Research Article

Compressed Measurements Based Spectrum Sensing for Wideband Cognitive Radio Systems

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Spectrum sensing is the most important component in the cognitive radio (CR) technology. Spectrum sensing has considerable technical challenges, especially in wideband systems where higher sampling rates are required which increases the complexity and the power consumption of the hardware circuits. Compressive sensing (CS) is successfully deployed to solve this problem. Although CS solves the higher sampling rate problem, it does not reduce complexity to a large extent. Spectrum sensing via CS technique is performed in three steps: sensing compressed measurements, reconstructing the Nyquist rate signal, and performing spectrum sensing on the reconstructed signal. Compressed detectors perform spectrum sensing from the compressed measurements skipping the reconstruction step which is the most complex step in CS. In this paper, we propose a novel compressed detector using energy detection technique on compressed measurements sensed by the discrete cosine transform (DCT) matrix. The proposed algorithm not only reduces the computational complexity but also provides a better performance than the traditional energy detector and the traditional compressed detector in terms of the receiver operating characteristics. We also derive closed form expressions for the false alarm and detection probabilities. Numerical results show that the analytical expressions coincide with the exact probabilities obtained from simulations.

1. Introduction

There is a remarkable growing demand on wireless devices and services that use the electromagnetic spectrum for communication. In static license regime, the spectrum bands are assigned to licensed holders on a long term basis for large geographical regions. However, a large portion of the assigned spectrum remains underutilized [1]. Cognitive radio (CR) can utilize the spectrum more efficiently in an opportunistic fashion. CR allows a secondary user (SU) to use a specific spectrum band as long as its licensed primary user (PU) is protected against harmful interference. The Federal Communications Commission (FCC) defined the CR as follows: a radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation [2]. The main functions for CRs are spectrum sensing, spectrum management, spectrum mobility, and spectrum sharing [3].

One key component of the CR system is spectrum sensing (SS), by which a SU radio can detect the presence or absence of the PUs and identify the available white spaces in the spectrum [4]. In order not to affect the performance of the PU, the SS process has to meet the sensing speed and accuracy requirements set by the FCC [2]. The problem becomes more challenging, particularly in wideband systems where the sampling rate has to be higher than or equal to the Nyquist rate. Consequently, the complexity and the cost of the hardware circuits as well as the power consumption would be high [5–7]. Moreover, the timing requirements for rapid sensing may only allow the acquisition of a small number of samples which may not provide accurate information about the existence of the PU.

Recently, compressive sensing (CS) is used to solve the high sampling rates problem in the CR systems. CS allows the sensing of sparse signals at sub-Nyquist sampling rates and a reliable recovery of the signal via computationally efficient
algorithms [8, 9]. In CS, the signal can be recovered from a
small number of projections over a sensing basis (measure-
ments) as long as it is sparse over the representation basis that
is incoherent with the sensing basis. Examples of the sensing
basis matrices are Gaussian, Bernoulli, partial Hadamard,
and partial Fourier matrices. In order to reconstruct the
signal, the sensing basis should satisfy the restricted isometry
property (RIP) [10]. Signal reconstruction is the solution of
an $\ell_1$-norm optimization problem to recover the high dimen-
sional signal from the low dimensional measurements.

In previous work of using CS in CR applications, SS is
completed in three steps: sampling using an analog to infor-
mation converter (AIC) at a sub-Nyquist rate, reconstructing
the Nyquist rate signal or reconstructing the PU frequency
response, and then applying one of the spectrum detection
techniques on the reconstructed signal [11, 12, 16]. The SS
process can be considered as a binary hypothesis test, where
the detection algorithm has to decide which one of the
two hypotheses is probably true. The mostly used detection
schemes are matched filter detection [13], energy detection
[14], and feature detection [15]. Although the algorithms
proposed in [11, 12, 16] reduce the hardware complexity of
acquiring the signal, they increase the computational com-
plexity through the reconstruction process. The fundamental
task of SS is to detect the presence of the PUs. Therefore, the
full reconstruction of the wideband signal is not necessary
as long as the PU existence can be accurately detected from
the compressed measurements. In [17], it was shown that the
detection and estimation problem can be solved using
the compressed measurements without reconstructing the
original signal. However, the observation matrix chosen is the
random observation matrix which serves the purpose of the
reconstruction process not the detection process. The authors
showed that the detection probability increases as the number
of the compressed measurements increases. The analytical
results of [17] show that the traditional detector outperforms
the compressed measurements based detection algorithm
in terms of the detection probability and the proposed
detection algorithm will provide the same performance as the
traditional detector only when the number of the compressed
measurements equals the number of Nyquist rate samples.

