Research Article

Joint DOD and DOA Estimation for High Speed Target Using Bistatic MIMO Radar

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In bistatic multiple-input multiple-output (MIMO) radar, range migration and invalidly synthesized virtual array resulting from the serious mismatch of matched filter make it difficult to estimate direction of departure (DOD) and direction of arrival (DOA) of high speed target using the traditional superresolution algorithms. In this study, a method for joint DOD and DOA estimation of high speed target using bistatic MIMO radar is proposed. After multiplying the received signals with the conjugate of the delayed versions of the transmitted signals, Fourier transform (FT) of the multiplied signals over both fast time and slow time is employed. Then, the target components of radar return corresponding to the different transmitted waveforms can be perfectly separated at the receivers by extracting the target frequency-domain data along slow-time frequency dimension when the delay between the transmitted signals and their subsequent returns is timed. By splicing the separated target components distributed along several range cells, the virtual array can be formed, and then DOD and DOA of high speed target can be estimated using the superresolution algorithm with the range migration and the mismatch of matched filter properly removed. Simulation results have proved the validity of the proposed algorithm.

1. Introduction

Bistatic radar with the widely separated transmitter and receiver has the advantages of covert receivers, giving increased immunity to antiradiation missiles and electronic countermeasures, and possible antistealth capabilities [1]. However, the need for synchronization between the transmitter and receiver leads to an increase in complexity. Unlike bistatic phased-array radar, bistatic multiple-input multiple-output (MIMO) radar is characterized by using multiple transmit antennas to simultaneously transmit orthogonal waveforms. Therefore, bistatic MIMO radar will simultaneously illuminate a very broad angular sector instead of a focused angular sector. The virtual array can be formed by matching the received signals with the transmitted signals, and then the direction of arrivals (DOAs) and the direction of

departures (DODs) of targets can be estimated by processing the output of virtual array using the existing angle estimation algorithms, that is, the algorithms proposed in [2–4]. Due to the broad illuminated angular sector and the target location determined by its DOA and DOD in this case, the beam scanning and range information of the target are redundant and then the time and space synchronization are relaxed in bistatic MIMO radar [2, 3].

With the military technology evolution and the increasing exploitation of space resources, high speed maneuvering targets, that is, aircraft, missile, and space debris, pose severe challenges to modern radar in recent years. Joint DOD and DOA estimation methods have been widely investigated in bistatic MIMO radar [2–8]. But all of them, which are designed for low speed target, will be seriously deteriorated when dealing with the target moving at a high speed.

The successful virtual array formation and effective target energy accumulation during the long observation time are vital for angle estimation in bistatic MIMO radar. However, the variety of the phase caused by the large-scale Doppler frequency of high speed target within the repetition interval of radar signal results in the severe distortion of target echo. Then significant mismatch loss would occur in matched filters. That is, the virtual array cannot be effectively formed for high speed target [9]. Meanwhile, during the long observation time, the target with high speed easily goes through several range cells so that the target energy is distributed along multiple range cells [10–14]. As a consequence, for high speed target, the angle estimation performances of the existing methods, that is, the proposed methods in [6–8], are limited with both the mismatch loss in matched filter and the target residence time in a single range cell. Persy and Dipietro [15] have introduced the keystone transform to compensate for the linear range migration during the long integration time for synthetic aperture radar (SAR) ground moving targets imaging without knowing the target motion information. For high speed target, it is highly possible that both Doppler frequency ambiguity and target radial acceleration exist. However, in this situation, the detection performance of the keystone transform would be seriously reduced. At the same time, the data interpolation operator is employed to implement the keystone transform, which results in the large computational load. Dorp [16] introduced the keystone transform into MIMO radar for correcting the envelope migration of high speed target. It is worth pointing out that time division, rather than code division, strategy for transmitting signals is applied in MIMO radar to synthesize the virtual elements one by one at different time. The basic idea of the transmitter time division is to avoid the effect of the large-scale Doppler frequency on the virtual array formation. However, for high speed target, the switching time between the transmitters is normally longer than the inverse of the relative Doppler frequency of the target. Thus, the phase variety of the return signal caused by the target's Doppler frequency during the virtual array forming time alters the array manifold and then degrades the accuracy of the angle estimation. In [17], a long-time integration method based on Hough transform is proposed to detect the weak target without acceleration. This method is capable of overcoming the problem of the range migration, but it performs poorly under the condition that the virtual array is unsuccessfully formed because of the large-scale Doppler frequency of target, and its computational burden is also very huge. A novel long-time coherent detection method, Radon-Fourier transform (RFT), is proposed in [18–20], which realizes the echo spatial-temporal decoupling via joint searching along range and velocity dimensions. RFT may obtain the significant coherent integration gain without the Doppler ambiguity restriction, but, for the high speed maneuvering target detection using bistatic MIMO radar, a huge computational complexity is needed for five-dimensional joint searching of range, velocity, acceleration, DOD, and DOA. In [21], the space-time RFT for wideband digital array radar is proposed for high speed target detection by jointly realizing digital beamforming (DBF), range

compression, and long-time coherent integration. However, prior to the spatial and temporal information processing for bistatic MIMO radar, the virtual array should be effectively formed when dealing with the target moving at a high speed.

