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Research Article

A Hybrid Solvers Enhanced Integral Equation Domain Decomposition Method for Modeling of Electromagnetic Radiation

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The hybrid solvers based on integral equation domain decomposition method (HS-DDM) are developed for modeling of electromagnetic radiation. Based on the philosophy of "divide and conquer," the IE-DDM divides the original multiscale problem into many closed nonoverlapping subdomains. For adjacent subdomains, the Robin transmission conditions ensure the continuity of currents, so the meshes of different subdomains can be allowed to be nonconformal. It also allows different fast solvers to be used in different subdomains based on the property of different subdomains to reduce the time and memory consumption. Here, the multilevel fast multipole algorithm (MLFMA) and hierarchical (H-) matrices method are combined in the framework of IE-DDM to enhance the capability of IE-DDM and realize efficient solution of multiscale electromagnetic radiating problems. The MLFMA is used to capture propagating wave physics in large, smooth regions, while H-matrices are used to capture evanescent wave physics in small regions which are discretized with dense meshes. Numerical results demonstrate the validity of the HS-DDM.

1. Introduction

Nowadays the simulation of practical electromagnetic (EM) problems is of vital importance in many areas of electric engineering. Because of its obvious advantages, the integral equation method (IEM) has attracted many attentions during the past decades. Unfortunately, the IEM always leads to an ill-conditioned dense matrix equation when it is used to solve the practical EM problems, because of the multiscale features of the geometry in the real world. When iterative method is used to solve this problem, it will converge very slowly or even fail to converge finally.

The construction of an effective preconditioner to improve the condition number of ill-posed matrix of multiscale system is an urgent task. The preconditioners based on the mathematical investigation of the matrix property [1] and developed from the physical insight of the integral operator [2, 3] have been successfully used in the EM problems. Based on the philosophy of "divide and conquer," the domain decomposition method (DDM) provides an efficient framework to solve the multiscale problems. It has been successfully incorporated into the finite element method (FEM) [4, 5], finite difference method (FDM) [6], integral equation method (IEM) [7, 8], and hybrid FEM and IEM (MS-DDM) [9]. The integral equation domain decomposition method (IE-DDM) [7, 8] divides the original problem into many nonoverlapping closed subdomains. Transmission conditions are enforced on the touching faces to maintain the continuity of current across the interfaces. Because of the nonconformal property of the IE-DDM, each subdomain can be meshed independently.

The small monopole antennas mounted on a large complex platform are a typical multiscale problem. The large smooth region can be discretized with large smooth elements, but the small tiny region needs to be discretized with very fine elements to describe the geometry and model the field properly. There are two different natures of physics, propagating wave physics and evanescent wave physics, corresponding to different electrical size of the elements. In this paper, hybrid solvers, multilevel fast multipole algorithm (MLFMA) [10], and hierarchical matrices (H-matrices) method [11, 12] are incorporated into the framework of IE-DDM to capture the

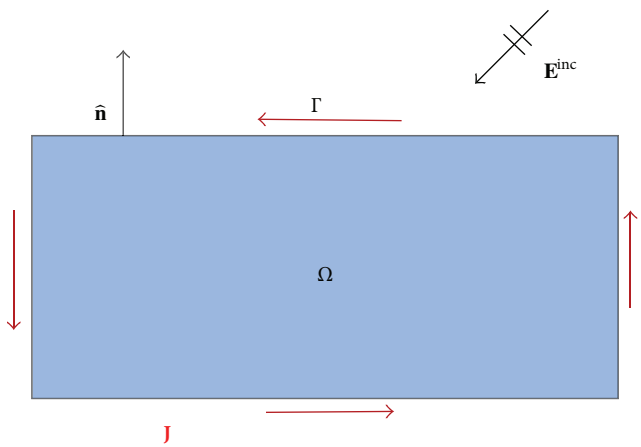

Figure 1: EM scattering from a PEC object.

two different physics in different subregions, respectively. Different from the MS-DDM in [9] incorporating the IEM and FEM into the framework of DDM to evaluate the effects of antennas-on-platform, here the IE-based HS-DDM is developed to analyze the multiscale EM radiating problems of antennas mounted on a large platform.

