Research Article

An Improved Antenna Array Pattern Synthesis Method Using Fast Fourier Transforms

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An improved antenna array pattern synthesis method using fast Fourier transform is proposed, which can be effectively applied to the synthesis of large planar arrays with periodic structure. Theoretical and simulative analyses show that the original FFT method has a low convergence rate and the converged solution can hardly fully meet the requirements of the desired pattern. A scaling factor is introduced to the original method. By choosing a proper value for the scaling factor, the convergence rate can be greatly improved and the final solution is able to fully meet the expectations. Simulation results are given to demonstrate the effectiveness of the proposed algorithm.

1. Introduction

In order to solve complex antenna pattern synthesis problems, various methods using various optimization algorithms have been developed. In [1], a quadratic program is formed for arbitrary array pattern synthesis. In [2], a convex optimization problem [3] is formulated for pattern synthesis subject to arbitrary upper bounds. For certain cases, the convex programming problem can be reduced to a linear programming problem [4]. In [5] an effective hybrid optimization method is proposed for footprint pattern synthesis of very large planar antenna arrays [6]. The methods mentioned above all adopt conventional optimization algorithms [7]. Global optimization algorithms such as genetic algorithms [8–10] and particle swarm optimization algorithms [11–13] have also been successfully applied in pattern synthesis problems. To approximate the desired pattern for an array, we need to discretize the angular space. The bigger the elements number is, the greater the required discrete density needs to be. Usually, the computational complexity would grow greatly as the elements number increases. As a consequence, normal synthesis techniques using local or global optimization algorithms are usually not suitable for large planar arrays.

As we know, fast Fourier transforms (FFT) are able to quickly compute the radiation pattern of an array with periodic structure. Once the number of FFT points is specified, the computation time is barely affected by the element number. In [14], an FFT method suitable for large planar arrays is proposed. The operation is very straightforward, which mainly involves direct and inverse fast Fourier transforms. In [15], a modified iterative FFT technique is proposed for leaky-wave antenna pattern synthesis. In [16], FFT is used for the pattern synthesis of nonuniform antenna arrays. The iterative FFT method is very efficient as shown in [14], and many examples are presented, but why the method is effective has not been explained.

In this paper, both theoretical and simulative analyses of the FFT method are presented. It is found that, though effective, it is a slow-convergence method and can hardly converge to the optimum solution. Based on the analyses, we introduce a scaling factor and the performance can be greatly improved.

2. Original FFT Method for Pattern Synthesis

First, we establish the relationship between a certain point in the FFT result and the corresponding angle of the radiation

pattern. Consider a planar array with $M \times N$ elements arranging in a rectangular grid and spacing d_x and d_y between rows and columns. Assume the element pattern is isotropic. The array factor is given by

$$F(\theta, \phi) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I_{mn} e^{j2\pi[m((d_x/\lambda) \sin \theta \cos \phi) + n((d_y/\lambda) \sin \theta \sin \phi)]}, \quad (1)$$

where I_{mn} is the complex excitation of the mn th element and λ is the wavelength. If the (u, v) coordinates ($u = \sin \theta \cos \phi$, $v = \sin \theta \sin \phi$) are used, the array pattern can be written as

$$F(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I_{mn} e^{j2\pi[m((d_x/\lambda)u) + n((d_y/\lambda)v)]}. \quad (2)$$

Performing $K_{\text{fft}} \times L_{\text{fft}}$ points 2D inverse fast Fourier transform (IFFT) on the excitations, we get

$$IF(k, l) = \frac{1}{K_{\text{fft}} L_{\text{fft}}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I_{mn} e^{j2\pi[m(k/K_{\text{fft}}) + n(l/L_{\text{fft}})]} \quad (3)$$

$$(k = 0, 1, \dots, K_{\text{fft}} - 1; l = 0, 1, \dots, L_{\text{fft}} - 1).$$

If we want to represent the array factor using $IF(k, l)$, then the coordinates (u, v) and (k, l) are related by

$$\begin{aligned} \frac{d_x}{\lambda} u &= \frac{k}{K_{\text{fft}}} + S_k \\ \frac{d_y}{\lambda} v &= \frac{l}{L_{\text{fft}}} + S_l, \end{aligned} \quad (4)$$

where S_k and S_l are integers, making sure both sides of the equations have the same value range. Suppose that both K_{fft} and L_{fft} are even numbers. Define

$$\begin{aligned} k' &= \begin{cases} k + \frac{K_{\text{fft}}}{2}, & k < \frac{K_{\text{fft}}}{2} \\ k - \frac{K_{\text{fft}}}{2}, & k \geq \frac{K_{\text{fft}}}{2} \end{cases} \\ l' &= \begin{cases} l + \frac{L_{\text{fft}}}{2}, & l < \frac{L_{\text{fft}}}{2} \\ l - \frac{L_{\text{fft}}}{2}, & l \geq \frac{L_{\text{fft}}}{2} \end{cases} \end{aligned} \quad (5)$$