In bistatic MIMO radar, suffering from the range migration and the invalidly synthesized virtual array when dealing with high speed target, the performances of the existing angle estimation methods will rapidly degrade. This paper makes an effort to provide a method for overcoming the effect of the large-scale Doppler frequency on the virtual array formation and removing range migration during the long-time integration, thus facilitating the angle estimation for high speed target in bistatic MIMO radar. In order to overcome the effect of the large-scale Doppler frequency on the virtual array formation, the two-dimensional (2D) Fourier transform (FT) is employed to transform the multiplied signals, which are obtained by multiplying the received signals with the conjugate of the delayed versions of the transmitted signals, into the fast-time frequency and slow-time frequency domains, and then the target components of the radar return corresponding to the different transmitted waveforms perfectly separated by extracting the target frequency-domain data. By splicing the separated target components that are distributed along several range cells, the virtual array is synthesized with the range migration removed. Thus, the DOD and DOA of high speed target can be obtained from the output of virtual array using the superresolution algorithm. Simulation results demonstrate the effectiveness of the proposed method.

The remainder of this paper is organized as follows. In Section 2, the signal model for high speed target in bistatic MIMO radar is established. In Section 3, the proposed angle estimation algorithm for high speed target is described. Moreover, the simulation results of the proposed algorithm are presented and the performances are investigated in Section 4. Finally, Section 5 concludes the paper.

2. Signal Model

Consider a bistatic MIMO radar system [2] with M closely spaced transmit elements and N closely spaced receive elements, as shown in Figure 1. Both the transmit array and the receive array are uniform linear arrays and the spacing between adjacent elements of the transmit array and the receive array is denoted by d_t and d_r , respectively. We assume that the range between the target and the transmit array or receive array is much larger than the aperture of the transmit array or receive array. M different continuous coded periodic signals with identical bandwidth are employed, which are temporally orthogonal. The vector including M orthogonal transmitted baseband coded signals can be expressed as [2]

$$\mathbf{S}(\tilde{t}, t_l) = [S_1(\tilde{t}, t_l) \ S_2(\tilde{t}, t_l) \ \cdots \ S_M(\tilde{t}, t_l)]^T, \quad (1)$$

where $[\cdot]^T$ denotes the vector/matrix transpose, $\tilde{t} = t + t_l$, $t_l = lT$ is the slow time, t ($0 \leq t < T$) is the fast time, T is the period of the transmitted signal, and $S_m(\tilde{t}, t_l)$, $m = 1, 2, \dots, M$, denotes the transmitted baseband coded signal of the m th transmitter.

FIGURE 1: Bistatic MIMO radar scenario.

We assume that there are P high speed targets with different Doppler frequencies located at the same initial range cell. The directions of P targets with respect to the transmit array normal (i.e., DODs) are denoted by $\theta_{t1}, \theta_{t2}, \dots, \theta_{tP}$, respectively, and the directions of P targets with respect to the receive array normal (i.e., DOAs) are denoted by $\theta_{r1}, \theta_{r2}, \dots, \theta_{rP}$, respectively. Thus, $(\theta_{tp}, \theta_{rp})$ denotes the location of the p th target. Because the range walk of target within the observation time is much smaller than the target range, DODs and DOAs of targets are assumed to be constant during the observation time. The radial velocities of the p th target ($p = 1, 2, \dots, P$) with respect to the transmit array and the receive array are v_{tp} and v_{rp} , respectively, and then $v_p = v_{tp} + v_{rp}$ are the sum of two radial velocities for the p th target. The radial accelerations of the p th target with respect to the transmit array and the receive array are a_{tp} and a_{rp} , respectively, and then $a_p = a_{tp} + a_{rp}$ are the sum of two radial accelerations for the p th target. The received baseband signal through reflections of P targets can be written as [2, 3, 13, 14]

$$\mathbf{X}(\tilde{t}, t_l) = \sum_{p=1}^P \mathbf{a}_r(\theta_{rp}) \rho_p \mathbf{a}_t^T(\theta_{tp}) \mathbf{S} \left(\tilde{t} - \frac{v_p \tilde{t}}{c} - \frac{a_p \tilde{t}^2}{2c}, t_l \right) \times e^{-j2\pi(f_{dp} \tilde{t} + (a_p/2\lambda) \tilde{t}^2)} + \boldsymbol{\omega}(\tilde{t}, t_l), \quad (2)$$

where $\mathbf{X}(\tilde{t}, t_l) = [X_1(\tilde{t}, t_l) X_2(\tilde{t}, t_l) \dots X_N(\tilde{t}, t_l)]^T \in \mathbb{C}^{N \times 1}$ is the received signal vector; ρ_p denotes the complex amplitude of the reflected signal of the p th target involving the reflection coefficients and path losses of the target; $\mathbf{a}_r(\theta_{rp}) = [1 e^{-j(2\pi/\lambda)d_r \sin \theta_{rp}} \dots e^{-j(2\pi/\lambda)(N-1)d_r \sin \theta_{rp}}]^T$ and $\mathbf{a}_t(\theta_{tp}) = [1 e^{-j(2\pi/\lambda)d_t \sin \theta_{tp}} \dots e^{-j(2\pi/\lambda)(M-1)d_t \sin \theta_{tp}}]^T$ are the steering vectors of the receive array and transmit array, respectively, where λ denotes the wavelength; $f_{dp} = f_1(v_{tp} + v_{rp})/c = f_1 v_p/c$ denotes the Doppler frequency of the p th target, where f_1 is the radar carrier frequency; and the noise vector $\boldsymbol{\omega}(\tilde{t}, t_l)$ is assumed to be independent and identically distributed, and zero-mean complex white Gaussian distribution with $\boldsymbol{\omega}(\tilde{t}, t_l) \sim N^c(0, \sigma_\omega^2 \mathbf{I}_N)$, where σ_ω^2 is noise variance and $\mathbf{I}_N$ is the identity matrix with the size of $N \times N$. The contribution of the target's acceleration to the envelope migration during the observation time can be neglected and the effect of the target's velocity on the envelope migration during the fast time can also be ignored. The reason is that