2. Theory of the HS-DDM

Consider the problem of electromagnetic scattering from a PEC object with domain Ω, as shown in Figure 1. Γ is the exterior surface of Ω. Based on the equivalent principle, when the EM wave illuminates on the PEC object, the equivalent current $\mathbf{J} = \hat{\mathbf{n}} \times \mathbf{H}^{\text{tol}}$, $\mathbf{H}^{\text{tol}} = \mathbf{H}^{\text{inc}} + \mathbf{H}^{\text{sca}}$. The original domain Ω can be decomposed into two closed nonoverlapping subdomains, Ω_1 and Ω_2, and they satisfy $\Omega = \Omega_1 \cup \Omega_2$. The Γ_1, Γ_2 are the exterior surface of Ω_1, Ω_2, which satisfy $\Gamma_1 = \overline{\Gamma}_1 \cup \Gamma_1^+$, $\Gamma_2 = \overline{\Gamma}_2 \cup \Gamma_2^-$. $\overline{\Gamma}_1$ and $\overline{\Gamma}_2$ are the surface of Ω_1 and Ω_2, which satisfy $\overline{\Gamma}_1 = \Gamma_1 \cap \Gamma$ and $\overline{\Gamma}_2 = \Gamma_2 \cap \Gamma$.

$\mathbf{J}_1$ and $\mathbf{J}_2$ are the surface currents on Γ_1 and Γ_2. $\mathbf{J}_1^+$ and $\mathbf{J}_2^-$ are the surface currents on the touching faces Γ_1^+ and Γ_2^-. The decomposed problem is equivalent to the original problem by enforcing the transmission condition on the touching faces:

$$\mathbf{J}_1^+ = -\mathbf{J}_2^-. \tag{1}$$

Through many derivations, the system matrix equation can be got like the following:

$$[\mathbf{M}]\,\mathbf{x} = [\mathbf{b}] + [\mathbf{N}]\,\mathbf{x}, \tag{2}$$

$$[\mathbf{M}] = \begin{bmatrix} \mathbf{M}_1 & \\ & \mathbf{M}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\overline{\Gamma}_1\overline{\Gamma}_1} & \mathbf{A}_{\overline{\Gamma}_1\Gamma_1^+} & & \\ \mathbf{A}_{\Gamma_1^+\overline{\Gamma}_1} & \mathbf{A}_{\Gamma_1^+\Gamma_1^+} & & \\ & & \mathbf{A}_{\Gamma_2^-\Gamma_2^-} & \mathbf{A}_{\Gamma_2^-\overline{\Gamma}_2} \\ & & \mathbf{A}_{\overline{\Gamma}_2\Gamma_2^-} & \mathbf{A}_{\overline{\Gamma}_2\overline{\Gamma}_2} \end{bmatrix}. \tag{3}$$

The submatrix blocks $\mathbf{A}_{\overline{\Gamma}_j\overline{\Gamma}_j}$, $\mathbf{A}_{\overline{\Gamma}_j\Gamma_j^+}$, $\mathbf{A}_{\Gamma_j^+\overline{\Gamma}_j}$, and $\mathbf{A}_{\Gamma_j^+\Gamma_j^+}$ stand for the self-coupling of each subdomain Ω_j, $j = 1, 2$ itself:

$$[\mathbf{N}] = \begin{bmatrix} & & \mathbf{C}_{\overline{\Gamma}_1\Gamma_2^-} & \mathbf{C}_{\overline{\Gamma}_1\overline{\Gamma}_2} \\ \mathbf{B}_{\Gamma_1^+\overline{\Gamma}_1} & \mathbf{B}_{\Gamma_1^+\Gamma_1^+} & \mathbf{D}_{\Gamma_1^+\Gamma_2^-} & \\ & \mathbf{D}_{\Gamma_2^-\Gamma_1^+} & \mathbf{B}_{\Gamma_2^-\Gamma_2^-} & \mathbf{B}_{\Gamma_2^-\overline{\Gamma}_2} \\ \mathbf{C}_{\overline{\Gamma}_2\overline{\Gamma}_1} & \mathbf{C}_{\overline{\Gamma}_2\Gamma_1^+} & & \end{bmatrix},$$