In this paper, we propose an algorithm that enables the
detection of the PU existence from the compressed meas-
urements directly without going into the intermediate pro-
cess of reconstructing the signal. This can be realized by
designing a compressive sensing process that guarantees that
the information used in the detection process is preserved in
the compressed measurements. To achieve this, the sensing
matrix has to be designed with constraints different from
those used in the conventional CS [10] and the detector
proposed in [17]. The algorithm we propose in this paper
is based on the well-known energy detection techniques.
Therefore, the sensing matrix should preserve the energy of
PU signal. The discrete cosine transform (DCT) [18] has the
property of energy compaction where most of the energy of the
time domain signal will be concentrated in few samples of
the DCT domain signal. Therefore, we adopt the DCT
matrix as the sensing matrix in our algorithm. In addition
to the reduction in the computational complexity due to the
elimination of the reconstruction process, the results show
that the proposed algorithm provides a better performance
than the traditional energy detector that uses the Nyquist rate
signal and accordingly the compressed detector proposed in
[17]. Moreover, the detection algorithm we propose does not
require a priori knowledge of estimation of the PU signal
sparsity. We also derived closed form expressions for the
detection and false alarm probabilities. Simulation results are
provided to validate the derived expressions.

The remainder of this paper is organized as follows. The
system model and the detection algorithms are described in
Section 2. The derivation of the false alarm and detection
probabilities is provided in Section 3. Section 4 presents the
numerical results and discussions. Finally, the conclusions are
drawn in Section 5.

2. System Model and Detection Algorithms

In this section, we present the system model, compres-
sive sensing based energy detection (CSBED) algorithm,
and the compressed measurements based energy detection
(CMBED) algorithm. Let us first summarize some notations
and definitions that will be used through the paper:

(i) $\mathcal{H}_0$: it is the null hypothesis which states that there
is no transmission from the PU in the band under
consideration,
(ii) $\mathcal{H}_1$: it is the alternative hypothesis which states that
there is a transmission from the PU in the band under
consideration,
(iii) $\mathcal{D}_0$: the SU accepts $\mathcal{H}_0$ after applying the detection
algorithm,
(iv) $\mathcal{D}_1$: the SU rejects $\mathcal{H}_0$ after applying the detection
algorithm,
(v) $P(\mathcal{D}_1 | \mathcal{H}_0)$: it is the probability that the SU decides
that the PU exists while it does not (probability of false
alarm).

2.1. System Model. In this paper, the primary network is
assumed to be the television broadcasting system. According
to the IEEE 802.22 WRAN standards [19], the PU signal is
transmitted using orthogonal frequency division multiplex-
ing (OFDM) with the following specifications: 54 MHz: main
carrier frequency, 4.45 Mbps: data rate, 16-QAM: payload
modulation, and 2048 FFT: size. The channel is modeled
as an additive white Gaussian noise (AWGN) channel. The
hypothesis model is defined as follows:

$$\mathcal{H}_0 : x(t) = n(t),$$  \hspace{1cm} (1)
$$\mathcal{H}_1 : x(t) = s(t) + n(t),$$  \hspace{1cm} (2)

where $x(t)$ is the signal received by the SU, $s(t)$ is the PU signal
with bandwidth $W$, and $n(t)$ is an (AWGN) with zero mean
and one-sided power spectral density of $N_0/2$. The received
signal is sensed using CS to get the low dimensional data vec-
tor $y$ which has a sub-Nyquist sampling rate and is given by

$$y = \Phi x,$$  \hspace{1cm} (3)
where $\mathbf{x}$ is an $N \times 1$ vector which represents the Nyquist rate samples of $x(t)$, $\mathbf{y}$ is the $M \times 1$ compressed measurements vector, and $\mathbf{Φ}$ is an $M \times N$ sensing matrix ($M < N$).