Combining (4) and (5) and considering the value ranges of u and v , we have

$$\begin{aligned} u &= \frac{\lambda}{d_x} \left(\frac{k'}{K_{\text{fft}}} - \frac{1}{2} \right) \\ v &= \frac{\lambda}{d_y} \left(\frac{l'}{L_{\text{fft}}} - \frac{1}{2} \right). \end{aligned} \quad (6)$$

So, the relation between the array factor and the IFFT of the array excitation is

$$F(u, v) = K_{\text{fft}} L_{\text{fft}} \cdot IF(k', l'). \quad (7)$$

Finally, the visible space is given by

$$\left(\frac{\lambda}{d_x} \right)^2 \left(\frac{k'}{K_{\text{fft}}} - \frac{1}{2} \right)^2 + \left(\frac{\lambda}{d_y} \right)^2 \left(\frac{l'}{L_{\text{fft}}} - \frac{1}{2} \right)^2 \leq 1. \quad (8)$$

Note that the indices change in (5) is in fact the fftshift operation in Matlab.

Once the corresponding relationship is established, the procedure of the FFT method for pattern synthesis is given as follows.

- (1) Specify dimension of the array $M \times N$, the initial excitation $I_{mn}^{(1)}$, the FFT points $K_{\text{fft}} \times L_{\text{fft}}$, and the maximum iteration times N_{max} .
- (2) Perform IFFT on the excitation of the i th iteration $I_{mn}^{(i)}$ and obtain the array factor $F_{kl}^{(i)}$.
- (3) Extract the amplitude $|F_{kl}^{(i)}|$ and phase $\varphi_{kl}^{(i)}$ of $F_{kl}^{(i)}$.
- (4) Compare $|F_{kl}^{(i)}|$ with the desired pattern F_{kl}^e . If the computed pattern fully meets the requirements or the maximum iteration time is reached, terminate the procedure; otherwise go to the next step.
- (5) Obtain the new pattern $F_{kl}^{(i)'} by replacing the undesired $F_{kl}^{(i)}$ with $F_{kl}^e \cdot e^{j\varphi_{kl}^{(i)}}$ as follows:$

$$F_{kl}^{(i)'} = \begin{cases} F_{kl}^e \cdot e^{j\varphi_{kl}^{(i)}}, & (k, l) \in U \\ F_{kl}^{(i)}, & (k, l) \notin U, \end{cases} \quad (9)$$

where U is a set containing all the points where the pattern is undesired. For example, if (k^*, l^*) is a point within the side lobe region and $|F_{k^*l^*}^{(i)}| > F_{k^*l^*}^e$, then it means that the side lobe level exceeds the desired level and that $(k^*, l^*) \in U$.

- (6) Perform $K_{\text{fft}} \times L_{\text{fft}}$ points 2-D FFT on $F_{kl}^{(i)'}$ and choose the first $M \times N$ points as the initial excitation for the next iteration $I_{mn}^{(i+1)}$. Constraints can be easily made for amplitude-only or phase-only synthesis.
- (7) Go to step (2).

The procedure is also illustrated in Figure 1.

3. Algorithm Analysis

The procedure is simple yet effective. In this section, the method is carefully examined. Without loss of generality, take an N -element linear array for example. Suppose that in the i th iteration the excitation sequence is $\mathbf{I}^{(i)}$ which has been extended to M_{fft} ($M_{\text{fft}} > N$) points by zero padding. M_{fft} is the number of the FFT points. After comparing the obtained pattern $\mathbf{F}^{(i)}$ with the desired pattern, we can obtain the error pattern $\Delta \mathbf{F}^{(i)}$. Now we will find the relation between the error pattern in the next iteration $\Delta \mathbf{F}^{(i+1)}$ and $\Delta \mathbf{F}^{(i)}$. It is difficult to

FIGURE 1: Flowchart of the FFT procedure for pattern synthesis.

precisely compute $\Delta \mathbf{F}^{(i+1)}$ through the above assumption. Let $\Delta \mathbf{I}^{(i)}$ be the corresponding error excitation for $\Delta \mathbf{F}^{(i)}$. Then

$$\Delta \mathbf{F}^{(i)} = \text{IFFT} [\Delta \mathbf{I}^{(i)}], \quad (10)$$

where $\Delta \mathbf{I}^{(i)}$ contains M_{fft} points. Consider the elements in $\Delta \mathbf{F}^{(i)}$

$$\Delta F_m^{(i)} = \begin{cases} (F_m^e - |F_m^{(i)}|) \cdot e^{j\varphi_m^{(i)}}, & m \in U \\ 0, & m \notin U. \end{cases} \quad (11)$$