$$[\mathbf{b}] = \begin{bmatrix} \mathbf{b}_{\overline{\Gamma}_1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{b}_{\overline{\Gamma}_2} \end{bmatrix}; \tag{4}$$

$$[\mathbf{x}] = \begin{bmatrix} \mathbf{J}_{\overline{\Gamma}_1} \\ \mathbf{J}_{\Gamma_1^+} \\ \mathbf{J}_{\Gamma_2^-} \\ \mathbf{J}_{\overline{\Gamma}_2} \end{bmatrix}.$$

The matrix blocks $\mathbf{C}_{\overline{\Gamma}_1\Gamma_2^-}$, $\mathbf{C}_{\overline{\Gamma}_1\overline{\Gamma}_2}$, $\mathbf{C}_{\overline{\Gamma}_2\overline{\Gamma}_1}$, and $\mathbf{C}_{\overline{\Gamma}_2\Gamma_1^+}$ stand for coupling among different subdomains, and $\mathbf{D}_{\Gamma_1^+\Gamma_2^-}$, $\mathbf{D}_{\Gamma_2^-\Gamma_1^+}$ are the cement matrices which are used to enforce the transmission condition. The detailed expression of the matrix elements can be found in [7, 8]. But the IE-based domain decomposition method was only used to solve the EM scattering from multiscale nonpenetrable objects and conducting body of translation in [7, 8]. Here, the IE-DDM is extended to solve the EM radiation problems, and the right-hand-side vectors in (2) are replaced by voltage sources.

An effective preconditioner can be implemented in the IE-DDM as follows:

$$\mathbf{M}^{-1}\,[\mathbf{M} - \mathbf{N}]\,\mathbf{x} = \mathbf{M}^{-1}\,[\mathbf{b}]\,. \tag{5}$$

To realize this preconditioner effectively, the Krylov subspace-based method [13] can be used to solve this equation by an inner-outer iterative strategy. For practical engineering applications, fast solvers are needed to accelerate the matrix-vector multiplication products (MVPs). In [7, 8], only the MLFMA was used to accelerate the MVPs in the inner and outer iterations. Here, based on the different properties of different submatrices, both MLFMA and hierarchical matrices method are incorporated into the framework of the IE-DDM to accelerate the MVPs in the inner iterations of different subdomains. Because the original multiscale problem is decomposed into many subdomains, different fast solvers can be used in different subdomains. For subdomains which are discretized with very tiny elements, evanescent wave physics dominates there, and traditional MLFMA based on the expansion of plane wave spectrum is inefficient. By resorting to the H-matrices method, which is one of fast matrix compression techniques, the evanescent wave physics can be efficiently captured. For subdomains which are discretized with standard elements, propagating wave physics dominates there, and traditional MLFMA works very well, so it can be used to speed up the MVPs

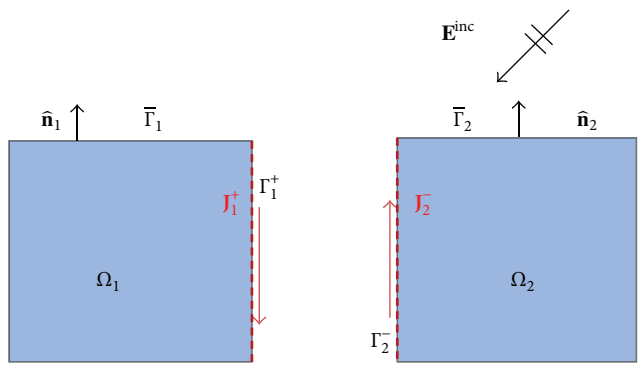

Figure 2: The notation of the decomposition of the original PEC object.