2.2. Compressive Sensing Based Energy Detection Algorithm (CSBED). In CSBED, the Nyquist rate signal is reconstructed from the compressed measurements vector $\mathbf{y}$ of (3) and then the energy detector is applied to the reconstructed signal. The reconstruction process is a linear inverse problem with sparseness constraint. It was shown that this problem is an NP-hard [20]. Basis Pursuit (BP) [21] is a reconstruction technique that transforms the problem to a convex optimization problem that can be solved by linear programming as follows:

$$
\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{Φ}\mathbf{x}.
$$

(4)

Several reconstruction techniques have been proposed in the literature. Examples of these techniques are Matching Pursuit (MP) [22] and Orthogonal Matching Pursuit (OMP) [23]. These techniques differ in the computational complexity and memory requirements. For instance, the computational complexity and memory requirements of the OMP algorithms as a function of the iteration number $k$ are summarized in Table 1. In QR-1, only after the final iteration $K$ does OMP find the solution [24]. The accuracy of the reconstructed signal depends on the number of compressed measurements $M$ and the number of iterations. In order to detect the existence of the PU, the energy of the reconstructed signal $\xi_\mathbf{x}$ is calculated and compared with a predetermined threshold $\lambda$ as follows:

$$
\xi_\mathbf{x} \overset{\mathcal{D}_0}{\gtrless} \lambda.
$$

(5)

2.3. Compressed Measurements Based Energy Detection Algorithm (CMBED). In this paper, we propose to detect the existence of the PU directly from the compressed measurements without reconstructing the Nyquist rate signal. Hence, the computational complexity will be dramatically reduced. Figure 1 shows the block diagram of the proposed detection algorithm. In ordinary CS, the sensing matrix is chosen with specific constraints such that the Nyquist rate signal can be accurately reconstructed. Since the Nyquist rate signal will not be reconstructed in the proposed algorithm, the constraints of the sensing matrix are different. The sensing matrix has to preserve specific parameters of the signal such that the PU can be detected. In this paper, the energy detector will be applied to the compressed measurements. Therefore, the sensing matrix has to preserve the energy of PU signal. The discrete cosine transform (DCT) expresses a finite sequence of data points in terms of a sum of cosine functions oscillating at different frequencies. One of the properties of the DCT is the strong energy compaction where most of the signal information tends to be concentrated in the few low-frequency coefficients [18]. In the proposed algorithm, the CS is performed using the DCT sensing matrix. The entry of the sensing matrix that is located at the $k$th row and the $i$th column is given by

$$
\phi(k, i) = c(k) \cos \left( \frac{\pi (2i + 1) k}{2N} \right),
$$

(6)

where $k = 0, 1, \ldots, M - 1$, $i = 0, 1, \ldots, N - 1$, and

$$
c(k) = \begin{cases} 
\frac{1}{N}, & k = 0, \\
\frac{2}{N}, & k \neq 0.
\end{cases}
$$

(7)

It is worth mentioning that the first $M$ rows of the DCT matrix are chosen as they correspond to the high-energy coefficients. After CS, according to Parseval’s theorem [18], the energy of the signal can be calculated from the compressed measurements as follows [25]:

$$
\xi_{\mathbf{y}_M} = \sum_{k=0}^{M-1} \left( \frac{y_k}{\sqrt{N_0 \mathcal{W}}} \right)^2.
$$

(8)

Afterwards, the energy detector decision variable $\xi_{\mathbf{y}_M}$ is compared with a predetermined threshold $\lambda$ to decide $\mathcal{D}_0$ or $\mathcal{D}_1$ as follows:

$$
\xi_{\mathbf{y}_M} \overset{\mathcal{D}_0}{\gtrless} \lambda.
$$

(9)
3. False Alarm and Detection Probabilities

In this section, we derive closed form expressions for the false alarm and detection probabilities of the proposed CMBED algorithm. The probability of false alarm $P_F$ is defined as the probability that the SU declares that the PU exists while it does not and is given by

$$P_F = P(\mathcal{H}_0 | \mathcal{H}_0) = P(\xi_{\mathcal{H}_0} \geq \lambda | \mathcal{H}_0).$$

(10)

When the PU does not exist, the signal received by the SU is an AWGN as given by (1). Therefore, the output of CS would be a vector of $M$ Gaussian random variables (linear combination of i.i.d Gaussian RV’s is a Gaussian RV) which is given by

$$y = [v_1 \ v_2 \ \cdots \ v_M]^T,$$

(11)

where $v_1, v_2, \ldots, v_M$ are independent and identically distributed Gaussian RV’s each has zero mean and variance $N_{01} W$. Substituting from (11) into (8) yields

$$\xi_{\mathcal{H}_0} = \sum_{k=0}^{M-1} \left( \frac{v_k}{\sqrt{N_{01} W}} \right)^2.$$  

(12)