the range change caused by the above two factors is much less than the radar range resolution. Under this condition, (2) can be simplified as

$$\mathbf{X}(\tilde{t}, t_l) \approx \sum_{p=1}^P \mathbf{a}_r(\theta_{rp}) \rho_p \mathbf{a}_t^T(\theta_{tp}) \mathbf{S} \left(\tilde{t} - \frac{v_p \tilde{t}}{c}, t_l \right) e^{-j2\pi f_{dp} \tilde{t}} \cdot e^{-j2\pi f_{dp} t_l} e^{-j\pi a_p t_l^2 / \lambda} + \boldsymbol{\omega}(\tilde{t}, t_l). \quad (3)$$

By (3), in general cases, the envelope migration of high speed target echo caused by the high velocity of the target would exceed one range cell during the integration time, thus affecting the accumulation of target energy. There are three exponential terms in (3). The first and the second are the Doppler terms with respect to the fast time and the slow time induced by the target's radial velocity. It is highly possible that the variety of the phase caused by the large-scale Doppler frequency of high speed target within the fast time is more than half of a turn of the unit circle; that is, $f_{dp} > 1/2T$. So the sharp distortion of target echo will occur, which results in the severe mismatch loss in matched filters, and then virtual array cannot be effectively formed. The third is the frequency modulation term because of the target's radial acceleration ignoring the effect of the target's radial acceleration on the phase variety during the fast time. Due to the range migration and the unsuccessfully formed virtual array, bistatic MIMO radar is incapable of effectively estimating DOD and DOA of high speed target.

3. Joint DOD and DOA Estimation for High Speed Target

Assume that the number of the range cells that the p th high speed target has gone through at the slow time t_l is denoted by $z_p(t_l)$, $p = 1, 2, \dots, P$, which is an integer and can be written as

$$z_p(t_l) = \left\lceil \frac{v_p t_l}{\delta} \right\rceil, \quad (4)$$

where $\delta = c/B$ is the range resolution of bistatic radar, B is the bandwidth of the transmitted signal, and $\lceil \cdot \rceil$ means the ceil operator of integer. Ignoring the truncated effect of the integer operator on the envelope migration of echo, (3) can be rewritten as

$$\mathbf{X}(\tilde{t}, t_l) \approx \sum_{p=1}^P \mathbf{a}_r(\theta_{rp}) \rho_p \mathbf{a}_t^T(\theta_{tp}) \mathbf{S} \left(\tilde{t} - \frac{z_p(t_l) \delta}{c}, t_l \right) e^{-j2\pi f_{dp} \tilde{t}} \cdot e^{-j2\pi f_{dp} t_l} e^{-j\pi a_p t_l^2 / \lambda} + \boldsymbol{\omega}(\tilde{t}, t_l). \quad (5)$$

Assume that during the observation time the total number of the across range cells of target is element of $[-Z, Z]$, where Z is an integer corresponding to the maximum possible number of the across range cells and minus

denotes that the target moves towards the radar. Multiplying the received signal of the m th receive element with the conjugate of the delayed version of the m th transmitted signal $S_m(\bar{t} - (z\delta/c), t_l)$, $z \in [-Z, Z]$, one can construct a signal $Y_{mn}(t, t_l, z)$ corresponding to the z th searching across range cell as follows:

$$\begin{aligned}
 Y_{mn}(t, t_l, z) = & \sum_{\substack{p=1 \\ p \in C_{zl}}}^P \beta_p(z) e^{-j(2\pi/\lambda)(n-1)d_r \sin \theta_{rp}} \\
 & \cdot e^{-j(2\pi/\lambda)(m-1)d_t \sin \theta_{tp}} e^{-j2\pi f_{dp} t} \\
 & \cdot e^{-j2\pi f_{dp} t_l} e^{-j\pi a_p t_l^2 / \lambda} \\
 & + \phi_{mn}(t, t_l, z) + W_{mn}(t, t_l, z), \\
 m = 1, 2, \dots, M, \quad n = 1, 2, \dots, N,
 \end{aligned} \tag{6}$$

where C_{zl} denotes the set for the variable p , that is, when $p \in C_{zl}$ the equality $z_p(t_l) = z$ should hold, $\beta_p(z) = \rho_p e^{j\psi(z)}$, $\psi(z)$ is the initial phase of the p th target reflected signal when this target is located at the z th range cell, and $W_{mn}(t, t_l, z) = \omega_n(t, t_l) S_m^*(t + t_l - (z\delta/c), t_l)$, where $[\cdot]^*$ denotes complex conjugate. The first term in the right hand side of (6) holds under the condition $S_m(t + t_l - (z_p(t_l)\delta/c), t_l) S_m^*(t + t_l - (z\delta/c), t_l) = 1$ with $z_p(t_l) = z$, which becomes a sinusoid signal over the fast time. The second term in the right hand side of (6) is given by

