The Radiation Problem from a Vertical Hertzian Dipole Antenna above Flat and Lossy Ground: Novel Formulation in the Spectral Domain with Closed-Form Analytical Solution in the High Frequency Regime

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1. Introduction

The so-called “Sommerfeld radiation problem” is a well-known problem in the area of propagation of electromagnetic (EM) waves above flat lossy ground for obvious applications in the area of wireless telecommunications [1–6]. The classical Sommerfeld solution to this problem is provided in the physical space by using the so-called “Hertz potentials” and it does not end up with closed-form analytical solutions. Norton [7, 8] concentrated in subsequent years more on the engineering application of the above problem with obvious application to wireless communications and provided approximate solutions to the above problem, which are represented by rather long algebraic expressions for engineering use, in which the so-called “attenuation coefficient” for the propagating surface wave plays an important role.

In this paper, the authors take advantage of previous research work of them for the EM radiation problem in free space [9] by using the spectral domain approach. Furthermore, in [10], the authors provided the fundamental formulation for the problem considered here, that is, the solution in spectral domain for the radiation from a dipole moment at a specific angular frequency (\(\omega\)) in isotropic media with a flat infinite interface. In that paper, the authors end up with integral representations for the received electric and magnetic fields above or below the interface (line-of-sight (LOS) plus reflected field-transmitted fields, resp.), where the integration takes place over the radial spectral coordinate...
2. Geometry of the Radiation Problem

The geometry of the problem is given in Figure 1. Here, a Hertzian (small) dipole with dipole moment \( p \) directed to positive \( x \)-axis, at altitude \( x_0 \) above the infinite, flat, and lossy ground, radiates time-harmonic electromagnetic (EM) waves at angular frequency \( \omega = 2\pi f \) (exp(−iωt) time dependence is assumed in this paper). Here, the relative complex permittivity of the ground (medium 2) is \( \varepsilon' = \varepsilon / \varepsilon_0 = \varepsilon_r + i \varepsilon_i \), where \( \varepsilon = \sigma / \omega \varepsilon_0 = 18 \times 10^9 \sigma / f \), with \( \sigma \) being the ground conductivity and \( f \) the frequency of radiation, and \( \varepsilon_0 = 8.854 \times 10^{-12} \) F/m is the absolute permittivity in vacuum or air. Then, the wavenumbers of propagation of EM waves in air and lossy ground, respectively, are given by the following:

\[
\begin{align*}
k_{01} &= \frac{\omega}{c_1} = \omega \sqrt{\varepsilon_0 \mu_1} = \omega \sqrt{\varepsilon_0 \mu_0}, \\
k_{02} &= \frac{\omega}{c_2} = \omega \sqrt{\varepsilon_2 \mu_2} = \omega \sqrt{\varepsilon_2 \mu_0} = k_{01} \sqrt{\varepsilon_r + i \varepsilon_x}.
\end{align*}
\]

(1)

The Maxwell equations for the time-harmonic EM fields considered above are given by

\[
\begin{align*}
\text{rot} \, E - i \omega \mu_0 \mu_1 H &= 0, \\
\text{rot} \, H + i \omega \varepsilon_0 \varepsilon_1 H &= j,
\end{align*}
\]

(2)

where \( j \) is current density (source of EM fields considered here).

3. Formulation of the Sommerfeld Radiation Problem in the Spectral Domain: Integral Representation for the Received Electric and Magnetic Fields

3.1. EM Fields in terms of Spectral Domain Current Densities.

Following [9, 10], the EM field in physical space is derived from current density \( J \) in spectral domain and Green's function \( \tilde{\Psi} \), also in the spectral domain, through inverse three-dimensional (3D) Fourier transformation as follows:

\[
H = -i F^{-1} \left[ \tilde{\Psi} \cdot \left( k \times \tilde{J} \right) \right],
\]

(3)

\[
E = -\frac{i}{\omega \varepsilon_0 \varepsilon_1} F^{-1} \left[ \tilde{\Psi} \left[ \varepsilon_1 \mu_1 k_{01}^2 \tilde{J} - \left( k \cdot \tilde{J} \right) k \right] \right],
\]

(4)

where the symbol \( \langle \rangle \) denotes the inner product, \( F^{-1} \) is the inverse 3D Fourier transform (FT) operator, and

\[
\tilde{\Psi} = (k_{01}^2 - k^2)^{-1} = (k_{01}^2 - k_x^2 - k_z^2)^{-1}
\]