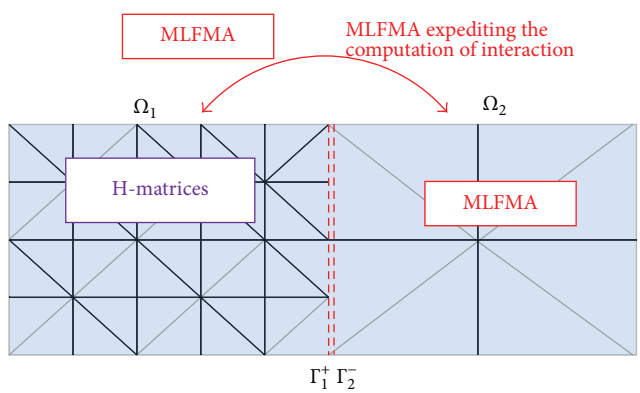

Figure 3: The systematic description of HS-DDM. In the first subdomain (left) with tiny meshes, hierarchical matrices method is used. In the second subdomain (right) with larger meshes, MLFMA is used.

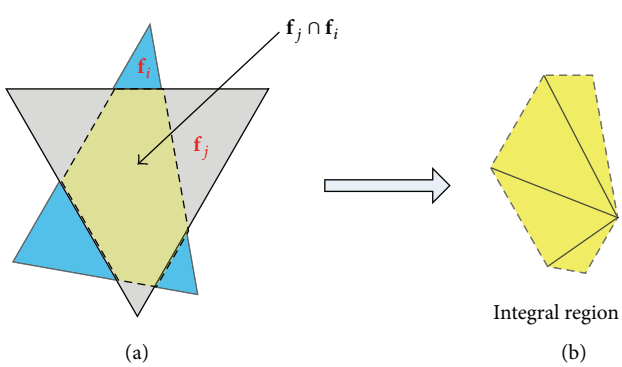

Figure 4: (a) An example of the nonconformal patches on the interface. (b) The description of evaluation of the cement matrix (union mesh).

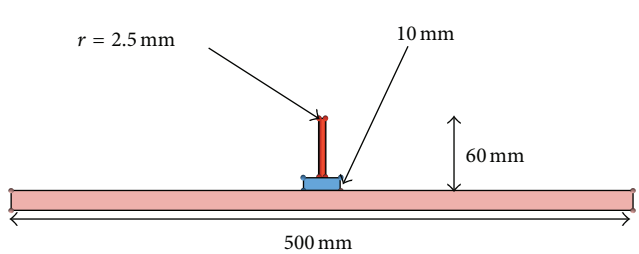

Figure 5: The geometry of the monopole antenna mounted on a PEC plate.

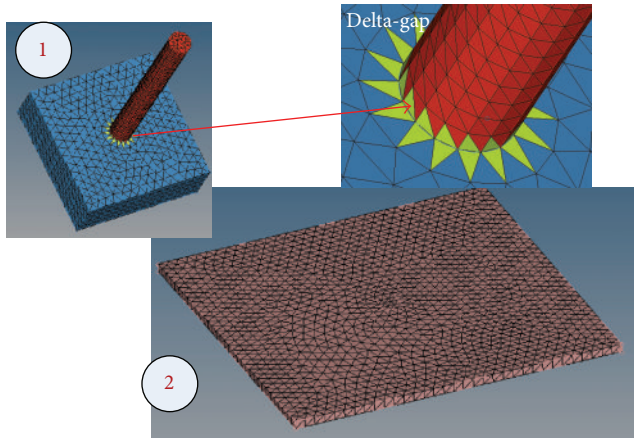

Figure 6: The domain partition of the monopole antenna mounted on a PEC tablet.