$$\begin{aligned}
 \phi_{mn}(t, t_l, z) = & \sum_{\substack{p=1 \\ p \in C_{zl}}}^P \beta_p(z) e^{-j(2\pi/\lambda)(n-1)d_r \sin \theta_{rp}} \\
 & \cdot \sum_{\substack{i=1 \\ i \neq m}}^M e^{-j(2\pi/\lambda)(i-1)d_t \sin \theta_{tp}} S_i\left(t + t_l - \frac{z\delta}{c}, t_l\right) \\
 & \times S_m^*\left(t + t_l - \frac{z\delta}{c}, t_l\right) \\
 & \cdot e^{-j2\pi f_{dp} t} \cdot e^{-j2\pi f_{dp} t_l} e^{-j\pi a_p t_l^2 / \lambda} \\
 & + \sum_{\substack{p=1 \\ p \notin C_z}}^P \beta_p e^{-j(2\pi/\lambda)(N-1)d_r \sin \theta_{rp}} \mathbf{a}_t^T(\theta_{tp}) \\
 & \cdot \mathbf{S}\left(t + t_l - \frac{z_p(t_l)\delta}{c}, t_l\right) S_m^*\left(t + t_l - \frac{z\delta}{c}, t_l\right) e^{-j2\pi f_{dp} t} \\
 & \cdot e^{-j2\pi f_{dp} t_l} e^{-j\pi a_p t_l^2 / \lambda}.
 \end{aligned} \tag{7}$$

Following that, a Fourier transform (FT) to the variable t is employed to transform the signal $Y_{mn}(t, t_l, z)$ into the fast-time frequency domain, which can be written as

$$\begin{aligned}
 Y_{mn}(f, t_l, z) = & \int_0^T Y_{mn}(t, t_l) e^{-j2\pi f t} dt \\
 = & \sum_{\substack{p=1 \\ p \in C_z}}^P \beta_p(z) e^{-j(2\pi/\lambda)(n-1)d_r \sin \theta_{rp}} \\
 & \cdot e^{-j(2\pi/\lambda)(m-1)d_t \sin \theta_{tp}} \\
 & \cdot \frac{\sin[\pi(f + f_{dp})T]}{\pi(f + f_{dp})T} \\
 & \cdot e^{-j\pi(f + f_{dp})T} \cdot e^{-j2\pi f_{dp} t_l} e^{-j\pi a_p t_l^2 / \lambda} \\
 & + \phi_{mn}(f, t_l, z) + W_{mn}(f, t_l, z).
 \end{aligned} \tag{8}$$

Because of the high target speed and low repetition frequency of radar signal, it is highly possible that under-sampling will occur [13, 14]. In this situation, the target's true Doppler frequency can be expressed as [13, 14]

$$f_{dp} = f_{dp0} + \frac{n_p}{T}, \tag{9}$$

where f_{dp0} is the ambiguous Doppler frequency and n_p is the fold factor. Substituting (9) into (8), we can obtain

$$\begin{aligned}
 Y_{mn}(f, t_l, z) = & \sum_{\substack{p=1 \\ p \in C_{zl}}}^P \beta_p(z) e^{-j(2\pi/\lambda)(n-1)d_r \sin \theta_{rp}} \\
 & \cdot e^{-j(2\pi/\lambda)(m-1)d_t \sin \theta_{tp}} \cdot \frac{\sin[\pi(f + f_{dp})T]}{\pi(f + f_{dp})T} \\
 & \cdot e^{-j\pi(f + f_{dp})T} \cdot e^{-j2\pi f_{dp0} t_l} \cdot e^{-j2\pi(n_p/T)t_l} \\
 & \cdot e^{-j\pi a_p t_l^2 / \lambda} + \phi_{mn}(f, t_l, z) + W_{mn}(f, t_l, z).
 \end{aligned} \tag{10}$$

It is worth pointing out that $2\pi(n_p/T)t_l$ in the fifth exponential term is integer times of 2π because of $t_l = lT$ and the integer n_p . This term in (10) becomes $e^{-j2\pi(n_p/T)t_l} = 1$. So (10) can be rewritten as

$$\begin{aligned}
 Y_{mn}(f, t_l, z) = & \sum_{\substack{p=1 \\ p \in C_{zl}}}^P \beta_p(z) e^{-j(2\pi/\lambda)(n-1)d_r \sin \theta_{rp}} \\
 & \cdot e^{-j(2\pi/\lambda)(m-1)d_t \sin \theta_{tp}} \cdot \frac{\sin[\pi(f + f_{dp})T]}{\pi(f + f_{dp})T} e^{-j\pi(f + f_{dp})T} \\
 & \cdot e^{-j2\pi f_{dp0} t_l} \cdot e^{-j\pi a_p t_l^2 / \lambda} + \phi_{mn}(f, t_l, z) + W_{mn}(f, t_l, z).
 \end{aligned} \tag{11}$$

Assume that the reflected signal of the p th target located at the z th range cell is distributed along the repetition period index $l = L_{p\min}, L_{p\min} + 1, \dots, L_{p\max}$. Following that, a Fourier transform is taken to the signal $Y_{mn}(f, t_l, z)$ with variable t_l , and with the principle of stationary phase [22] we have

$$\begin{aligned}
 Y_{mn}(f, f_l, z) \approx & \sum_{\substack{p=1 \\ p \in C_{zl}}}^P \beta_p(z) e^{-j(2\pi/\lambda)(n-1)d_r \sin \theta_{rp}} \\
 & \cdot e^{-j(2\pi/\lambda)(m-1)d_t \sin \theta_{tp}} \\
 & \cdot \frac{\sin[\pi(f + f_{dp})T]}{\pi(f + f_{dp})T} e^{-j\pi(f + f_{dp})T} \\
 & \cdot \text{rect}\left[\frac{f_l + f_{dp0}}{(L_{p\max} - L_{p\min})T \cdot a_p/\lambda}\right] \\
 & \times e^{j\pi(f_l + f_{dp0})^2/(a_p/\lambda)} \\
 & \times e^{-j\pi f_l(L_{p\max} + L_{p\min})T} + \zeta_{mn}(f, f_l, z),
 \end{aligned} \quad (12)$$

where $\text{rect}[u] = \begin{cases} 1, & |u| \leq 1/2 \\ 0, & |u| > 1/2 \end{cases}$, $\zeta_{mn}(f, f_l, z) = \phi_{mn}(f, f_l, z) + W_n(f, f_l, z)$, and $\phi_{mn}(f, f_l, z)$, $W_n(f, f_l, z)$ are the Fourier transform of $\phi_{mn}(f, t_l, z)$ and $W_{mn}(f, t_l, z)$, respectively. Thus, Fourier transforms over both fast time and slow time have transformed the multiplied signals into the fast-time frequency and slow-time frequency domains. However, in the slow-time frequency domain, the target's acceleration broadens the spectrum and the Doppler frequency ambiguity occurs.