(5)

is 3D Green’s function in spectral domain and cylindrical coordinates. Furthermore, by noting that, for the problem considered here, current density \( \tilde{J} = \tilde{J}(k_x, 0, 0) \) has only \( x \)-component and that wavevector \( \tilde{k} = (k_x, k_y = 0, k_z) \) does not possess azimuthal \( \alpha \) component, by performing the cross product and inverse FT operation of (3), we obtain

\[
H(r) = -\frac{i}{(2\pi)^3} \tilde{e}_x \int_{k_x = 0}^{\infty} \int_{k_y = -\infty}^{\infty} \int_{k_z = -\infty}^{\infty} k_x \tilde{J}(k_x) \tilde{\Psi} k_x \exp \left( i k_x r \right) dk_x dk_y dk_z,
\]

(6)

Similarly, by performing the inner product and inverse FT operation of (4), we obtain

\[
E(r) = -\frac{i}{(2\pi)^3} \varepsilon_0 \varepsilon_1 \omega \int_{k_x = 0}^{\infty} \int_{k_y = -\infty}^{\infty} \int_{k_z = -\infty}^{\infty} (e_1 \mu_1 k_{01}^2 \tilde{e}_x - k_x \tilde{J}(k_x) \tilde{\Psi} k_x \exp \left( i k_x r \right) dk_x dk_y dk_z
\]

(7)
where
\[ k = (k_\rho, 0, k_x) = k_\rho \hat{e}_\rho + k_x \hat{e}_x \]  
(8)
is the wavevector of propagation and \( r = (\rho, \alpha, x) \) is the point of observation (see Figure 1), all in cylindrical coordinates. Furthermore, by taking (8) into account, (7) for the received electric field can also be written in the following form:

\[ E(r) = -\frac{i}{2\pi \omega e_0 \varepsilon} \int_0^\infty \int_0^{2\pi} \left( (\varepsilon \mu k_\rho^2 - k_x^2) \hat{e}_\rho - k_x k_x \hat{e}_x \right) \cdot \hat{T}(k_\rho) \psi k_\rho \times \exp (ik \cdot r) \, dk_\rho \, d\alpha \, dk_x, \]

(9)

Furthermore, in order to integrate expressions (6) and (9) with respect to azimuthal angle \( \alpha \) (see Figure 1), we take into account the fact that

\[ k \cdot r = k_x x + k_\rho \rho \cdot \cos (\alpha - \beta), \]

(10)

where \( \beta \) is the azimuth angle of the projection of vector \( k \) on the \( yz \)-plane (see Figure 1). Then, by using the following identities for Bessel functions:

\[ \frac{1}{2\pi} \int_0^{2\pi} \exp (ik \cdot r \cos \alpha) \, d\alpha = I_0(k_\rho \rho), \]

\[ \int_0^\infty I_0(k_\rho \rho) \, dk_\rho = \frac{1}{2} \int_{-\infty}^\infty H_0^{(1)}(k_\rho \rho) \, dk_\rho, \]

(11)

where \( I_0 \) is the Bessel function of first kind and zero order and \( H_0^{(1)} \) is the Hankel function of first kind and zero order, we obtain

\[ H(r) = -\frac{i}{8\pi^2 \omega e_0 \varepsilon} \int_{k_\rho = -\infty}^{k_\rho = \infty} \int_{k_x = -\infty}^{k_x = \infty} k_x \hat{T}(k_\rho) \psi k_\rho \times \hat{T}(k_\rho) \exp (ik \cdot r) \, dk_\rho \, dk_x, \]

\[ E(r) = -\frac{i}{8\pi^2 \omega e_0 \varepsilon} \int_{k_\rho = -\infty}^{k_\rho = \infty} \int_{k_x = -\infty}^{k_x = \infty} \left( (\varepsilon \mu k_\rho^2 - k_x^2) \hat{e}_\rho - k_x k_x \hat{e}_x \right) \times \hat{T}(k_\rho) \psi k_\rho H_0^{(1)}(k_\rho \rho) \times \exp (ik \cdot r) \, dk_\rho \, dk_x, \]

(12)

