

Research Article

Sidelobe Suppression with Null Steering by Independent Weight Control

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Received 20 April 2015; Accepted 18 May 2015

Academic Editor: Vincenzo Galdi

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A uniform linear array of n antenna elements can steer up to $n - 1$ nulls. In situations where less than $n - 1$ nulls are required to be steered, the existing algorithms have no criterion to utilize the remaining weights for sidelobe suppression. This work combines sidelobe suppression capability with null steering by independent weight control. For this purpose, the array factor is transformed as the product of two polynomials. One of the polynomials is used for null steering by independent weight control, while the second one is for sidelobe suppression whose coefficients or weights are determined by using convex optimization. Finally, a new structure is proposed to incorporate the product of two polynomials such that sidelobe suppression weights are decoupled from those of null steering weights. Simulation results validate the effectiveness of the proposed scheme.

1. Introduction

Null steering for adaptive beamforming is an important area of research due to its military and commercial applications [1–6]. Null steering techniques can broadly be classified as Direction of Arrival (DOA) based beamformers and optimal adaptive beamformers [1]. Existing DOA based null steering techniques either control the excitation of elements' amplitude only [2, 3], phase only [4, 5], or both [6]. There are also techniques that achieve null steering by changing the position and/or elevations of the array elements [7–10].

Elements' phase and amplitude excitation techniques are slow for large arrays because even if a single null changes its position, the whole set of weights is required to be reevaluated which is obviously time consuming and complicated. On the other hand, null steering by controlling the excitation amplitude only is easy to implement and less sensitive to quantization error but reduces the number of steerable nulls [2, 3]. The techniques like phase only and position perturbations of the array elements are nonlinear

problems and cannot be solved directly by analytical methods [11]. Besides, position perturbations methods require servo motors for the movement of the elements and, in case of large arrays, the complexity to control element position increases due to increase in computational time to find new position perturbations.

The solution for this problem is the null steering by independent weight control where if a single null changes its position, only the weight set corresponding to that null is evaluated and changed [3, 6]. Unfortunately, these DOA based beamformers do not have sidelobe suppression capability. Although this problem has been addressed for optimal adaptive beamforming algorithms [12, 13] the method cannot be used in DOA based beamformers because in these algorithms pattern synthesis weights are not determined by using optimization techniques.

This paper presents a technique where the array factor is transformed as product of two polynomials such that one polynomial denoted by AF_1 provides independent steering of all available nulls while the other denoted by AF_2 suppresses

sidelobes by utilizing remaining weights. The coefficients of AF_1 are determined by forcing the zeros of AF_1 to lie on unit circle in the complex plane [6] while the coefficients of AF_2 are evaluated using convex optimization. Furthermore a structure is presented to implement this array factor where the sidelobe suppression weights are decoupled from the weights meant for independent null steering. Therefore the coefficients of AF_2 will not be required to change with the change in position of any interference.

The rest of the work is organized as follows. Section 2 presents problem formulation, while proposed technique and structure are discussed in Sections 3 and 4, respectively. Simulation results are given in Section 5. Finally, conclusion and future work recommendation are given in Section 6.

2. Problem Formulation

Consider a uniform linear array (ULA) of N omnidirectional elements. Let d be the element spacing, α the progressive phase, and θ the angle of arrival of the plane wave impinging on the array. Figure 1 shows the path difference of this wave on the adjacent elements of the ULA; that is, $x = d\cos\theta$. The output of the array elements is multiplied by the properly selected coefficients A_0, A_1, \dots, A_{N-1} to steer the nulls in the directions of interferences. We take the first element (element 0) of the array as reference and $\psi = \alpha + (2\pi/\lambda)d\cos\theta$, where $k = 2\pi/\lambda$ is known as wave number. The signal at element 2 is the delayed version of the signal at element 1 and is expressed as $e^{j\psi}$. Let $e^{j\psi} = z$; then the array factor can be given as

$$\text{AF}_1 = A_0 + A_1 z + \dots + A_{N-1} z^{(N-1)}. \quad (1)$$

This AF_1 is the $N - 1$ -degree polynomial and has $N - 1$ roots (nulls) dependent on coefficients A_0, A_1, \dots, A_{N-1} . In factorized form, (1) becomes

$$\text{AF}_1 = (z - z_1)(z - z_2) \cdots (z - z_{N-1}), \quad (2)$$

where z_1, z_2, \dots, z_{N-1} are zeros forced to exist on the unit circle in the complex plane [2, 3, 6, 14] and their position on the circle depends on coefficients A_0, A_1, \dots, A_{N-1} .