Due to the P target echoes existing in the initial range cell, that is, $z = 0$, the frequency-domain data $Y_{mn}(f, f_l, z = 0)$ ($m = 1, 2, \dots, M$, $n = 1, 2, \dots, N$) corresponding to the initial range cell can be exploited to estimate the Doppler frequencies and the ambiguous Doppler frequencies of P targets. When $f = -f_{dp}$ and $f_l = -f_{dp0}$, $p = 1, 2, \dots, P$, $Y_{mn}(f, f_l, z = 0)$ will generate the peaks. In order to improve the signal-to-noise ratio (SNR) for detection, the frequency-domain data with all transmitters and receivers of the bistatic MIMO radar is incoherently accumulated. Therefore, f_{dp} and f_{dp0} can be estimated as

$$(\hat{f}_{dp}, \hat{f}_{dp0}) = \arg \max_{f, f_l} \sum_{m=1}^M \sum_{n=1}^N |Y_{mn}(-f, -f_l, z = 0)|. \quad (13)$$

Unfortunately, because the estimated $\hat{f}_{dp}$ is integer times of the repetition frequency of radar signal, the fold factor n_p in (9) cannot be solved by the estimated $\hat{f}_{dp}$ and $\hat{f}_{dp0}$. Then the p th target's velocity v_p can be obtained only by the estimated unambiguous Doppler frequency corresponding to the fast-time frequency domain, which can be expressed as

$$\hat{v}_p = \lambda \cdot \hat{f}_{dp}. \quad (14)$$

Using the very short echo time, the Fourier transform over fast-time domain has poor frequency resolution so

that the Doppler frequency shifting value caused by target's acceleration during the observation time is much less than half of the fast-time frequency resolution. Then the target's energy can be approximately regarded as being concentrated in the same fast-time frequency cell during the observation time. However, due to the high frequency resolution of Fourier transform over slow-time domain, the target's energy may be distributed along several slow-time frequency cells.

The frequency-domain data of the p th target can be extracted from $Y_{mn}(f, f_l, z)$ along the slow-time frequency dimension when $f = -f_{dp} = -\hat{v}_p/\lambda$, which can be written as

$$\begin{aligned}
 Y_{mn}\left(-\frac{\hat{v}_p}{\lambda}, f_l, z\right) = & e^{-j(2\pi/\lambda)(n-1)d_r \sin \theta_{rp}} \\
 & \cdot e^{-j(2\pi/\lambda)(m-1)d_t \sin \theta_{tp}} \\
 & \cdot H_p\left(-\frac{\hat{v}_p}{\lambda}, f_l, z\right) \\
 & + \zeta_{mn}\left(-\frac{\hat{v}_p}{\lambda}, f_l, z\right), \quad p \in C_z,
 \end{aligned} \quad (15)$$

where $H_p(-\hat{v}_p/\lambda, f_l, z) = \beta_p(z) e^{-j\pi(\hat{v}_p/\lambda + f_{dp})T} \cdot \text{rect}[(f_l + f_{dp0})/((L_{p\max} - L_{p\min})T \cdot a_p/\lambda)] \cdot e^{-j\pi(f_l + f_{dp0})^2/(a_p/\lambda)} e^{-j\pi f_l(L_{p\max} + L_{p\min})T}$.

In fact, $Y_{mn}(-\hat{v}_p/\lambda, f_l, z)$ can be considered as the separated target component from the m th transmitter to the n th receiver via the p th target located at the z th range cell, $m = 1, 2, \dots, M$, $n = 1, 2, \dots, N$. The separated target components for the p th target with all transmitters and receivers of bistatic MIMO radar can be expressed as

$$\begin{aligned}
 \mathbf{Y}\left(-\frac{\hat{v}_p}{\lambda}, f_l, z\right) = & \mathbf{A}(\theta_{rp}, \theta_{tp}) H_p\left(-\frac{\hat{v}_p}{\lambda}, f_l, z\right) \\
 & + \boldsymbol{\zeta}\left(-\frac{\hat{v}_p}{\lambda}, f_l, z\right), \quad p \in C_z,
 \end{aligned} \quad (16)$$

where $\mathbf{Y}(-\hat{v}_p/\lambda, f_l, z) = [Y_1(-\hat{v}_p/\lambda, f_l, z), Y_2(-\hat{v}_p/\lambda, f_l, z), \dots, Y_{MN}(-\hat{v}_p/\lambda, f_l, z)]^T$, $\mathbf{A}(\theta_{rp}, \theta_{tp}) = \mathbf{a}_{r1}(\theta_{rp}) \otimes \mathbf{a}_{t1}(\theta_{tp})$, $\otimes$ is the Kronecker product, and the vector $\mathbf{W}_l \in \mathbb{C}^{MN \times 1}$ represents a noise vector.