3.2. Formulation of the Boundary Value Problem. For the problem considered in this work (Figure 1), we now use (12), to write the appropriate expressions for the reflected (\( R \)) and transmitted (\( T \)) EM field, as follows:

\[ H_R(r) = -\frac{i}{8\pi^2 \omega e_0 \varepsilon} \int_{k_\rho = -\infty}^{k_\rho = \infty} \int_{k_x = -\infty}^{k_x = \infty} k_x \hat{T}_R(k_\rho) \psi k_\rho \times \hat{T}_R(k_\rho) \exp (ik \cdot r) \, dk_\rho \, dk_x, \]

\[ E_R(r) = -\frac{i}{8\pi^2 \omega e_0 \varepsilon} \int_{k_\rho = -\infty}^{k_\rho = \infty} \int_{k_x = -\infty}^{k_x = \infty} \left( (\varepsilon \mu k_\rho^2 - k_x^2) \hat{e}_\rho - k_x k_x \hat{e}_x \right) \times \hat{T}_R(k_\rho) \psi k_\rho H_0^{(1)}(k_\rho \rho) \times \exp (ik \cdot r) \, dk_\rho \, dk_x, \]

\[ H_T(r) = -\frac{i}{8\pi^2 \omega e_0 \varepsilon} \int_{k_\rho = -\infty}^{k_\rho = \infty} \int_{k_x = -\infty}^{k_x = \infty} k_x \hat{T}_T(k_\rho) \psi k_\rho \times \hat{T}_T(k_\rho) \exp (ik \cdot r) \, dk_\rho \, dk_x, \]

\[ E_T(r) = -\frac{i}{8\pi^2 \omega e_0 \varepsilon} \int_{k_\rho = -\infty}^{k_\rho = \infty} \int_{k_x = -\infty}^{k_x = \infty} \left( (\varepsilon \mu k_\rho^2 - k_x^2) \hat{e}_\rho - k_x k_x \hat{e}_x \right) \times \hat{T}_T(k_\rho) \psi k_\rho H_0^{(1)}(k_\rho \rho) \times \exp (ik \cdot r) \, dk_\rho \, dk_x, \]

(13)

where \( k_{01} \) and \( k_{02} \) are given by (1) and

\[ \psi_1 = \frac{1}{k_{01}^2 - k_\rho^2 - k_x^2}, \]

\[ \psi_2 = \frac{1}{k_{02}^2 - k_\rho^2 - k_x^2}, \]

(14)

\[ \hat{T}_R = [\hat{T}_R(k_\rho), 0, 0], \quad \hat{T}_T = [\hat{T}_T(k_\rho), 0, 0] \] are the Fourier components of surface current density. Furthermore, the line-of-sight (LOS) EM field of the Hertzian dipole in the far field is given by [11, 15]

\[ H_{\alpha}^{\text{LOS}}(r, \theta) = \frac{\omega k_{01} \rho \exp (ikr)}{4\pi} \frac{\exp (ikr)}{r} \sin \theta, \]

(15)

where spherical coordinates \((r, \theta)\) are given in terms of cylindrical coordinates \((\rho, \alpha)\) (see Figure 1) by

\[ r = \rho + \left( x - x_0 \right)^2, \]

(16)
\[ \theta = \pi - \tan^{-1} \left( \frac{\rho}{(x_0 - x)} \right), \quad \text{for} \ x_0 > x, \quad (17a) \]

or

\[ \theta = \tan^{-1} \left( \frac{\rho}{(x - x_0)} \right), \quad \text{for} \ x > x_0, \quad (17b) \]

\[ E_{\text{LOS}} (r, \theta) = \zeta H_{\alpha}^{\text{LOS}} \cos \theta \bar{e}_p - \zeta H_{\alpha}^{\text{LOS}} \sin \theta \bar{e}_x, \quad (18) \]

where \( H_{\alpha}^{\text{LOS}} \) is given by (15)–((17a) and (17b)).

Then, the total EM field in the regions \( x > 0 \) and \( x < 0 \) (see Figure 1) is given by

\[ H (r) = \begin{cases} H_{\alpha}^{\text{LOS}} (r) + H_{\alpha}^{R} (r), & x > 0, \\ H_{\alpha}^{T} (r), & x < 0, \end{cases} \]

\[ E (r) = \begin{cases} E_{\alpha}^{\text{LOS}} (r) + E_{\alpha}^{R} (r), & x > 0, \\ E_{\alpha}^{T} (r), & x < 0. \end{cases} \]

Furthermore, by performing the integrations of expressions (13) over \( k_x \), by using the residue theorem [16], we obtain the following integral expressions for the EM fields.