The decision variable of (12) follows the central chi-square distribution with $M$ degrees of freedom and unity variance with the following probability density function [13, eq. (2.3-21)]:

$$f_{H0}(\xi_{\mathcal{H}_0}) = \frac{1}{2^{M/2} \Gamma(M/2)} \xi^{M/2-1} e^{-\xi/2},$$

(13)

where $\Gamma(\cdot)$ is the gamma function. From (10) and (13), the probability of false alarm would be given by

$$P_F = \frac{1}{2^{M/2} \Gamma(M/2)} \int_{\lambda}^{\infty} \xi^{M/2-1} e^{-\xi/2} d\xi$$

$$= \frac{\Gamma(M/2, \lambda/2)}{\Gamma(M/2)},$$

(14)

where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function. The probability of detection is given by

$$P_D = P(\mathcal{H}_1 | \mathcal{H}_1) = P(\xi_{\mathcal{H}_1} \geq \lambda | \mathcal{H}_1).$$

(15)

When the PU exists, the signal received by the SU is the summation of the PU signal and a white Gaussian noise as given by (2). The output of CS would be

$$y = [s_1 + v_1 \ s_2 + v_2 \ \cdots \ s_M + v_M]^T,$$

(16)

where $s_1, s_2, \ldots, s_M$ are the $M$ DCT coefficients of the PU signal. Substituting from (16) into (8) yields

$$\xi_{\mathcal{H}_1} = \sum_{k=0}^{M-1} \left( \frac{s_k + v_k}{\sqrt{N_{01} W}} \right)^2.$$  

(17)

The decision variable of (17) is a noncentral chi-square random variable with $M$ degrees of freedom and has the following probability density function [13, eq. (2.3-29)]

$$f_{H1}(\xi_{\mathcal{H}_1}) = \frac{1}{2^{(M-1)/2} \mu^{M/2}} e^{-\xi/(2\mu)} I_{M/2-1} \left( \sqrt{\frac{\xi}{\mu}} \right),$$

(18)

where $I_m(\cdot)$ is the $m$th order modified Bessel function of the first kind and $\mu$ is the noncentrality parameter which is given by [14]

$$\mu = \sum_{k=0}^{M-1} \left( \frac{s_k}{\sqrt{N_{01} W}} \right)^2.$$  

(19)

Let $E_s = \sum_{k=0}^{N-1} s_k^2$ be the total energy of the PU signal and $\eta_E$ be the fraction of energy contained in the $M$ DCT coefficients relative to $E_s$, i.e., $\eta_E = 1$ when $M = N$. Then

$$\eta_E = \frac{1}{E_s} \sum_{k=0}^{M-1} s_k^2.$$  

(20)

Substituting from (20) into (19) yields

$$\mu = \frac{\eta_E}{E_s} \frac{\eta_E y}{N_{01} W} = \eta_E y,$$

(21)

where $y$ is the signal to noise ratio. From (15), (18), and (21), the probability of detection can be calculated as follows:

$$P_D = \frac{1}{2} \int_{\lambda}^{\infty} \left( \frac{\xi}{\mu} \right)^{(M-2)/2} e^{-\xi/(2\mu)} I_{M/2-1} \left( \sqrt{\frac{\xi}{\mu}} \right) d\xi$$

$$= \frac{Q_{M/2}(\sqrt{\eta_E y}, \sqrt{\lambda})}{\sqrt{\eta_E y}}.$$  

(22)

where $Q_{M/2}(\cdot, \cdot)$ is the generalized Marcum Q-function [13, eq. (2.3-36)]. The probability of misdetection is given by

$$P_M = P(\mathcal{H}_0 | \mathcal{H}_1) = 1 - P_D$$

$$= 1 - Q_{M/2}(\sqrt{\eta_E y}, \sqrt{\lambda}).$$  

(23)

The performance of the proposed algorithm is compared with that of the traditional energy detector (TED) and the compressed detector presented in [17] where the probability of detection is approximated by

$$P_D = Q \left( \frac{Q^{-1}(P_F) - \sqrt{M}}{N} \sqrt{\frac{\eta_E y}{\sqrt{\lambda}}} \right).$$  

(24)

In the compressed detector [17], decreasing the compression ratio (i.e., decreasing $M$) causes a degradation in performance but the algorithm we propose in this paper provides a better performance at low compression ratios as will be shown and intuitively interpreted in Section 4.

