing in the fringing field. Consequently, some of antenna parameters such as resonance frequency, input impedance, voltage standing wave ratio, return loss, quality factor, and antenna bandwidth are functions of curvature. In this work, we study the effect of curvature on the performance of circular patch microstrip antenna. A mathematical model for the antenna parameters as functions of curvature is also introduced. The model is applied in case of using two substrates of different refractive index values. By extension, the antenna performance was studied through simulation by using method of moments (MoM) which is reliable in solving Maxwell's integral equations in the frequency domain. The results from simulation compare very favorably with the described analytical results.
Mathematical and experimental results for a slot coupled cylindrical-circular microstrip antenna for input impedance as a function of curvature using cavity model were introduced in [11]. The mathematical model assumed that there is no change in resonance frequency, but the experimental results showed a little shift in frequency due to curvature. A physical explanation for this experimental result had never been given. We have introduced the first physical explanation of this shift in frequency in [17]. The curvature effects on the effective dielectric constant and consequently on the operating frequency as cleared from (1) are introduced in [18].

2. The Proposed Model

The fringing field effect of a microstrip line on the effective dielectric constant is firstly introduced by Wheeler in 1961 [19]. Applying this model, we have produced an expression of the dielectric constant as a function of radius of curvature for a rectangular microstrip printed antenna conformed on a cylindrical surface [20]. Starting from Maxwell’s equations, expressions for electric and magnetic fields intensities, quality factor, input impedance, VSWR, and return loss are obtained as a function of radius of curvature [21].

Here, we extend our work and study the performance of a conformal circular patch antenna.

First, we obtained dielectric constant $\varepsilon_{\text{eff}}$ for different curvatures, as shown in [20], and then substituted the dielectric constant in (1) to get the resonance frequency for TM$_{01}$ as a function of curvature. In this case, the electric field components $E_\theta$ and $E_\varphi$, for a far field are obtained as shown below:

$$E_\theta = E_0 e^{-jk_0\varphi} \sum_{n=-\infty}^{\infty} e^{jna_0} f_n(-k_0 \cos \theta),$$

$$E_\varphi = -E_0 \frac{\omega \mu \varepsilon_0}{\mu_0 \pi r} \sum_{n=-\infty}^{\infty} e^{jna_0} g_n(-k_0 \cos \theta),$$

where

$$f_n = \int_{-\infty}^{\infty} M(n, p) e^{-j\varphi_0} e^{-j\varphi \cos \theta} \, dz$$

$$g_n = \int_{-\infty}^{\infty} \frac{M(n, p) e^{-j\varphi_0} e^{-j\varphi \cos \theta} \, dz}{\sqrt{k_0^2 - n^2}} H_p^2 \left( a \sqrt{k_0^2 - n^2} \right)$$

$$\times \left( \frac{np}{a \sqrt{k_0^2 - n^2}} - 1 \right).$$

$M(n, p)$ is the equivalent magnetic currents along the circumference of the circular path, $\omega$ is the angular frequency, and $k_0 = \omega_0 \sqrt{\mu_0 \varepsilon_0}$.

Boundary conditions were applied on the elementary wave function. So, the homogenous wave vanishes at edges and satisfies the normalized conditions. Then, coaxial feed is considered as a rectangular current source with equivalent cross-sectional area $S_Z \times S_\Phi$ centered at $(Z_0, \Phi_0)$. 

a cylindrical surface. The fringing field effect on the performance of a conformal circular patch antenna is also presented in this paper.

In the rectangle microstrip printed antenna, the effective dielectric constant, depending on fringing field, is varying with curvature and microstrip patch length [12]. However, in circular patch microstrip antenna, it depends on the distance between the central axis and the microstrip edge. The value of that distance is varying from zero to the circular patch radius.

The cylindrical-circular patch microstrip antenna is very famous and popular in many military applications [13]. The circular patch microstrip antenna has a number of advantages over the rectangular one. It has a symmetric radiation pattern, higher directivity, smaller VSWR, and higher return loss [14].

The resonance frequency $f_{010}$ for TM$_{01}$ mode as a function of an effective radius of antenna $a_{\text{eff}}$ and effective dielectric constant $\varepsilon_{\text{eff}}$ has been presented by Balanis as [11]

$$f_{010} = \frac{3.8318 \times c}{2\pi a_{\text{eff}} \sqrt{\varepsilon_{\text{eff}}}},$$

where $c$ is the speed of light.

Luk and Lee studied the curvature effect on the performance of circular disc microstrip antenna conformed on a cylindrical body, as shown in Figure 1, for TM$_{01}$ [15]. Quality factor, input impedance, and power are introduced for different values of curvatures. The authors noted that all studied parameters are functions of curvature except the operating frequency, according to their model. In their model, the authors did not consider the fringing filed effect on the antenna parameters and the effect of radius of curvature on effective dielectric constants.