Assume that the p th target moves away from radar. Analyzing the results of (13), we find that the indexes of the range cells that the p th high speed target has gone through during the long observation time are $z_p = 0, 1, \dots, Z_p$, respectively. Then the separated target components distributed along all the across range cells for the p th target are spliced as follows:

$$\begin{aligned}
 \tilde{\mathbf{Y}}_{pl} = & \left[\mathbf{Y}\left(-\frac{\hat{v}_p}{\lambda}, f_l, 0\right), \mathbf{Y}\left(-\frac{\hat{v}_p}{\lambda}, f_l, 1\right), \dots, \right. \\
 & \left. \mathbf{Y}\left(-\frac{\hat{v}_p}{\lambda}, f_l, Z_p\right) \right],
 \end{aligned} \quad (17)$$

where $\tilde{\mathbf{Y}}_{pl} \in \mathbb{C}^{MN \times Z_p}$ is the output of the formed virtual array with MN virtual elements at the slow-time index l . Using (17),

we can form the covariance matrix of $\tilde{\mathbf{Y}}_{pl}$ for the p th target from the spliced data of L snapshots, which is given by

$$\mathbf{R}_p = \frac{1}{Z_p L} \sum_{l=1}^L \tilde{\mathbf{Y}}_{pl} \cdot \tilde{\mathbf{Y}}_{pl}^H = \frac{1}{Z_p} \sum_{z=0}^{Z_p} \frac{1}{L} \cdot \sum_{l=1}^L \mathbf{Y} \left(-\frac{\hat{v}_p}{\lambda, f_l, z} \right) \mathbf{Y}^H \left(-\frac{\hat{v}_p}{\lambda, f_l, z} \right), \quad (18)$$

where $(\cdot)^H$ denotes the Hermitian transpose. It is worth pointing out that the estimation precision of covariance matrix after splicing the separated target components that are distributed along all the across range cells can be remarkably improved due to the average of the covariance matrixes corresponding to all the across range cells. Thus, the angle estimation precision can also be increased using the more accurate covariance matrix.

Assume that the p th target moves towards radar. Analyzing the results of (13), we find that the indexes of the range cells that the p th high speed target has gone through during the long observation time are $z_p = 0, -1, \dots, -Z_p$, respectively. Then the separated target components for the p th target distributed along all the across range cells are spliced as follows:

$$\tilde{\mathbf{Y}}_{pl} = \left[\mathbf{Y} \left(-\frac{\hat{v}_p}{\lambda, f_l, 0} \right), \mathbf{Y} \left(-\frac{\hat{v}_p}{\lambda, f_l, -1} \right), \dots, \mathbf{Y} \left(-\frac{\hat{v}_p}{\lambda, f_l, -Z_p} \right) \right]. \quad (19)$$

After synthesizing the virtual array with the range migration removed, most of the existing superresolution angle estimation algorithms can be applied to obtain DOD and DOA of the high speed target. In this paper, only the ESPRIT algorithm is used for angle estimation because of its simplicity and efficiency. Using (19), we can form the covariance matrix of $\tilde{\mathbf{Y}}_{pl}$ for the p th target from the spliced data of L snapshots, which can be written as

$$\mathbf{R}_p = \frac{1}{Z_p L} \sum_{l=1}^L \tilde{\mathbf{Y}}_{pl} \cdot \tilde{\mathbf{Y}}_{pl}^H = \frac{1}{Z_p} \sum_{z=-Z_p}^0 \frac{1}{L} \cdot \sum_{l=1}^L \mathbf{Y} \left(-\frac{\hat{v}_p}{\lambda, f_l, z} \right) \mathbf{Y}^H \left(-\frac{\hat{v}_p}{\lambda, f_l, z} \right). \quad (20)$$

Performing the eigendecomposition on the $MN \times MN$ sample covariance matrix $\mathbf{R}_p$ [5], we then have

$$\mathbf{R}_p = \mathbf{U}_s \Sigma_s \mathbf{U}_s^H + \mathbf{U}_n \Sigma_n \mathbf{U}_n^H, \quad (21)$$

where Σ_s is a scalar representing the largest eigenvalue because of only one existing target in $\mathbf{R}_p$, Σ_n is a diagonal matrix constructed by the remaining $MN - 1$ eigenvalues, $\mathbf{U}_s \in \mathbb{C}^{MN \times 1}$ is the signal subspace composed of the eigenvector corresponding to the largest eigenvalue, and $\mathbf{U}_n \in \mathbb{C}^{MN \times (MN-1)}$ is the noise subspace containing the remaining

$MN - 1$ eigenvectors of $\mathbf{R}_p$. Because the signal subspace spans the same space with the steering matrix, we have $\mathbf{U}_s = \mathbf{A}(\theta_{rp}, \theta_{tp})T$, where T is a nonzero scalar.

Define $\mathbf{A}'(\theta_{rp}, \theta_{tp}) = \mathbf{a}_{t1}(\theta_{tp}) \otimes \mathbf{a}_{r1}(\theta_{rp})$, which is row equivalent to $\mathbf{A}(\theta_{rp}, \theta_{tp})$ [7]. Suppose that $\mathbf{U}'_s$ is a $MN \times 1$ signal subspace vector formed from $\mathbf{U}_s$ by the same row interchange operations as $\mathbf{A}'(\theta_{rp}, \theta_{tp})$ is formed from $\mathbf{A}(\theta_{rp}, \theta_{tp})$. Let $\mathbf{U}_{s1}$ and $\mathbf{U}_{s2}$ be the first and the last $(N - 1)M$ rows of $\mathbf{U}_s$, and let $\mathbf{U}'_{s1}$ and $\mathbf{U}'_{s2}$ be the first and the last $(M - 1)N$ rows of $\mathbf{U}'_s$.