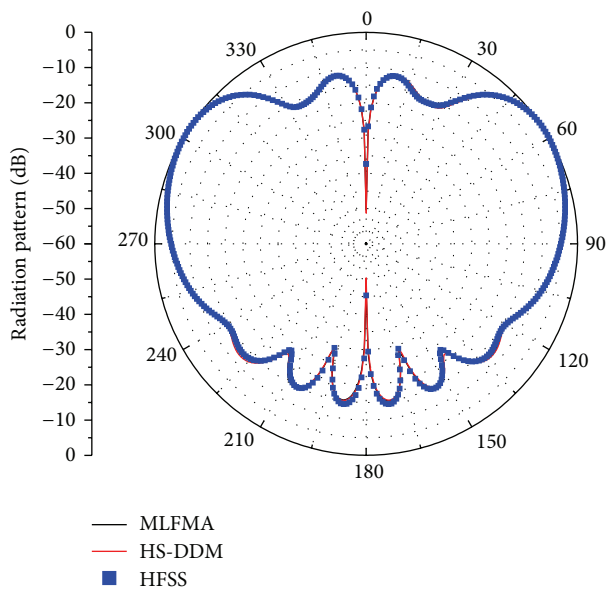

Figure 7: The normalized radiating pattern of the monopole antenna on a plate at 2.0 GHz.

in inner iterations. And the MLFMA is also applied for acceleration of interactions among different subdomains.

Considering the domain partition of the original PEC object Ω as an example (Figure 2), if the Γ_1 is discretized with very tiny elements and Γ_2 is discretized with large smooth elements, the H-matrices method can be used to speed up the matrix-vector multiplication product $[\mathbf{M}_1][\mathbf{J}_1]$ while the MLFMA can be used to effectively speed up the matrix-vector multiplication products $[\mathbf{M}_2][\mathbf{J}_2]$, $[\mathbf{N}][\mathbf{J}]$ in (5). A systematic description on the HS-DDM is demonstrated in Figure 3.

Solver A: Multilevel Fast Multipole Algorithm. Multilevel fast multipole algorithm (MLFMA) is the extension of the fast multipole method (FMM). It is a very efficient algorithm for accelerating the matrix-vector multiplication. The FMM based on the addition theory of Green's function divides the interaction of matrix element into two categories: the near region couplings are computed with method of moment (MoM) and the far region couplings are computed through three steps, aggregation, translation, and the disaggregation. More details about the MLFMA can be found in [10].

Solver B: Hierarchical Matrices Method. Although the MLFMA is very efficient for iterative solution of the IEM, it does not work well when treating very tiny meshes. As a purely algebraic and kernel independent arithmetic, the H-matrices algorithm is a good supplement to the MLFMA for the case of tiny meshes. It provides a highly compressed representation of a full matrix by using of low-rank property [10]. For a basis functions set which is discretized from a given object, firstly it is subdivided into two clusters recursively which contains approximately equal number of basis functions. By rearranging the basis functions by their indices, they are numbered consecutively and concentrated geometrically. If two clusters are well separated geometrically, Green's function which connects the interaction of them is varying very slowly in their domains. That means that the interaction matrix is low rank, and only a few patterns of interactional vectors can represent the whole patterns. So the interaction matrix is admissible which can be compressed by low-rank represented algorithms, such as adaptive cross approximation (ACA) algorithm [14]. Otherwise, it is nonadmissible, which needs to be computed by MoM directly. Here, it is necessary to clarify that the Krylov subspace method is also needed to solve (5) in the corresponding subdomains in which the submatrices are compressed by H-matrices method.

Based on the physical consideration, the HS-DDM combined the advantages of the MLFMA and H-matrices method in the framework of domain decomposition can achieve more flexible mesh generation, higher computational efficiency, and less memory requirement, compared with MLFMA or H-matrices method. In the HS-DDM, nonconformal meshes are available in different subdomains, and it greatly increases the ability of geometry modeling. It also leads to a well-conditioned matrix as the preconditioner is easier to be constructed in the DDM, so it achieves faster convergence