If the set U only contains points representing the side lobe region, then the magnitude values of the elements in $\Delta \mathbf{F}^{(i)}$ are very small, as well as $\Delta \mathbf{I}^{(i)}$. So the variation in $\mathbf{I}^{(i+1)}$ from $\mathbf{I}^{(i)}$ is very little. Then we can use $\mathbf{F}^{(i)} + \Delta \mathbf{F}^{(i)}$ as the desired pattern in the $i + 1$ th iteration and obtain the error pattern

$$\Delta \mathbf{F}^{(i+1)} \approx \mathbf{F}^{(i)} + \Delta \mathbf{F}^{(i)} - \mathbf{F}^{(i+1)} = \text{IFFT} [\mathbf{I}^{(i)} + \Delta \mathbf{I}^{(i)} - \mathbf{I}^{(i+1)}]. \quad (12)$$

For complex weighting using both amplitude and phase, $\mathbf{I}^{(i+1)}$ is given as

$$\mathbf{I}^{(i+1)} = \mathbf{I}^{(i)} + [\Delta \mathbf{I}^{(i)} - \Delta \mathbf{I}^{(i)'}], \quad (13)$$

where $\Delta \mathbf{I}^{(i)'}$ is a sequence by setting the first N elements in $\Delta \mathbf{I}^{(i)}$ to zero. So

$$\Delta \mathbf{F}^{(i+1)} \approx \text{IFFT} [\Delta \mathbf{I}^{(i)'}]. \quad (14)$$

As a result, we have

$$\|\Delta \mathbf{F}^{(i+1)}\|_2^2 \approx \frac{1}{M_{\text{fft}}} \|\Delta \mathbf{I}^{(i)'}\|_2^2 \leq \frac{1}{M_{\text{fft}}} \|\Delta \mathbf{I}^{(i)}\|_2^2 = \|\Delta \mathbf{F}^{(i)}\|_2^2, \quad (15)$$

where $\|\cdot\|_2$ denotes the l_2 -norm.

It is seen from (15) that the method has the ability to converge. However, as M_{fft} increases the difference between $\|\Delta \mathbf{I}^{(i)'}\|_2^2$ and $\|\Delta \mathbf{I}^{(i)}\|_2^2$ decreases. We can conclude that the method has a lower converge rate for larger FFT points. For amplitude-only synthesis, the analysis is similar and (15) still holds. For phase-only analysis, the excitation of the $i + 1$ th iteration is

$$\mathbf{I}^{(i+1)} = \mathbf{I} \cdot e^{j \arg[\mathbf{I}^{(i)} + \Delta \mathbf{I}^{(i)} - \Delta \mathbf{I}^{(i)'}]}, \quad (16)$$

where $\mathbf{I}$ is the given amplitude. Since the magnitude value of $\Delta \mathbf{I}^{(i)}$ can be very small compared to $\mathbf{I}$, $\mathbf{I}^{(i+1)}$ can be approximated by

$$\mathbf{I}^{(i+1)} \approx \mathbf{I}^{(i)} + [\Delta \mathbf{I}^{(i)} - \Delta \mathbf{I}^{(i)'}]. \quad (17)$$

So the convergence properties are similar to amplitude-only or complex tapering.

To verify our analysis, we will give an example. Consider a linear array with 60 elements spacing by half wavelength. For the amplitude-only synthesis, the desired pattern has a maximum side lobe level of -40 dB. For the phase-only case, the maximum side lobe level is -18 dB. We use the uniform taper as the initial excitation and set the maximum iteration

FIGURE 2: Norm of the error pattern: (a) amplitude-only and (b) phase-only.

number to 5000 and 10000 for the two cases, respectively. Figure 2 shows the convergence properties of the norm of the error pattern. We can see that the method has a low convergence rate, as it converges logarithmically in the latter part of the iteration. We can also see that the bigger the FFT points number is, the lower the method converges. Figure 2 also illustrates that although the norm of the error pattern can converge to a very low value, it does not reach zero. It means that the method can hardly synthesize a pattern that fully meets the requirements of the desired pattern.