In the upper half space (\( x > 0 \)),

\[ H (r) = \frac{1}{8\pi} \int_{-\infty}^{\infty} k_x \int_{-\infty}^{\infty} k_x \bar{J}_T (k_x) H_0^{(1)} (k_x) e^{ik_x r} dk_x \]

\[ + \frac{1}{8\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k_x \bar{J}_T (k_x) H_0^{(1)} (k_x) e^{ik_x r} dk_x \]

\[ \cdot H_0^{(1)} (k_x) e^{ik_x r} dk_x \]

(20)

while for the lower half space (\( x < 0 \))

\[ H^T (r) = \frac{1}{8\pi} \int_{-\infty}^{\infty} k_x \int_{-\infty}^{\infty} k_x \bar{J}_T (k_x) H_0^{(1)} (k_x) e^{-ik_x r} dk_x \]

\[ + \frac{1}{8\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k_x \bar{J}_T (k_x) H_0^{(1)} (k_x) e^{-ik_x r} dk_x \]

(21)

3.3. Application of the Boundary Conditions (BCs): Solution for the Unknown Current Densities at the Interface in Spectral Domain. We now apply the BCs that at the interface (\( x = 0 \)) the tangential components of electric field \( E \) and magnetic field \( H \) must be continuous; namely,

\[ H_{\alpha}^{\text{LOS}} + H_{\alpha}^{R} = H_{\alpha}^{T}, \]

\[ E_{\alpha}^{\text{LOS}} + E_{\alpha}^{R} = E_{\alpha}^{T}, \]

(23)

where

\[ H_{\alpha}^{\text{LOS}} = -\frac{1}{8\pi} \int_{-\infty}^{\infty} i\omega \psi k_p e^{ik_x x_0} H_0^{(1)} (k_x \rho) dk_x, \]

\[ E_{\alpha}^{\text{LOS}} = \frac{1}{8\pi} \int_{-\infty}^{\infty} i\omega \psi k_p e^{ik_x x_0} H_0^{(1)} (k_x \rho) dk_x, \]

(24)

Then, from (23), we find

\[ \frac{1}{8\pi} \int_{-\infty}^{\infty} \left( \frac{i\omega \psi k_p e^{ik_x x_0}}{k_1} + \bar{T}_R (k_p) \right) H_0^{(1)} (k_x \rho) k_x dk_x \]

\[ = \frac{1}{8\pi} \int_{-\infty}^{\infty} \bar{T}_R (k_p) H_0^{(1)} (k_x \rho) k_x dk_x, \]

\[ \frac{1}{8\pi} \int_{-\infty}^{\infty} \left( -i\omega k_p e^{ik_x x_0} + \bar{T}_R (k_p) k_1 \right) H_0^{(1)} (k_x \rho) k_x dk_x \]

\[ = \frac{1}{8\pi} \int_{-\infty}^{\infty} \bar{T}_R (k_p) k_2 H_0^{(1)} (k_x \rho) k_x dk_x. \]

(25)

Therefore, from (25), we obtain the following system of algebraic equations:

\[ i\omega k_p e^{ik_x x_0} \frac{k_1}{k_1} + \bar{T}_R (k_p) = \bar{T}_T (k_p), \quad (26a) \]

\[ -i\omega k_p e^{ik_x x_0} + \bar{T}_R (k_p) k_1 = \frac{\xi_1 k_1}{\xi_2} \bar{T}_T (k_p) k_2. \quad (26b) \]

The solutions of systems of (26a) and (26b) are the unknown Fourier components of surface current densities, as follows:

\[ \bar{T}_R (k_p) = i\omega k_p e^{ik_x x_0} \frac{\xi_2 k_1 - \xi_1 k_2}{\xi_1 \xi_2 k_1 + \xi_1 k_2}, \quad (27a) \]

\[ \bar{T}_T (k_p) = -i\omega k_p e^{ik_x x_0} \frac{2\xi_2}{\xi_1 \xi_2 k_1 + \xi_1 k_2}. \quad (27b) \]
3.4. Expressions for the Reflected and Transmitted EM Fields in Integral Representations. Substituting expressions of (27a) and (27b) for the unknown current densities (at the interface, in spectral domain) in (20)–(21), we obtain the reflected and transmitted EM fields in integral representations, as follows.