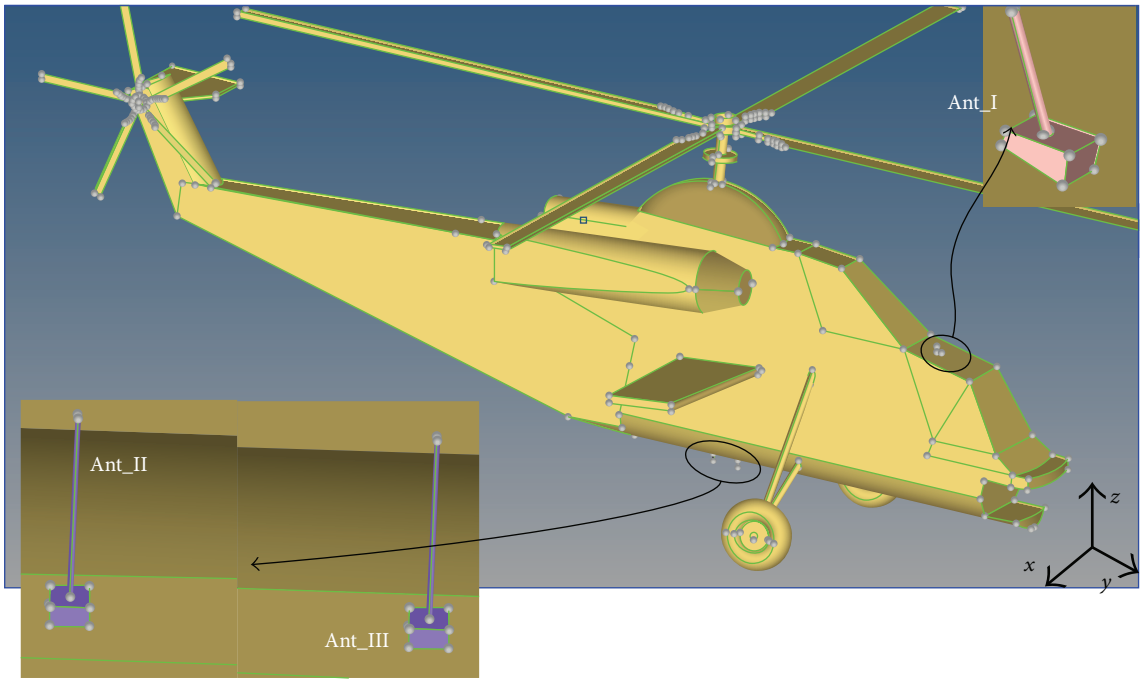

Figure 8: The geometry of the antennas mounted on a complex platform.

compared with traditional MLFMA for complicated objects like multiscale objects.

For nonconformal meshes in the HS-DDM, it is important to fast and accurately compute the cement matrices $\mathbf{D}_{\Gamma_1^+\Gamma_2^-}$, $\mathbf{D}_{\Gamma_2^-\Gamma_1^+}$. As shown in Figure 4, the union mesh technique [15, 16] is adopted for the treatment of nonconformal meshes on the touching faces between adjacent subdomains to determine the common integral region for cement matrix. Then the common integral region is divided into several triangles, the Gaussian quadrature formula is used over each triangle, and the integral results are finally attained based on the summation of the integral result over each triangle.

3. Numerical Results

In this section, some numerical results are presented to demonstrate the validation and capability of the HS-DDM.

3.1. Example for Validation. In the first numerical example, a monopole antenna mounted on a perfectly electric conductor (PEC) plate is investigated to demonstrate the validation of the HS-DDM, at 2.0 GHz. The geometry and the dimension of the antenna-on-platform are shown in Figure 5. The commonly used delta-gap source [15] is used as the feed port, as shown in Figure 6. The domain partition of the monopole antenna mounted on plate is also shown in Figure 6. The HS-DDM decomposed the antenna and plate into two closed subdomains. The convergence tolerance of both HS-DDM and traditional MLFMA is 0.01. The radiation pattern results of the HS-DDM, commercial software HFSS, and traditional single domain MLFMA are shown in Figure 7, respectively. It can be seen that they agree well with each other.

3.2. Example for Antennas Mounted on Large Complex Platform. In the second numerical example, three monopole antennas mounted on the simplified PEC helicopter as shown in Figure 8. The length of the helicopter is about 20 meters.

The radius and length of antennas I, II are 5 mm, 104 mm and 2.5 mm, 141 mm, respectively. The dimension of antenna III is same as that of antenna II.