To steer the main beam in the desired direction θ_s , the progressive phase shift is given by

$$\alpha = -\left(\frac{2\pi}{\lambda}\right) d \cos \theta_s. \quad (3)$$

In order to incorporate expression (1), factors of (2) are to be multiplied iteratively by combining the coefficients of the same powers of z at each step. This process of multiplication is complicated and time consuming for large arrays. The algorithms proposed in [3, 6] avoid this multiplication by decoupling null steering weights, while the current work presents a method to add sidelobe suppression capability to these algorithms by adjusting and decoupling the weights of spare elements of the array from those of null steering weights.

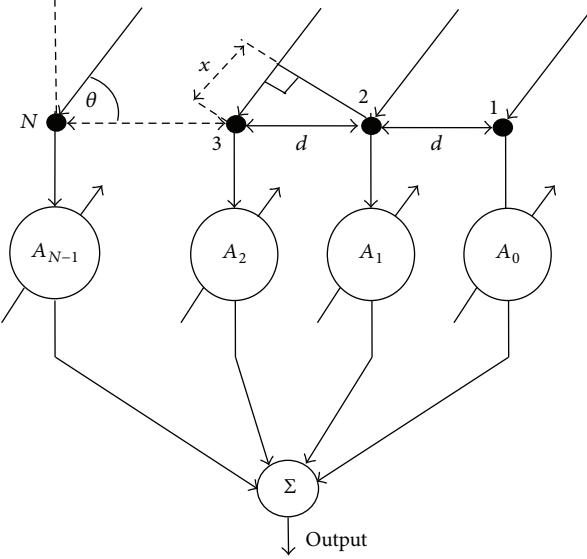


FIGURE 1: Uniform linear array of N elements with $N - 1$ steerable nulls.

3. Proposed Algorithm for Independent Null Steering (INS) with Suppressed Sidelobes

According to Schelkunoff [14], the product of two or more polynomials results in a polynomial and there exists a linear array corresponding to this resultant polynomial. Using the same fact, consider the desired array factor AF given below:

$$\text{AF} = \text{AF}_1 \cdot \text{AF}_2, \quad (4)$$

where AF_1 is meant for independent steering of N_1 nulls and can be given as

$$\text{AF}_1 = \prod_{i=1}^{N_1} (z - z_i). \quad (5)$$

The purpose of AF_2 is to suppress sidelobes and it can be expressed by

$$\text{AF}_2 = b_0 + b_1 z + \dots + b_{N_2-1} z^{N_2-1} = \mathbf{s}_{N_2} \mathbf{b}, \quad (6)$$

where $\mathbf{b} = [b_0, b_1, \dots, b_{N_2-1}]^T$ represents the complex weight vector and $\mathbf{s}_{N_2} = [1, z, \dots, z^{N_2-1}]$ is the steering vector. The condition on the vector \mathbf{b} is that it guarantees the sidelobe suppression and steers the main beam in the same direction as that of AF_1 . To find out \mathbf{b} , consider a set S_R that is union of two sets containing angles from the left and right sidelobe regions of the main beam with beam-width θ_{mb} ; that is,

$$S_R = \left\{ 0 < \theta \leq \theta_s - \frac{\theta_{mb}}{2} \right\} \cup \left\{ \theta_s + \frac{\theta_{mb}}{2} < \theta \leq 180^\circ \right\}. \quad (7)$$

Then p discrete angles are selected from S_R and placed in another set A_{S_R} given by

$$A_{S_R} = \{ \theta_i \mid \theta_i \in S_R \wedge \theta_i = \theta_0 + i\delta\theta \} \subseteq S_R, \quad (8)$$