In the higher half space (LOS field plus reflected field, \(x > 0\)),
\[
\hat{H}(r) = \hat{H}^{\text{LOS}} - \frac{i \omega \hat{p}_E}{8 \pi} \int_{-\infty}^{\infty} k^2 \cdot \frac{\varepsilon_2 K_1 - \varepsilon_1 K_2}{\varepsilon_2 K_1 + \varepsilon_1 K_2} e^{i \varepsilon_r \Delta x} \cdot H_0^{(1)}(k \rho) \cdot e^{i k r} \cdot \hat{e}_r \, dk \rho,
\]
\[
\hat{E}(r) = \hat{E}^{\text{LOS}} - \frac{i p}{8 \pi \varepsilon_r \varepsilon_0} \int_{-\infty}^{\infty} k^2 \cdot \frac{\varepsilon_2 K_1 - \varepsilon_1 K_2}{\varepsilon_2 K_1 + \varepsilon_1 K_2} \cdot e^{i \varepsilon_r \Delta x} \cdot H_0^{(1)}(k \rho) \cdot e^{i k r} \cdot \hat{e}_r \, dk \rho,
\]
where
\[
I_1 = \int_{k \rho = -\infty}^{\infty} \frac{\varepsilon_2 K_1 - \varepsilon_1 K_2}{\varepsilon_2 K_1 + \varepsilon_1 K_2} \cdot k^2 \cdot H_0^{(1)}(k \rho) \cdot e^{i k r} \cdot \hat{e}_r \, dk \rho,
\]
\[
I_2 = \int_{k \rho = -\infty}^{\infty} \frac{\varepsilon_2 K_1 - \varepsilon_1 K_2}{\varepsilon_2 K_1 + \varepsilon_1 K_2} \cdot k^2 \cdot H_0^{(1)}(k \rho) \cdot e^{i k r} \cdot \hat{e}_r \, dk \rho,
\]
\[
I_3 = \int_{k \rho = -\infty}^{\infty} \frac{\varepsilon_2 K_1 - \varepsilon_1 K_2}{\varepsilon_2 K_1 + \varepsilon_1 K_2} \cdot k^2 \cdot H_0^{(1)}(k \rho) \cdot e^{i k r} \cdot \hat{e}_r \, dk \rho.
\]

Moreover, in order to calculate integral \(I_1\) (in an almost identical manner, integrals \(I_2\) and \(I_3\) will be calculated, using the SPM method [11–13, 17]), let us assume large argument approximation for the Hankel functions of (32)–(34); namely, let us assume that
\[
k \rho \cdot \rho \gg 1
\]
for which case function \(H_0^{(1)}(k \rho)\) becomes a highly oscillating function of \(k \rho\). Then, since stationary phase method (SPM) is to be applied, we just replace \(H_0^{(1)}(k \rho)\) in (32) by its asymptotic large argument approximation:
\[
H_0^{(1)}(k \rho) = \sqrt{-\frac{2i}{\pi k \rho}} \cdot e^{i k r \rho}.
\]

Then, integral \(I_1\) of (32) takes the following form:
\[
I_1 = \sqrt{-\frac{2i}{\pi}} \cdot \frac{1}{\sqrt{p}} \int_{k \rho = -\infty}^{\infty} k^\rho \cdot \frac{\varepsilon_2 K_1 - \varepsilon_1 K_2}{\varepsilon_2 K_1 + \varepsilon_1 K_2} \cdot e^{i k r \rho} \cdot \hat{e}_r \, dk \rho.
\]

4. Electromagnetic (EM) Fields Reflected from Infinite, Flat, and Lossy Ground in the Far Field Region: Analytical High Frequency Expressions Obtained through the Application of the Stationary Phase Method (SPM)