The domain partition of the HS-DDM is shown in Figure 9, the geometry is decomposed into 6 subdomains, the main body of simplified helicopter is decomposed into 3 subdomains, and each antenna is a closed subdomain. The mesh size of the helicopter subdomains and antennas subdomains is 30 mm and 1 mm, respectively. The MLFMA and H-matrices method are used in the helicopter subdomains 1–3 and antennas subdomains 4–6, respectively. The delta-gap source is used to feed antenna I at 1.0 GHz. The inner and outer convergence tolerance are 0.001 and 0.01, respectively. The convergence tolerance of the traditional MLFMA is 0.01. The current distributions and radiation patterns of IE-DDM, HS-DDM, and MLFMA are shown in Figures 10 and 11. It can be seen that they agree with each other very well. The input impedance of the antenna evaluated by IE-DDM, HS-DDM, and traditional MLFMA is $34.3 + 45.3i$, $35.2 + 44.2i$, and $32.3 + 49.4i$, respectively.

The impedance of the antenna reflects the near field current state of the EM radiation. When the convergence threshold of traditional MLFMA is enhanced to 0.001, the input impedance changes to $33.4 + 46.1i$, which is more close to the results of the DDM. From this, it can be seen that the near field current and far field radiation pattern results of the DDM are more reliable and stable. It means that the preconditioner of the DDM is very effective. It can improve the condition number of the matrix greatly. Compared to

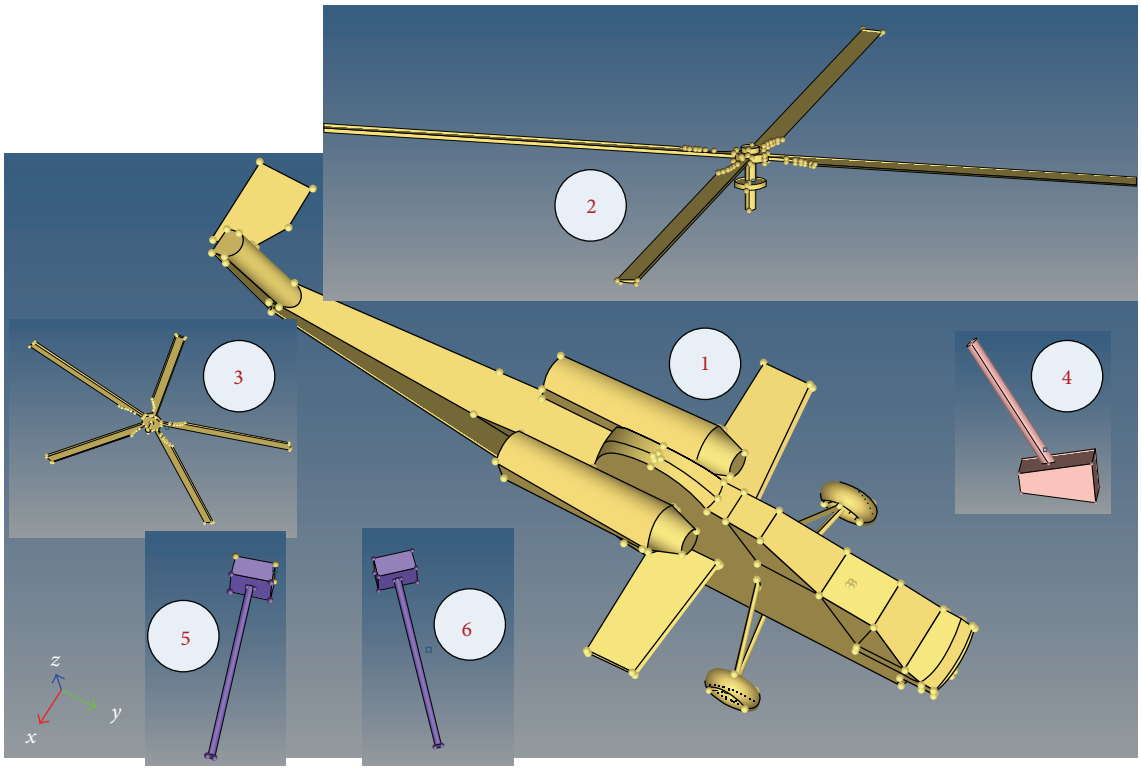

Figure 9: The domain partition of the antennas-on-platform: 6 subdomains.