4. Numerical Results and Discussions

In this section, we present numerical results to study the performance of the proposed sensing technique. Unless otherwise stated, the PU signal is assumed to follow the IEEE
802.22 WRAN standards described in Section 2.1. Figure 2 compares the complementary receiver operating characteristics (ROC) of the traditional energy detector (TED) (i.e., the energy detector is applied to signal that is sampled at a rate higher than the Nyquist rate) with that of the CSBED algorithm. Gaussian and Bernoulli matrices are used to perform the CS process. In this comparison, the compression ratio \( M/N \) is 70% and the SNR equals 15 dB. In CSBED, the OMP algorithm is used to reconstruct the signal. We find that the performance of the CSBED technique is worse than that of the TED. That is because the PU signal was not perfectly reconstructed. Therefore, the TED will be considered when evaluating the performance of the CMBED technique proposed in this paper.

Figure 3 compares the complementary ROC of the proposed CMBED algorithm with that of the TED algorithm, using compressed measurements sensed with Gaussian and partial Fourier ensemble matrices, and the compressed detector in [17], when the compression ratio \( M/N \) is 10% and the SNR equals 15 dB. The false alarm and the misdetection probabilities are obtained from both simulations and the analytical results of Section 3. We find that the analytical results coincide with the simulation results which validates (14) and (22). We also find that the proposed CMBED algorithm provides a better performance than the TED algorithm and other algorithms. That is because the compressed measurements contain most of the PU signal energy and only a fraction of the noise energy. Figure 4 shows the percentage of the energy \( \eta_E \) contained in \( M \) compressed measurements of both PU and noise signals against the compression ratio \( M/N \). We find that, at \( M/N = 10\% \), the percentage of the energy contained in the compressed measurements of the PU signal equals 91.5% and that of the noise signal equals 10%. That is because, on the one hand, the DCT-based CS preserves the energy of the PU signal, and, on the other hand, it would only preserve \( M/N \) of the AWGN energy.

Figure 5 shows the detection probability \( P_D \) against the compression ratio \( M/N \) when \( N = 128 \), \( \gamma = 15 \) dB, and target \( P_F = 0.01 \) for the proposed CMBED algorithm, TED, and compressed detector in [17]. We find that \( P_D \) increases as \( M/N \) increases until it reaches a maximum value of 0.903 at a compression ratio of 5.5% and then it decreases as \( M/N \) increases. This convex behavior of the detection probability of the CMBED algorithm can be interpreted as follows. Let \( E_{s_m} \) and \( E_{n_m} \) denote the energies contained in
the $M$ compressed measurements of the PU and the noise signals, respectively. It is clear that both $E_{s_M}$ and $E_{n_M}$ will increase as the compression ratio increases when $N$ is fixed. In order to keep the false alarm probability fixed (i.e., $P(E_{n_M} > \lambda) = 0.01$), increasing the compression ratio (or equivalently increasing $M$) necessitates an increase in the threshold $\lambda$. The detection probability is defined as the probability that the energy contained in the compressed measurements is greater than the threshold when the PU exists (i.e., $P(E_{s_M} + E_{n_M} > \lambda)$). When $M$ increases, both sides of the inequality will increase but not with the same rate. At lower compression ratios (i.e., smaller values of $M$), the growth of $E_{s_M}$ with respect to $M$ is much higher than that of $E_{n_M}$ (as shown in Figure 4) and accordingly $\lambda$. Therefore, the detection probability increases as $M/N$ increases. At higher compression ratios, increasing $M$ results in a negligible increase in $E_{s_M}$; however, a considerable increase occurs in $\lambda$. Hence, the detection probability decreases as $M/N$ increases.

Figure 6 shows the detection probability against the signal-to-noise ratio for CMBED, TED, and the compressed detector in [17] when $P_f = 0.01$ and compression ratio of 10% at $N = 128$. We find that the detection probability of the proposed CMBED algorithm is higher than other algorithms.

### 5. Conclusions

In this paper, we proposed a spectrum sensing technique for wideband cognitive radio systems. In order to detect the existence of the PU, the SU firstly samples the received signal at a sub-Nyquist rate using compressive sensing and then applies the energy detector on the compressed measurements. Consequently, the complexity required in reconstructing the Nyquist rate signal is eliminated. The DCT sensing matrix was used for the compressive sensing step due to its energy compaction property. In addition to the considerable complexity reduction, the results showed that proposed algorithm provides a better detection probability compared to traditional schemes at the same false alarm probability. That is because although the compressed measurements contain most of the energy of the PU signal, they only contain a fraction of the energy of the noise signal. We also derived analytical expressions for the false alarm and misdetection probabilities. These results are validated using simulations.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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