4. Improved FFT Method for Pattern Synthesis

For planar arrays, the elements in the error pattern $\Delta F^{(i)}$ are given by

$$\Delta F_{kl}^{(i)} = \begin{cases} (F_{kl}^e - |F_{kl}^{(i)}|) \cdot e^{j\varphi_{kl}^{(i)}}, & (k, l) \in U \\ 0, & (k, l) \notin U. \end{cases} \quad (18)$$

First we divide U into U_s and U_m , which are the sets representing the side lobe region and the main lobe region, respectively. When $(k, l) \in U_s$, $F_{kl}^e - |F_{kl}^{(i)}| < 0$ and the value tends to converge towards zero more slowly as the procedure iterates. So, to change the slowly varying characteristics, we propose the following updating equation for the error pattern:

$$\Delta F_{kl}^{(i)} = \begin{cases} (\alpha^{(i)} \cdot F_{kl}^e - |F_{kl}^{(i)}|) \cdot e^{j\varphi_{kl}^{(i)}}, & (k, l) \in U_s \\ (F_{kl}^e - |F_{kl}^{(i)}|) \cdot e^{j\varphi_{kl}^{(i)}}, & (k, l) \in U_m \\ 0, & (k, l) \notin U, \end{cases} \quad (19)$$

where $\alpha^{(i)}$ is a scaling factor within the range $[0, 1]$. The smaller the value of $\alpha^{(i)}$ is, the more intense the excitation changes. In (19), when $(k, l) \in U_m$, the error pattern remains unchanged, since for main lobe shaping synthesis, we need to approximate $|F_{kl}^{(i)}| - F_{kl}^e$ to a certain error level, whereas in the side lobe region, only $F_{kl}^e \geq |F_{kl}^{(i)}|$ is required. So, the final updating equation replacing (9) is given by

$$F_{kl}^{(i)'} = \begin{cases} \alpha^{(i)} \cdot F_{kl}^e \cdot e^{j\varphi_{kl}^{(i)}}, & (k, l) \in U_s \\ F_{kl}^e \cdot e^{j\varphi_{kl}^{(i)}}, & (k, l) \in U_m \\ F_{kl}^{(i)}, & (k, l) \notin U. \end{cases} \quad (20)$$

To analyze the convergence properties of the proposed method, we still use the prior examples and consider a linear array with 60 elements spacing by half wavelength. The maximum iteration times are 5000 and 10000 as before. As illustrated in Figures 3 and 4, although the curves are not smooth, the norm of the error pattern can converge to zero for all the cases. In both figures, we can also see that the iteration times of the $\alpha = 0$ case are much less than that of $\alpha = 0.99$. Figures 5 and 6 show the influence of α value on the maximum iteration times and the array directivity. It is shown that as α grows near one, the iterations times increase rapidly but the directivity only changes a little. So we can say that the proposed method can give a good performance by setting α to zero. The radiation patterns obtained using $\alpha = 0.0$ are shown in Figure 7, where θ is measured from one end of the linear array to the other end.

FIGURE 3: Norm of the error pattern for the amplitude-only tapering: (a) $\alpha = 0.99$ and (b) $\alpha = 0.0$.

FIGURE 4: Norm of the error pattern for the phase-only tapering: (a) $\alpha = 0.99$ and (b) $\alpha = 0.0$.

5. Example for Planar Array Pattern Synthesis

Consider a 60×60 planar array with elements spacing by 0.65λ in both directions. The desired pattern has a maximum side lobe level of -40 dB and two -60 dB notches

at rectangular sectors $\{-0.1 \leq u \leq 0.1, 0.2 \leq v < 0.3\}$ and $\{0.3 \leq u \leq 0.4, -0.1 \leq v < 0.1\}$. The procedure suggested in [14] is adopted. First the 256×256 points FFT is used. At the following two phases, 512×512 and 1024×1024 points FFT are performed, with the previous results being

FIGURE 5: Maximum iteration times (a) and directivity of the array (b) versus α for the amplitude-only tapering.

FIGURE 6: Maximum iteration times (a) and directivity of the array (b) versus α for the phase-only tapering.

FIGURE 7: Normalized pattern: (a) amplitude-only tapering and (b) phase-only tapering.

FIGURE 8: Number of undesired points versus iteration times.

FIGURE 9: Normalized pattern at u -cut and v -cut planes.

the initial excitations for the next phase. At each phase, the maximum iteration time is 1000, and the scaling factor α is set to zero. Figure 8 gives the convergence property of the number of undesired radiation points. It shows that, after the previous two phases, the final phase converges very quickly.

The 1D radiation patterns containing the two notch sectors are shown in Figure 9. Figure 10 presents the 2D radiation pattern, where the visible space is determined by (8). The normalized amplitude and phase of the excitations are given in Figure 11.

FIGURE 10: 2D radiation pattern.

FIGURE 11: Normalized amplitude and phase of the excitations.

6. Conclusion

The iterative FFT method is capable of synthesizing a large planar array. In this paper, the method is validated by theoretical and simulative analyses. But the original method has a low convergence rate, and the synthesized results are usually unable to achieve the optimal solution. A scaling factor is introduced to form an improved method, which can avoid the drawbacks of the original method. Analysis and simulation results showed the effectiveness of the improved method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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