In order to calculate the EM field above lossy ground (i.e., for \(x > 0\)), we write (28) in the following form:
\[
\hat{E}_{x>0} = \hat{E}^{\text{LOS}} - \frac{i p}{8 \pi \varepsilon_r \varepsilon_0} \cdot I_1 \cdot \hat{e}_r - \frac{i p}{8 \pi \varepsilon_r \varepsilon_0} \cdot I_2 \cdot \hat{e}_y,
\]
\[
\hat{H}_{x>0} = \hat{H}^{\text{LOS}} - \frac{i \omega \hat{p}_E}{8 \pi} \cdot I_3 \cdot \hat{e}_y.
\]
which finally yields the following expression for the "stationary point" (only one stationary point exists):

\[
k_{ps} = \frac{k_{01} \rho}{\left[ \left( (x + x_0)^2 + \rho^2 \right) \right]^{1/2}} = k_{01} \cos \varphi,
\]

where \( \varphi \) is the angle defined by the image point of the radiating Hertzian dipole, the observation point, and the horizontal line drawn from the above-mentioned image point and \( \cos \varphi \) is given by (41). Furthermore, note that angle \( \varphi \) is the well-known “grazing angle” in the literature [15], as shown in Figure 2.

Note here that for the air-lossy ground problem considered here \( k_{ps} \) is real and positive and \( k_{ps} < k_{01} \). Also, we can easily see that

\[
\lim_{\rho \to \infty} k_{ps} = \lim_{(x+x_0) \to 0} k_{ps} = k_{01}.
\]

Furthermore, according to the SPM method [11–13, 17], we also have to calculate the second derivative of the phase function, which in our case is calculated, from (38) and (40) as

\[
f''(k_{ps}) = -\frac{(x + x_0)}{\rho} \cdot \frac{k_{01}^2}{\left( k_{01}^2 - k_{ps}^2 \right)^{3/2}}.
\]

Note here that \( f''(k_{ps}) \) is always negative; that is,

\[
\text{sgn} \left[ f'' \left( k_{ps} \right) \right] = -1
\]

whose relation is needed in the application of SPM method.

Then, by actually applying the SPM method [11–13, 17], from (37), we find

\[
I_1 = i F \left( k_{ps} \right) e^{i \rho f(k_{ps})} \cdot e^{i (\pi/4) \text{sgn}[f''(k_{ps})]} \sqrt{\frac{2\pi}{\rho |f''(k_{ps})|}}
\]

\[
\cdot \exp \left( \frac{i \pi}{4} \right)
\]

or

\[
I_1 = \frac{i2}{\rho} \frac{1}{|f''(k_{ps})|^{1/2}} F \left( k_{ps} \right) e^{i \rho f(k_{ps})}.
\]

Then, by using expressions (33)-(34) and (45), we finally end up with the following expressions:

\[
I_1 = \frac{i2}{k_{01} \rho^{1/2}} \frac{1}{(x + x_0)^{1/2}} \frac{\cos \varphi}{k_{ps}^2 \kappa_{ps}^2 \kappa_{1s}^2} \frac{F_{2s}^2}{\kappa_{2s}^2} e^{i \rho f(k_{ps})},
\]

\[
I_2 = \frac{i2}{k_{01} \rho^{1/2}} \frac{1}{(x + x_0)^{1/2}} \frac{\cos \varphi}{k_{ps}^2 \kappa_{ps}^2 \kappa_{1s}^2} \frac{F_{2s}^2}{\kappa_{2s}^2} e^{i \rho f(k_{ps})},
\]

\[
I_3 = \frac{i2}{k_{01} \rho^{1/2}} \frac{1}{(x + x_0)^{1/2}} \frac{\cos \varphi}{k_{ps}^2 \kappa_{ps}^2 \kappa_{1s}^2} \frac{F_{2s}^2}{\kappa_{2s}^2} e^{i \rho f(k_{ps})},
\]

where

\[
\kappa_{1s} = \sqrt{k_{01}^2 - k_{ps}^2} = k_{01} \sin \varphi,
\]

where angle \( \varphi \) is defined in Figure 2, and

\[
\kappa_{2s} = \sqrt{k_{01}^2 - k_{ps}^2}.
\]

Then, our final solution in the high frequency regime (i.e., by using the SPM method) consists of (30)-(31) and (47)–(51), where \( k_{ps} \) is given by (41).
5. Final Formulae for the Received Electric and Magnetic Field Vector: Fields Reflected from the Lossy Ground