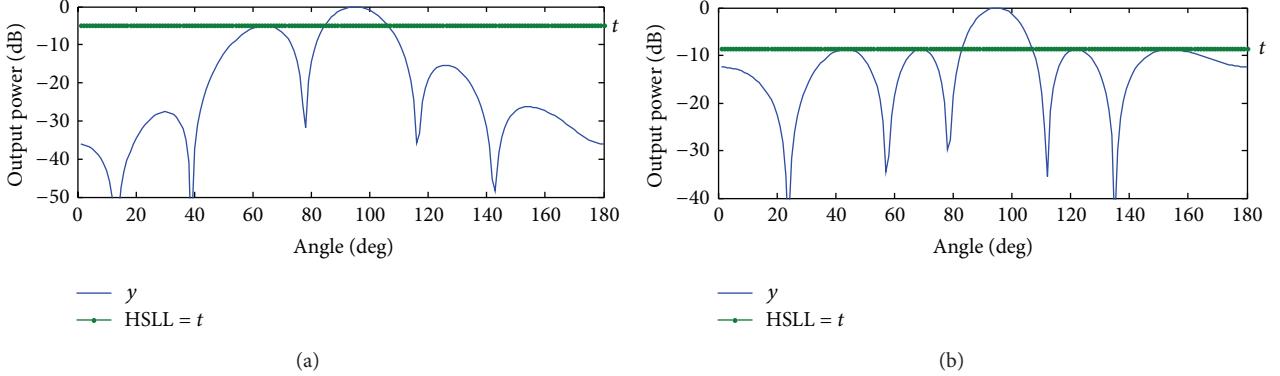


FIGURE 2: (a) Highest sidelobe level (HSLL = t) is required to be minimized when $N_2 = 6$, $\theta_s = 95^\circ$; (b) $y = \text{AF}_2$ pattern when highest sidelobe level (HSLL = t) has been minimized for $N_2 = 6$, $\theta_s = 95^\circ$, and $\theta_{\text{mb}} = 24^\circ$.

where $i = 0, 1, \dots, p - 1$ and θ_0 is the starting angle and $\delta\theta$ is the step size.

Now consider another matrix \mathbf{A} , containing steering vectors corresponding to the angles contained in A_{S_R} ; that is,

$$\mathbf{A} = \left[\mathbf{s}_{N_2}(\theta_0) \mathbf{s}_{N_2}(\theta_1) \cdots \mathbf{s}_{N_2}(\theta_{p-1}) \right]^T, \quad (9)$$

where $s_{N_2}(\theta_i) = [1, z_i, \dots, z_i^{N_2-1}]$ with $z_i = \exp(j(\alpha + (2\pi/\lambda)d\cos\theta_i))$. Now the requirement is to minimize the array output power along the angles $\theta_i \in A_{S_R}$ subject to the unit output along the desired direction θ_s .

The requirement is fulfilled if we restrict the peak output power in the sidelobe region, that is, the highest sidelobe level (HSLL) to have minimum value as shown in Figure 2. Minimizing this power along the angles $\theta_i \in A_{S_R}$ is the same as minimizing the absolute output along these angles; that is,

$$\min_{\mathbf{b}} \quad \max \left(\left| \mathbf{s}_{N_2}(\theta_i) \mathbf{b} \right| \right), \quad \theta_i \in A_{S_R} \quad (10)$$

subject to $\mathbf{s}_{N_2}(\theta_s)\mathbf{b} = 1$.

Since the steering vectors along these angles are contained in matrix \mathbf{A} the absolute output along these angles is the vector $|\mathbf{Ab}|$ and the above minimization problem becomes

$$\min_{\mathbf{b}} \max |A\mathbf{b}|$$

(11)

$$\text{subject to } \mathbf{s}_{N_2}(\boldsymbol{\theta}_s) \mathbf{b} = 1.$$

In (II) the objective function controls the sidelobe level and the constraint forms the main beam in the desired direction. This problem can be cast as the second-order cone program in the following manner:

$$\begin{aligned} & \text{Minimize} && t \\ & \text{subject to} && |\mathbf{Ab}| \leq t \\ & && \mathbf{s}_{N_s}(\theta_s) \mathbf{b} = 1. \end{aligned} \quad (12)$$

The inequalities in (12) are called second-order cone constraints and $(\mathbf{Ab}, t) \in$ second-order cone in \mathbb{R}^{p+1} [15]. Second-order cone program is the subclass of convex optimization and hence problem (11) can be solved using [16].