Then let us define the average estimators as follows [7]:

$$\begin{aligned} r_{rp} &= \frac{1}{M(N-1)} \sum_{i=1}^{M(N-1)} \frac{\mathbf{U}_{s2}(i)}{\mathbf{U}_{s1}(i)}, \\ r_{tp} &= \frac{1}{N(M-1)} \sum_{i=1}^{N(M-1)} \frac{\mathbf{U}'_{s2}(i)}{\mathbf{U}'_{s1}(i)}, \end{aligned} \quad (22)$$

where $\mathbf{U}_{s1}(i)$ and $\mathbf{U}_{s2}(i)$ are the i th elements in $\mathbf{U}_{s1}$ and $\mathbf{U}_{s2}$, respectively, and $\mathbf{U}'_{s1}(i)$ and $\mathbf{U}'_{s2}(i)$ are the i th elements in $\mathbf{U}'_{s1}$ and $\mathbf{U}'_{s2}$, respectively. Therefore, the DOA θ_{rp} and DOD θ_{tp} for the p th target, $p = 1, 2, \dots, P$, can be written as

$$\begin{aligned} \hat{\theta}_{rp} &= \arcsin \left(-\lambda \cdot \frac{\text{angle}(r_{rp})}{2\pi d_r} \right), \\ \hat{\theta}_{tp} &= \arcsin \left(-\lambda \cdot \frac{\text{angle}(r_{tp})}{2\pi d_t} \right). \end{aligned} \quad (23)$$

The DOAs and DODs for the other targets can also be estimated by the same method. In summary, the DOAs and DODs of high speed targets for bistatic MIMO radar can be estimated via the following procedure.

Step 1. The signal $Y_{mn}(t, t_l, z)$ corresponding to the z th searching across range cell is constructed by multiplying the received signal of the n th receive element with the conjugate of the delayed version of the m th transmitted signal $S_m(\bar{t} - (z\delta/c), t_l)$, $z \in [-Z, Z]$.

Step 2. Following that, a Fourier transform (FT) to the variable t is employed to transform the signal $Y_{mn}(t, t_l, z)$ into the fast-time frequency domain, thus obtaining the signal $Y_{mn}(f, t_l, z)$.

Step 3. A Fourier transform is taken to the signal $Y_{mn}(f, t_l, z)$ with variable t_l , and we have the signal $Y_{mn}(f, f_l, z)$.

Fourier transforms over both fast time and slow time shown in Steps 2 and 3 have transformed the multiplied signals into the fast-time frequency and slow-time frequency domains.

Step 4. The frequency-domain data of the p th target $Y_{mn}(-\hat{v}_p/\lambda, f_l, z)$ can be extracted from $Y_{mn}(f, f_l, z)$ along the slow-time frequency dimension, which can be considered as the separated target component from the m th transmitter to the n th receiver via the p th target located at the z th range

cell. Then the separated target components for the p th target with all transmitters and receivers of bistatic MIMO radar are expressed as $\mathbf{Y}(-\tilde{v}_p/\lambda, f_i, z)$.

Step 5. The separated target components for the p th target distributed along all the across range cells are spliced. Thus, the output of the formed virtual array for the p th high speed target is expressed as $\tilde{\mathbf{Y}}_{pl}$.

Step 6. The DOD and DOA of p th high speed target can be obtained from the output of virtual array $\tilde{\mathbf{Y}}_{pl}$ using the superresolution algorithm, that is, ESPRIT algorithm. The DOAs and DODs for the other targets can also be estimated by repeating Steps 4–6.

4. Simulation Result

Some numerical examples are presented to illustrate the performance of the proposed method. A bistatic MIMO radar with $M = 6$ transmitters and $N = 8$ receivers is adopted. Radar carrier frequency $f_1 = 10$ GHz, and the spacing between adjacent elements used for both transmit array and receive array is $d_t = d_r = 1.5$ cm. The Gold sequences are selected as the transmitted baseband signals of bistatic MIMO radar, which are nearly orthogonal binary codes with good autocorrelation and intercorrelation. The duration of a code is assumed to be $0.1 \mu\text{s}$, the period length of Gold sequence is 1023, and then the repetition period of the transmitted signals is $102.3 \mu\text{s}$. Suppose that the number of the repetition periods for the transmitted signals during the observation time is 128.

4.1. Results of 2D Fourier Transform of the Multiplied Signals. Assume that three high speed targets located at the same initial range cell and their DODs and DOAs are $(\theta_{t1}, \theta_{r1}) = (30^\circ, 60^\circ)$, $(\theta_{t2}, \theta_{r2}) = (5^\circ, 40^\circ)$, and $(\theta_{t3}, \theta_{r3}) = (25^\circ, 10^\circ)$, respectively. The sums of two radial velocities with respect to the transmit array and the receive array for three targets are given by 7500 m/s, 9000 m/s, and 6500 m/s, respectively. The sums of two radial accelerations with respect to the transmit array and the receive array for three targets are 400 m/s^2 , 500 m/s^2 , and 450 m/s^2 , respectively. The SNRs of three targets are all -25 dB. The received signals are multiplied with the conjugate of the delayed versions of the transmitted signals, and then the Fourier transform (FT) of the multiplied signals over both fast time and slow time is employed.