5.1. Electric Field Vector. By using (30), (47), and (48), we obtain the following result for the electric field vector, scattered from the lossy ground, at the observation point:

$$E_{sc}^{E}|_{x>0} = \frac{p}{4\pi\varepsilon_0\varepsilon_r}(x + \varepsilon_r)^{1/2} \frac{1}{k_0^{1/2}k_{ps}^{3/2}} \left( k_1^2 \varepsilon_0 + k_{ps} \varepsilon_x \right)$$

where angle $\varphi$ and distance $(A'A')$ are shown in Figure 2 (note that $(A'A')$ is the distance between the image point and the observation point and $\varphi$ is the so-called “grazing angle” [15]). Moreover, we observe that function

$$R_V = |R_V|e^{i\varphi_V} = \frac{k_2 - k_1}{k_2 + k_1}$$

is the usual (complex) “Fresnel reflection coefficient” for the “Sommerfeld radiation problem” considered in this paper (since $R_V$ is complex, this means change in magnitude and in phase of the incident EM wave upon reflection from the lossy ground).

Furthermore, in order to elaborate a little more in formula (52), we define the “amplitude factor” $F_0$ by

$$F_0 = \frac{p}{4\pi\rho^{1/2}} \frac{1}{(x + \varepsilon_r)^{1/2}} \frac{1}{k_0^{1/2}k_{ps}^{3/2}}$$

and the “phase factor” $\varphi_0$ by

$$\varphi_0 = k_{ps} \rho + k_1 (x + \varepsilon_r)$$

which is the phase in (52) in addition to the phase $\varphi_V$ originating from the complex “Fresnel reflection coefficient” $R_V$ of (53). Then, from (52)–(55), we obtain

$$E_{sc}^{E}|_{x>0} = \frac{1}{\varepsilon_0\varepsilon_r} F_0 R_V e^{i\varphi_0} \left( k_1 \varepsilon_0 + k_{ps} \varepsilon_x \right)$$

Finally, by taking (41) and (50) into account, we find the following expressions for horizontal (along $\hat{e}_x$) and vertical (along $\hat{e}_x$) components of electric field vector, respectively:

$$E_{sc}^{E}|_{x>0} = \frac{k_1}{\varepsilon_0\varepsilon_r} F_0 R_V e^{i\varphi_0}$$

$$E_{sc}^{E}|_{x>0} = \frac{k_{ps}}{\varepsilon_0\varepsilon_r} F_0 R_V e^{i\varphi_0}$$

5.2. Magnetic Field Vector. Similarly, by using (31) and (49), we find the following expression for the scattered magnetic field vector above the flat and lossy ground:

$$H_{sc}^{E}|_{x>0} = \frac{\omega p}{4\pi} \frac{1}{(x + \varepsilon_r)^{1/2}} \frac{1}{k_0^{1/2}k_{ps}^{3/2}} \left( k_1^2 \varepsilon_0 + k_{ps} \varepsilon_x \right)$$

Furthermore, by using the definitions of quantities $R_V$, $F_0$, and $\varphi_0$, (53)–(55), we obtain

$$H_{sc}^{E}|_{x>0} = \omega F_0 R_V e^{i\varphi_0} \varepsilon_x$$

Finally, note that from (58) and (61) it follows that

$$\left|H_{sc}^{E}|_{x>0}\right| = \zeta = \sqrt{\frac{\mu_0}{\varepsilon_0}},$$

where $\zeta$ is the free space impedance.

Expressions (52) and (59) are the classical expressions for the EM field reflected from the lossy ground and originating from the image point, as shown in Figure 2. Then, by using the newly derived expressions for the received EM field in spectral domain in this paper, (28), and by applying the SPM method, that is, in the high frequency regime, the classical “space wave” in region $x > 0$ [15] is derived. This result has the following two interesting consequences.

(1) The “space wave” [15], which corresponds to the complex summation (interference) of the reflected fields, (52) and (59), and the line-of-sight (LOS) fields (the latter not included in these equations, but shown in (15) and (18)), is the solution to the Sommerfeld radiation problem in the high frequency regime, where the so-called “surface wave” can be ignored [7, 8, 15].
Figure 3: Electric fields at observation point as a function of horizontal distance $\rho$ between transmitting Hertzian dipole and observation point, for frequency $f = 80$ MHz. Here, the various components of received electric field are shown as follows: line-of-sight (LOS) field (circle), field scattered from ground (asterisk), "space wave" (square), and "surface wave" (diamond). Note that in this case Norton’s “surface wave” is rather negligible as compared to the corresponding “space wave” [15].