The polynomial AF_2 suppresses sidelobe levels and has overlapping main beam with that of AF_1 . Due to pattern multiplication, $AF = AF_1 \cdot AF_2$ will give the independent null steering capability with suppressed sidelobes. The AF_2 is required to be calculated once and remains unchanged as long as the position of the main beam is unchanged. However if the position of the main beam is changed, the coefficients of both the polynomials, that is, AF_1 and AF_2 , will have to be recalculated and changed.

4. Proposed Structure

In this section, a structure is proposed to steer 5 nulls independently and to use three weights for sidelobe suppression; that is, $N_1 = 5$, $N_2 = 3$. This structure can easily be extended for any values of N_1 and N_2 . The structure starts with ULA of 8 elements, that is, $(N_1 + N_2)$. It can be seen from Figure 3 that, for first N_1 (which is 5 in this case) stages, each adder has 2 inputs. In the structure, first stage controls the position of first null where first input of each adder is multiplied by $(-z_1)$. The j th output of this stage is represented by $y_{1,j}$ with $j = 1, \dots, N_1 + N_2 - 1$ and is given by

$$y_{1,j} = z^{j-1} (z - z_1) \quad \text{for } j = 1, \dots, N_1 + N_2 - 1. \quad (13)$$

Similarly second stage controls second null by multiplying first input of each of its adders by $(-z_2)$. These outputs are

$$y_{2,j} = z^{j-1} (z - z_1)(z - z_2) \quad (14)$$

for $j = 1, \dots, N_1 + N_2 - 2$.

It is clear from Figure 3 that we will get N_2 outputs after N_1 stages and these outputs will be

$$y_{N_1,j} = z^{j-1} \prod_{i=1}^{N_1} (z - z_i) = z^{j-1} \text{AF}_1 \quad (15)$$

for $j = 1, \dots, N_2$.

It means these outputs are multiples of AF₁, as shown below:

$$\left[y_{N_1,1}, y_{N_1,2}, \dots, y_{N_1,N_2} \right]^T = \mathbf{A} \mathbf{F}_1 \left[1, z, \dots, z^{N_2-1} \right]^T. \quad (16)$$