A mathematical model for input impedance and far electric field of a coaxial fed circular patch microstrip printed antenna conformed on a cylindrical surface was introduced using cavity model by Luk and Lee [16]. The fringing filed effect on an effective circular patch radius and on an effective dielectric constant is considered in this paper. However, the curvature effects on the resonance frequency and on fringing field itself were not considered.
Figure 2: Impact of changing radius of curvature of a circular microstrip patch (a) on the effective dielectric constant and (b) on the resonance frequency. “Teflon substrate.”

Input impedance as function of effective dielectric constant $\varepsilon_{\text{eff}}$ and position of feeding ($Z_0$ and $\Phi_0$) is defined as

$$Z_{in} = j\omega h \sum_n \sum_m \frac{1}{k_n^2 - k^2} \frac{Re_{\text{eff}}}{2a_{\text{eff}}^3} \times \cos^2 \left( \frac{\pi}{a_{\text{eff}}} z_0 \right)$$

$$\times \sin c \left( \frac{\pi}{2a_{\text{eff}}} z_0 \right) \sin c \left( \frac{R\pi}{2a_{\text{eff}}} \theta_0 \right).$$

The total quality factor depends on the conduction quality factor $Q_c$, dielectric loss quality factor $Q_{d}$, and radiation quality factor $Q_{rad}$ as given in

$$\frac{1}{Q_t} = \frac{1}{Q_{rad}} + \frac{1}{Q_c} + \frac{1}{Q_d},$$

$$Q_c = h \sqrt{\pi \mu \sigma},$$

$$Q_d = \frac{1}{\tan \delta},$$

$$Q_{rad} = \frac{2\omega e_r}{hG},$$

where $K$ and $G_0$ are given in [11].

The bandwidth BW is given by

$$BW = \frac{f_0}{Q_t}$$

and hence the VSWR values as a function of curvature can be calculated as

$$\Delta f = f_0 \frac{\text{VSWR} - 1}{Q_t \sqrt{\text{VSWR}}}.$$
Impact of changing the radius of curvature on the resonance frequency is shown in Figure 2(b). There is a shift of 2 MHz in frequency for the whole change in curvature 2 cm and flat antenna. This value is very small compared to resonance frequency. However, it is very important in multichannel applications such as ZigBee RF4CE. It operates at 2.4 GHz and has a channel separation of 5 MHz. A small frequency shifting may lead to interference between channels.

Figure 3(a) shows the impact of changing the radius of curvature on the return loss as a function of frequency. The minimum values of $S_{11}$ are almost the same at $-91$ dB, but the minimum is shifted toward increasing the frequencies when the curvature decreases by the same values as shown in Figure 2(b). Figure 3(b) illustrates the change of VSWR as a function of curvature. The minimum value of VSWR is almost 1, but the frequency is shifted.

Figures 4(a) and 4(b) show the normalized $H$-plane at $\varphi = 90^\circ$ and 270$^\circ$ and $E$-plane at $\varphi = 0^\circ$ and 180$^\circ$ in dB as a function of curvature for 2 cm, 2 m, and flat surface, respectively. It should be noted that the radiation pattern is getting wider as the curvature increased, which is expected for more bending of the body.

The radiation quality factor $Q_{rad}$, total quality factor $Q_t$, bandwidth BW, and the real value of input impedance $R_{in}$ are calculated for different values of curvatures and listed in Table 1.

As radius of curvature increases, the value of $Q_{rad}$ decreases and consequently the $Q_t$ also decreases by a very small amount. There is a limited change in bandwidth about 400 Hz for a change in the radius of curvature from 2 cm to 2 m. Small change in input impedance about 0.08 $\Omega$ is also noted due to curvature change.

3.2. Epsilam-10 Ceramic-Filled Teflon Substrate. Epsilam-10 ceramic-filled Teflon substrate material has a dielectric constant equal to 10, tangent loss of 0.004, and the patch radius is 2.45 cm. The effect of changing the curvature of the cylindrical body on the effective dielectric constant and on the resonance frequency is illustrated in Figures 5(a) and 5(b), respectively.

It should be noted that 2.5 MHz overall change in frequency is achieved corresponding to change in curvature between 2 cm radius and flat antenna. The change in frequency is lower than that of the Teflon material shown in Figure 2(b).

Figures 6(a) and 6(b) show $S_{11}$ and VSWR, respectively, as a function of frequency for different curvatures. The minimum

<table>
<thead>
<tr>
<th>$R$</th>
<th>2 cm</th>
<th>20 cm</th>
<th>2 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{rad}$</td>
<td>67.5</td>
<td>67.4</td>
<td>67.37</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>1869</td>
<td>1868.7</td>
<td>1868.6</td>
</tr>
<tr>
<td>BW (MHz)</td>
<td>1.2929</td>
<td>1.2926</td>
<td>1.2925</td>
</tr>
<tr>
<td>$R_{in}$ ($\Omega$)</td>
<td>50.19</td>
<td>50.26</td>
<td>50.27</td>
</tr>
</tbody>
</table>
values of $S_{11}$ ($\approx -100$ dB) and VSWR (1) are independent of the curvature. However, the frequencies corresponding to these minima are shifted. The location of the minimum increases with increasing the curvature and follows the behavior shown in Figure 5(b).