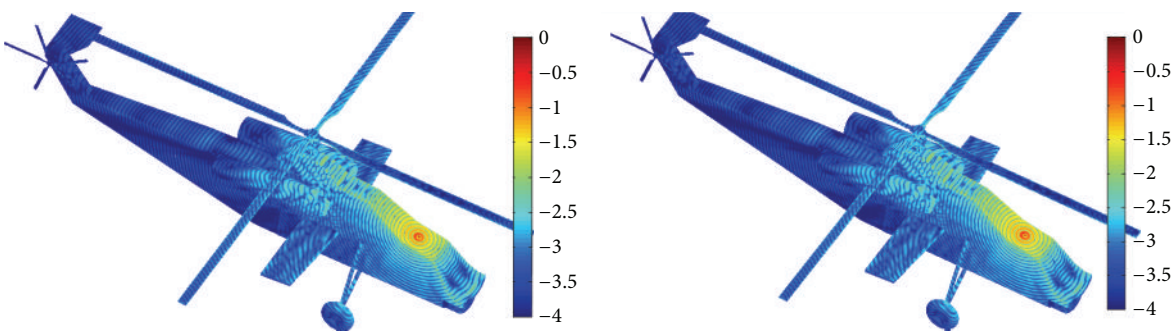

Figure 10: The surface current distribution of the monopole antenna on helicopter evaluated by the HS-DDM (a) and MLFMA (b) (logarithmic scale).

Table 1: Computational statics of antennas on helicopter.

Method	Unknowns	Mem.	Time (h:m)	Iter. number
MLFMA	922,017	31.5 GB	4:42	630
IE-DDM	897,597	36.9 GB	8:26	8
HS-DDM	897,597	7.3 GB	2:53	8

traditional MLFMA, the DDM only needs a relative larger threshold to get the accurate results.

The computational time and memory consumption of these methods are compared in Table 1. It can be seen that the HS-DDM greatly reduces the time and memory consumption compared to traditional MLFMA and the IE-DDM, by incorporating the advantage of MLFMA and H-matrices method. The DDM also provides an effective preconditioner from the iterative number of these two methods, as shown in Table 1.

4. Conclusion

In this paper, the HS-DDM is presented to solve the multiscale radiating problems, antennas mounted on a large platform. The HS-DDM inherits the advantages of IE-DDM and also incorporates the advantage of different fast solvers; here MLFMA and H-matrices method are implemented into the framework of IE-DDM to capture two different

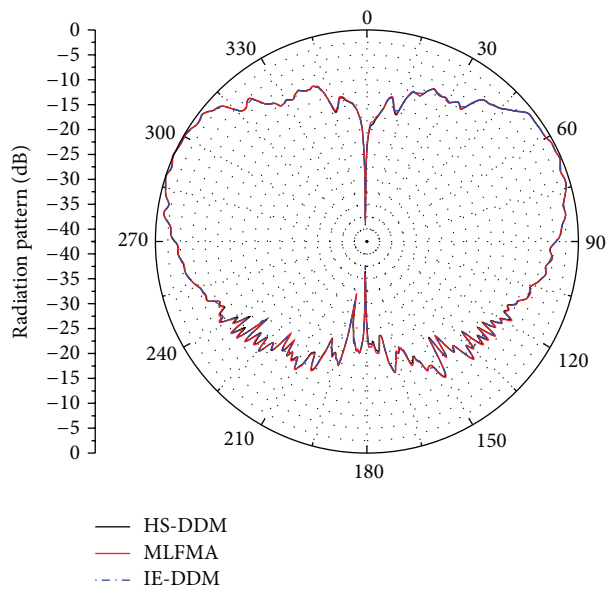

Figure 11: The normalized radiation pattern of the monopole antenna on helicopter at 1.0 GHz.

natures in different subregions, propagating wave physics and evanescent wave physics in the multiscale EM problems; the CPU time and memory consumption are reduced greatly.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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