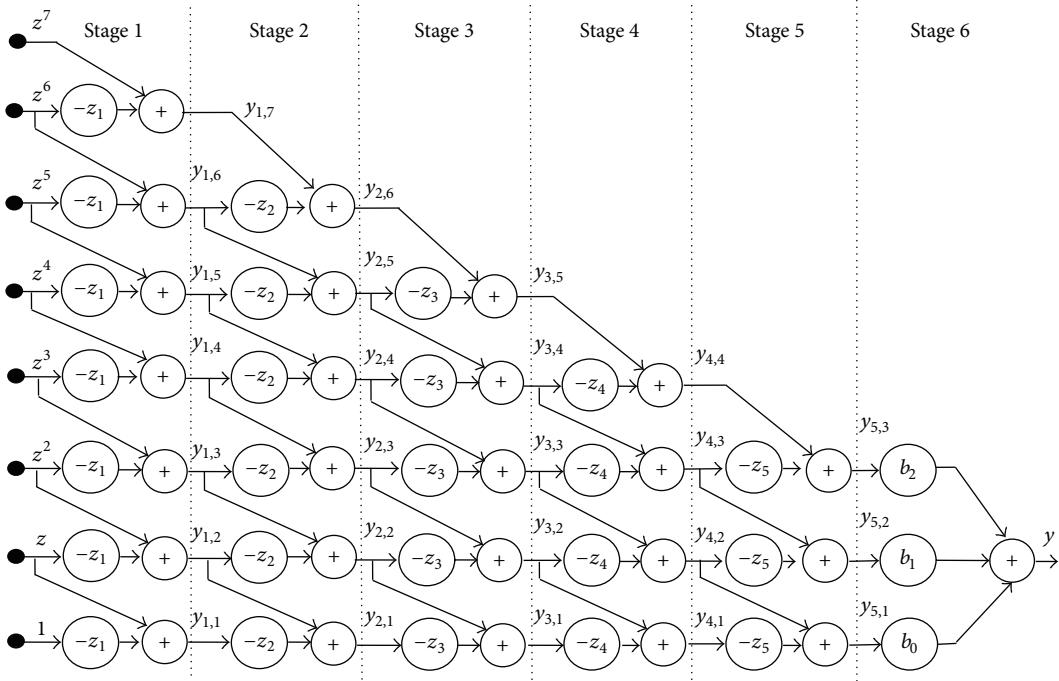


FIGURE 3: Structure for independent null steering with suppressed sidelobes.

Thus first N_1 stages will incorporate AF_1 and steer N_1 nulls by independent weight control. After that the resulting N_2 outputs are weighted to give AF_2 for sidelobe suppression. Finally these weighted outputs are summed to result in $\text{AF} = \text{AF}_1 \cdot \text{AF}_2$ given by (17) and the corresponding structure is shown in Figure 3. Consider

$$\begin{aligned} y &= \text{AF}_1 [1, z, \dots, z^{N_2-1}] [b_0, b_1, \dots, b_{N_2-1}]^T \\ &= \text{AF}_1 \text{AF}_2. \end{aligned} \quad (17)$$

As mentioned earlier, convex optimization is used to find out the N_2 complex weights. These N_2 weights remain unchanged even if one or more nulls change their position. On the other hand, if the main beam changes its position, the whole set of weights, including INS weights for N_1 stages and N_2 weights for sidelobe control, is changed. Even in such situation implementation of AF_1 in Figure 3 is not time consuming as compared to Figure 1, because it incorporates factors directly without multiplication and forming a single polynomial.

5. Simulation and Results

In this section, several simulation examples are carried out in MATLAB to verify the effectiveness of the proposed algorithm. In these examples element spacing d is taken as $\lambda/2$.

Example 1. This example compares the performance of the proposed technique with two algorithms, that is, INS by decoupling real weights (Realwt) and INS by decoupling complex weights (Cmplxwt). For this purpose, main beam is taken along 95° , while the four interferences are placed at 20° , 70° , 120° , and 160° . In this case, INS by decoupling real weights

and INS by decoupling complex weights require ULAs having 9 and 5 elements, respectively, while the proposed algorithm is utilized using a ULA of 9 elements with $N_1 = 4$ and $N_2 = 5$. One can clearly see in Figure 4 that the performance of the proposed algorithm is better for sidelobe suppression as compared to the other two algorithms.

Example 2. This example compares the performance of the proposed algorithm with INS by decoupling real weights when the second null is at 70° and also when it is shifted to 75° , while positions of all the other nulls remain unchanged. In Figure 5, again one can see that the proposed scheme outperforms Realwt, no matter whether the second null is at 70° or is shifted to 75° . Similar behavior can be observed if the position of multiple nulls is changed.

Example 3. This example demonstrates the effect of number of coefficients of AF_2 , that is, N_2 on the sidelobe level. For this purpose sidelobe levels are compared for the same set of signals as in Example 1, when $N_2 = 4, 6$, and 8 . It is clear from Figure 6 that the sidelobe suppression capability of the proposed algorithm increases as N_2 is increased.

Example 4. This example shows the capability of the proposed algorithm to decouple complex weights of AF_1 to steer nulls independently. For this purpose main beam is taken along 95° and four interferences are placed at 20° , 70° , 120° , and 160° . Then the position of interferences is changed one by one and the results are shown in Table 1 where the changed position of the nulls and corresponding weights are shown by bold entries. It can easily be deduced that when any interference changes its position, only the corresponding weight is changed, while the rest of the weights remain unaffected.