Figure 4: Similar to Figure 3, except that here the frequency of radiating Hertzian dipole is now equal to 30 MHz (lower frequency). In this case of lower frequency, the “surface wave” cannot be considered negligible, as compared to the “space wave” [15].

6. Numerical Results in the High Frequency Regime: Comparison with Norton’s Results

In this Section, indicative numerical results are provided for the electric field (magnitude) at the receiver point as a function of the horizontal distance ($\rho$) between transmitting Hertzian dipole and receiver position. These numerical results include the electric field scattered from the ground, magnitude of (52), the line-of-sight (LOS) field, the so-called “space wave” (which is just the complex summation (interference) of the two fields mentioned above), and, finally, the so-called “surface wave,” according to Norton [7, 8, 15]. Furthermore, these numerical results are provided for frequency of radiating dipole $f = 80$ MHz (Figure 3) or $f = 30$ MHz (Figure 4).

Comparison of numerical results for LOS electric field, scattered electric field, and space wave, derived from our formulation, and Norton’s results [7, 8, 15] shows very good agreement, as it can be seen in Figures 3 and 4. The surface wave represented in Figures 3 and 4 is the so-called “Norton surface wave” [7, 8, 15]. Note that at the higher frequency of 80 MHz (Figure 3) the surface wave, according to Norton’s formulation [7, 8, 15], is rather negligible, as compared to the “space wave,” while it becomes rather more important at the lower frequency of 30 MHz (Figure 4). Our proposed SPM method of Sections 4 and 5 (which is inherently a “high frequency method”) ignores this surface wave contribution in the high frequency regime.

Moreover, note that the problem parameters in Figures 3 and 4 are selected as follows: height of transmitting dipole $x_0 = 60$ m, height of observation point (receiver position) $x = 15$ m, current of the radiating Hertzian dipole $I = 1$ A, length of the Hertzian dipole $2h = 0.1$ m (much smaller than the wavelength $\lambda = c/f$ in both cases), relative dielectric constant of ground $\varepsilon_r = 20$, and ground conductivity $\sigma = 0.01$ S/m. Finally, note that the relation between current $I$ and dipole moment $p$ is given by $I(2h) = iwp$, where $\omega = 2\pi f$ and $i$ is the unit imaginary number.

7. Conclusions: Future Research

In this paper, we formulated the radiation problem from a vertical short (Hertzian) dipole above flat and lossy ground in the spectral domain, which resulted in an easy-to-manipulate integral expression for the received EM field above or below the ground. As also explained above, this formulation appears to have inherent advantages over the classical formulation by Sommerfeld [6], since it avoids the use of the so-called Hertz...
potential and its subsequent differentiation for the calculation of the received EM field. Subsequently, by applying the stationary phase method (SPM) in the high frequency regime, the classical solution for the “space wave” was rederived in a new fashion, thus showing that this is the dominant solution in this high frequency regime. Mathematical derivations regarding the application of our proposed method in spectral domain, as well as the application of the SPM method, were provided in reasonable detail above. Finally, numerical results in this high frequency limit were obtained and they were compared to Norton’s results [7, 8, 15].

Corresponding research in the near future by our research group will concentrate on the calculation of the received EM field below the ground at the high frequency regime (by using again the SPM method). Furthermore, we will calculate the received EM field, above or below the ground, for any frequency of the radiating dipole, in an exact and analytical manner [16] or in a numerical way (i.e., through the use of numerical integration techniques [18]). In this context, the behavior of surface waves will become evident through the use of the residue theorem, when applied to (28), in a way similar to [6].

Moreover, we intend also to investigate the formulation of the same radiation problem in spectral domain, but now in the case of a horizontal radiating Hertzian dipole above flat and lossy ground. In addition, further investigations will be performed in the case of rough (and not flat) ground and in the case of curvature of the earth’s surface for large distance communication applications. Finally, in the near future, our research group will focus on the design of a software product for accurate prediction of pass loss in different types of environment, like urban, suburban, and rural environments. The above software tool will be based on the exact electromagnetic (EM) method proposed in this paper, and therefore it is expected that it will exhibit important advantages over previously developed corresponding software tools. In this framework, comparisons with existing commercial software tools will also be performed [19].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References