Figures 7(a) and 7(b) show the normalized $H$-plane at $\varphi = 90^\circ$ and $270^\circ$ and $E$-plane at $\varphi = 0^\circ$ and $180^\circ$ in dB as a function of curvature for 2 cm, 2 m, and flat surface. It should be noted that the radiation pattern is getting wider as the curvature increases. The radiation pattern of Epsilam-10 ceramic-filled Teflon substrate is wider than that of Teflon substrate shown in Figure 4.

Table 2 shows the radiation quality factor $Q_{rad}$, total quality factor $Q_t$, bandwidth BW, and $R_{in}$ for different radius of curvatures.

<table>
<thead>
<tr>
<th>$R$</th>
<th>2 cm</th>
<th>20 cm</th>
<th>2 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{rad}$</td>
<td>411.5</td>
<td>410</td>
<td>409.7</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>2048.1</td>
<td>2047.5</td>
<td>2047.4</td>
</tr>
<tr>
<td>BW (MHz)</td>
<td>0.8642</td>
<td>0.864</td>
<td>0.862</td>
</tr>
<tr>
<td>$R_{in}$ (\Omega)</td>
<td>50.12</td>
<td>50.2</td>
<td>50.23</td>
</tr>
</tbody>
</table>

3.3. Simulation Results. In this subsection, using the MoM simulations, we thoroughly investigate the antenna performances including the return loss and VSWR of circular path antenna conformed on cylindrical body. Two different substrates (Teflon and Epsilam-10 ceramic-filled Teflon) were used. Results are compared to those obtained theoretically in the previous sections.

Figures 8 and 9 show the return loss and VSWR of a circular microstrip antenna conformed on a cylindrical body for 20 cm, 2 cm, and 2 m using Teflon and Epsilam-10 ceramic-filled Teflon substrate, respectively.

The results from FEKO EM solver compare very favorably with our analytical results described in the paper. Difference in return loss at curvature radius of 2 cm and 2 m is almost 3.4 MHz and 5 MHz for the cases of Teflon, Figure 8(a), and Epsilam-10 ceramic-filled Teflon, Figure 9(a), substrate, respectively. These shifts are very close to the reported theoretical results.

However, the change in the minimum value of return loss and resonance frequency on this value may be attributed to the approximations taken into account through the analytical calculations and the software simulations as well.

4. Conclusion

The originality of this paper is to introduce the change in the resonance frequency of a circular patch microstrip antenna conformed on a cylindrical body. This frequency change is mainly due the modification of the effective dielectric constant with the curvature. We study the associated effects
Figure 5: Impact of changing radius of curvature of a circular microstrip patch (a) on the effective dielectric constant and (b) on the resonance frequency. "Epsilam-10 ceramic-filled Teflon substrate."

Figure 6: Impact of changing radius of curvature (2 cm, 20 cm, and 2 m) of a circular microstrip patch (a) on the return loss in dB and (b) on the VSWR as a function of frequency. "Epsilam-10 ceramic-filled Teflon substrate."
Figure 7: (a) Normalized $H$-plane ($\varphi = 90^\circ, 270^\circ$) and (b) normalized $E$-plane ($\varphi = 0^\circ, 180^\circ$) in dB at radii of curvature 2 cm, 2 m and flat surface. “Epsilam-10 ceramic-filled Teflon substrate.”

Figure 8: Impact of changing radius of curvature (2 cm, 20 cm, and 2 m) of a circular microstrip patch (a) on the return loss in dB and (b) on the VSWR as a function of frequency. “Teflon substrate, FEKO”.
on some antenna parameters such as quality factor, input impedance, bandwidth, $S_{11}$, and VSWR at TM$_{01}$ mode. Teflon and Epsilam-10 ceramic-filled Teflon are used as substrate materials with different dielectric constants. The resonance frequencies increased by 2 MHz and 2.5 MHz for a curvature change from 2 cm to 2 m for Teflon and Epsilam-10 ceramic-filled Teflon, respectively. Consequently, the VSWR and $S_{11}$ for both dielectric materials are shifted in frequency with increasing curvature of a cylindrical body. The change in input impedance is 0.08 Ω for Teflon material and 0.11 Ω for Epsilam-10 ceramic-filled Teflon material for curvature changing from 2 cm to 2 m. The change in the bandwidth of Teflon and Epsilam-10 ceramic-filled Teflon is 2 kHz and 2.2 kHz, respectively. Total quality factor is decreasing with increasing the curvature by very limited values for both substrate materials. The antenna performance was studied through simulation by using FEKO 7.0 software from EMSS Inc. The results from simulation compare very favorably with the described analytical results.

In general, the curvature has a limited effect on the performance of a circular patch microstrip printed antenna in the case of single channel. However, a small shifting in frequency will be very effective when a multichannel system is used. This will lead to a high change in VSWR and $S_{11}$ values.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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