TABLE 1: Decoupling of weights by proposed algorithm for INS when $\theta_s = 95^\circ$, $N_1 = 4, N_2 = 5$.

S. number	θ_i	z_1	z_2	z_3	z_4
1	$20^\circ, 70^\circ, 120^\circ, 160^\circ$	$-0.9964 - 0.0842i$	$0.2207 + 0.9753i$	$0.2704 - 0.9627i$	$-0.8946 - 0.4469i$
2	$25^\circ, 70^\circ, 120^\circ, 160^\circ$	$-0.9998 + 0.0205i$	$0.2207 + 0.9753i$	$0.2704 - 0.9627i$	$-0.8946 - 0.4469i$
3	$20^\circ, 65^\circ, 120^\circ, 160^\circ$	$-0.9964 - 0.0842i$	$-0.0307 + 0.9995i$	$0.2704 - 0.9627i$	$-0.8946 - 0.4469i$
4	$20^\circ, 70^\circ, 125^\circ, 160^\circ$	$-0.9964 - 0.0842i$	$0.2207 + 0.9753i$	$0.0426 - 0.9991i$	$-0.8946 - 0.4469i$
5	$20^\circ, 70^\circ, 120^\circ, 155^\circ$	$-0.9964 - 0.0842i$	$0.2207 + 0.9753i$	$0.2704 - 0.9627i$	$-0.8429 - 0.5381i$

TABLE 2: Effect of change in θ_s on different parameters.

S. number	θ_s	Parameters	Value
1	95°	$b_0 - b_4$	$0.0989 + 0.0098i, 0.2426 + 0.0120i, 0.3171, 0.2426 - 0.0120i, 0.0989 - 0.0098i$
		$z_1 - z_4$	$-0.9964 - 0.0842i, 0.2207 + 0.9753i, 0.2704 - 0.9627i, -0.8946 - 0.4469i$
		α	0.2738
2	90°	$b_0 - b_4$	$0.0987, 0.2427, 0.3172, 0.2427, 0.0987$
		$z_1 - z_4$	$-0.9821 - 0.1883i, 0.4762 + 0.8793i, 0 - i, -0.9821 - 0.1883i$
		α	0

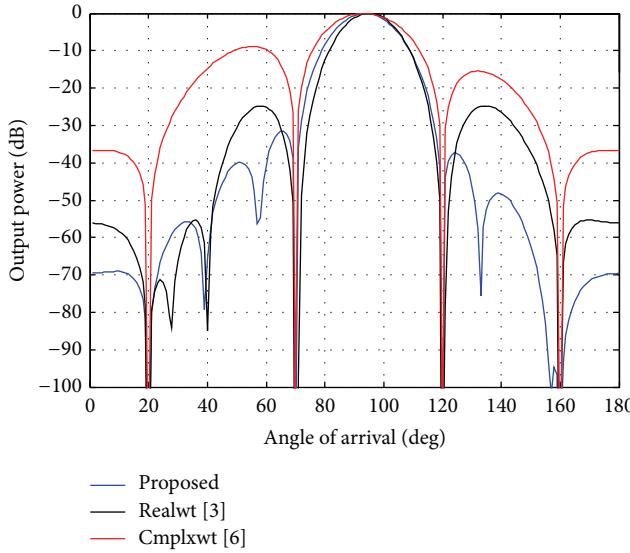
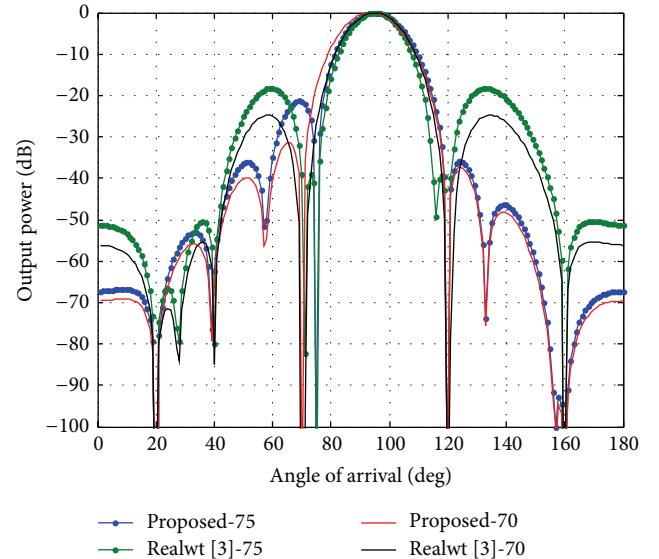


FIGURE 4: Performance comparison of the proposed algorithm with INS by decoupling real weights (Realwt) and INS by decoupling complex weights (Cmplxwt).