Figures 2(a)–2(f) show the results of 2D Fourier transform of the multiplied signals corresponding to the different searching across range cells, where the peak value indicates the existing target. By comparing the results of Figures 2(a)–2(f), we can show that the target's energy is concentrated in the same fast-time frequency cell when the target is located at the different range cells. The outputs are below noise level and unobservable as in Figures 2(a) and 2(f) with the searching range cell indexes $z = -1, 4$. However, there are three or two peaks that are much higher than the noise background and may easily be detected as in Figures 2(b), 2(c), 2(d), and 2(e) with the searching range cell indexes $z = 0, 1, 2, 3$. Thus, the energy of one target is distributed along the range cells

with the indexes $z = 0, 1, 2$, while the other two go through the range cells with the indexes $z = 0, 1, 2, 3$. The estimated target's velocities are 7625.8 m/s, 9092.3 m/s, and 6452.6 m/s, which are obtained by the estimated unambiguous Doppler frequency corresponding to the fast-time frequency domain. The relative errors of velocity estimation are large because of poor frequency resolution in fast-time frequency dimension. However, the Doppler frequency ambiguity occurs in the slow-time frequency dimension.

4.2. DOD and DOA Estimation for High Speed Target. In this simulation, we evaluate the DOD and DOA estimation performance of the proposed method and the standard ESPRIT method [5] that is employed in bistatic MIMO radar for angle estimation. The simulation parameters for the three targets are the same as those in Section 4.1. Figure 3 shows the obtained result by using the standard ESPRIT method for all three high speed targets with 150 Monte Carlo tests. We can observe that the high speed targets are unsuccessfully localized by the standard ESPRIT method due to the invalidly formed virtual array and the range migration. Figure 4 shows the obtained result by using the proposed method for all three high speed targets over 150 Monte Carlo tests. It is clear that the DOAs and DODs of the three high speed targets are well estimated and automatically paired. The crosses denote the true locations of the three high speed targets in Figures 3 and 4.

4.3. Angle Estimation Performance for High Speed Target versus SNR. In this simulation, the SNRs of three targets identically range from -30 dB to 0 dB, while the other parameters for the three targets are the same as those in Section 4.1. The root mean square error (RMSE) of two-dimensional angle estimation is defined as

$$\text{RMSE} = \sqrt{\frac{1}{\text{LC}} \sum_{i=1}^{\text{LC}} [(\hat{\theta}_{ti} - \theta_t)^2 + (\hat{\theta}_{ri} - \theta_r)^2]}, \quad (24)$$

where LC is the number of the independent trials and $\hat{\theta}_{ti}$ and $\hat{\theta}_{ri}$ are the estimates of true DOD θ_t and DOA θ_r in the i th Monte Carlo trial. Figure 5 shows the RMSEs of DOD and DOA estimation versus SNR for target 1, which is located at $(30^\circ, 60^\circ)$ with the velocity $v_1 = 7500$ m/s and $a_1 = 400 \text{ m/s}^2$. The number of Monte Carlo tests is 200. It is shown that the standard ESPRIT method fails to estimate the angles of the high speed target, while the proposed method has a marked performance improvement over the standard ESPRIT method.

Because only the DOD and DOA estimation problem for high speed target is considered in this paper, the Cramer-Rao bounds (CRBs) for target's motion parameters of different order [23] are not given. Moreover, the CRBs for target's DOD and DOA parameters in bistatic MIMO radar are derived under the condition that the virtual array can be successfully formed by matched filters [2], which cannot be used as a proper benchmark for evaluating the performance of the proposed method. The proposed method is provided for

FIGURE 2: Results of 2D Fourier transform of the multiplied signals.

overcoming the effect of the large-scale Doppler frequency on the virtual array formation and removing range migration during the long-time integration, thus facilitating the angle estimation for high speed target in bistatic MIMO radar. Therefore, it is a good way to make the angle estimation

performance of the standard ESPRIT method when dealing with the low speed target without acceleration serve as a benchmark for performance comparison as shown in Figure 5. It is clearly shown that the standard ESPRIT method has a good performance in the estimation of DOD and DOA

FIGURE 3: Estimation results of the standard ESPRIT method for three high speed targets.

FIGURE 4: Estimation results of the proposed method for three high speed targets.

of the low speed target due to the effective virtual array formed by conventional matched filters and absence of range walk, while the proposed method exploited to estimate angles of the high speed target performs only a little worse than the standard ESPRIT method used for dealing with the low speed target. Moreover, for the high speed target, the angle estimation performances of the proposed method in the case of target acceleration $a_1 = 0 \text{ m/s}^2$ and the proposed method using the target energy only in the initial range cell are also shown in Figure 5. These results illustrate that whether there exists target acceleration, the proposed method has a robust performance, and the proposed method using the target energy in all the across range cells attains better performance compared with that using the target energy only in the initial range cell. Thus, splicing the separated target components

FIGURE 5: RMSEs of DOD and DOA estimation versus SNR for target 1.

that are distributed along several range cells can improve the angle estimation accuracy.

5. Conclusions

A method for high speed target angle estimation using bistatic MIMO radar is proposed. For the target with high velocity, range migration always occurs and the serious mismatch induced by the target's large-scale Doppler frequency usually exists in matched filter, which creates difficulties for target parameter estimation. After 2D Fourier transform of the multiplied signals which are obtained by multiplying the received signals with the conjugate of the delayed versions of the transmitted signals, the proposed method perfectly separates the target components of the radar return corresponding to the different transmitted waveforms and then splices the separated target components that are distributed along several range cells to successfully synthesize the virtual array. Thus, the DOD and DOA of high speed target can be estimated by using the superresolution algorithm. The effectiveness of the proposed method has been demonstrated by the numerical experimental results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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