FIGURE 5: Performance comparison of proposed algorithm with INS by decoupling real weights [3], when second null changes its position from 70° to 75° .

Example 5. This example shows the effect of change in direction of main beam on z_i , \mathbf{b} , and α . For this purpose initially main beam is taken along 95° while four interferences are placed at $20^\circ, 70^\circ, 120^\circ$, and 160° . Then main beam is shifted at 90° while the interferences do not change their position as given in Table 2 for both the cases. It is clear that these parameters change with the change in θ_s .

6. Conclusion

A method for uniform linear array is proposed to steer the nulls by independent weight control and to suppress sidelobes. It uses the progressive phase shift for beam steering and product of two polynomials for independent null steering

with sidelobe suppression. The zeros of independent null steering polynomial are forced to lie on the unit circle in the complex plane. Similarly, the coefficients for sidelobe suppression are determined by convex optimization. Then a structure is proposed that incorporates the product of the two polynomials. In order to steer N_1 nulls independently and to use N_2 weights for sidelobe suppression, a ULA of $N_1 + N_2$ elements is required. In the structure, the number of weights for independent null steering and the total number of weights will be $0.5N_1(N_1 + 2N_2 - 1)$ and $0.5N_1(N_1 + 2N_2 - 1) + N_2$, respectively. Traditionally this array can steer $N_1 + N_2 - 1$ nulls but the proposed technique steers only N_1 nulls with suppressed sidelobes at the cost of $0.5N_1(N_1 + 2N_2 - 1) + N_2$

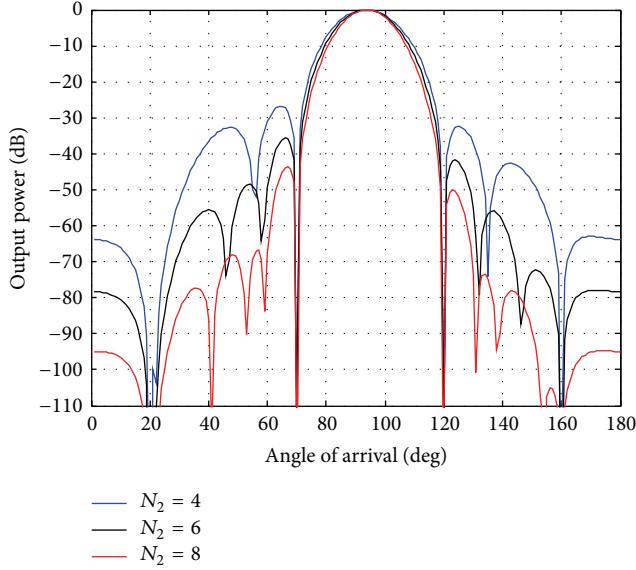


FIGURE 6: Performance of proposed algorithm for $N_2 = 4$, $N_2 = 6$, and $N_2 = 8$.

complex weights, $0.5N_1(N_1 + 2N_2 - 1) + 1$ adders, and $N_2 - 1$ nulls.

The coefficients for the sidelobe suppression are not affected by the change in position of one or more nulls with the condition that the main beam does not change its direction. On the other hand if the main beam changes its position, the whole set of weights of the structure along with the progressive phase shift will be changed. The sidelobes can be suppressed more by increasing the number of coefficients dedicated for this purpose.

In future one can use the same approach for two- and three-dimensional arrays.

Conflict of Interests

All the authors of this paper declare that there is no conflict of interests regarding the publication of this paper.

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