Research Article

Joint Multichannel Motion Compensation Method for MIMO SAR 3D Imaging

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The multiple-input-multiple-output (MIMO) synthetic aperture radar (SAR) system with a linear antenna array can obtain 3D resolution. In practice, it suffers from both the translational motion errors and the rotational motion errors. Conventional single-channel motion compensation methods could be used to compensate the motion errors channel by channel. However, this method might not be accurate enough for all the channels. What is more, the single-channel compensation may break the coherence among channels, which would cause defocusing and false targets. In this paper, both the translational motion errors and the rotational motion errors are discussed, and a joint multichannel motion compensation method is proposed for MIMO SAR 3D imaging. It is demonstrated through simulations that the proposed method exceeds the conventional methods in accuracy. And the final MIMO SAR 3D imaging simulation confirms the validity of the proposed algorithm.

1. Introduction

Conventional single-channel synthetic aperture radar (SAR) uses wideband signal and synthetic apertures to obtain high range resolution and high azimuth resolution, respectively. But it could not resolve along height due to the lack of baseline in the elevation direction. The imaging results are the projection from the scene in 3D to the range-azimuth plane. To resolve along height, the multiple-input-multiple-output (MIMO) SAR system [1–3] uses multichannel to form the baseline in the third dimension. Technically, the MIMO SAR system could place its sensor in many different layouts. Two possible geometries of MIMO SAR are along-track array (for the reduction of azimuth-ambiguities, moving target indication, superresolution, etc.) and across-track array (for the reduction of elevation-ambiguities, interferometry, 3D imaging, etc.) [4].

The MIMO SAR system has two main advantages: (1) the degrees of freedom can be greatly increased by the concept of virtual array provided by the multiple antennas [3]; (2) the MIMO SAR system can provide plenty of transmitting elements and receiving elements to satisfy the cross-track sampling [5]. Due to the second advantage, it can significantly improve the resolution of the third dimension. Thus it is widely used for 3D imaging. Profiting from the unique advantages, airborne MIMO SAR 3D imaging technique has become a field of intensive research in recent years. However, studies about the MIMO SAR are mostly concentrated on antenna arrangements [4, 6], waveform designs [7, 8], and imaging algorithms [4, 9]. The motion compensation (MOCO) has not been paid much attention. To deal with the motion errors, the most preferable way is using a high-precision navigation system. However, in many cases, due to the limit of accuracy of such systems, motion errors are unlikely to be compensated correctly. Thus MOCO methods based on raw data are widely used. Conventional MOCO methods [10–12] or autofocus methods [13, 14] only deal with the translational motion errors of the carrier aircraft. This is reasonable in single-channel SAR imaging because the sensor is normally placed at the centroid of the plane. The rotational motion errors only alter the beam direction and could be compensated through beam control. Nevertheless, for the MIMO SAR system, the rotational motion errors could no longer be ignored because they affect the relative positions of the sensors. Conventionally the motion errors are estimated and compensated separately for each channel. We refer to
2. Signal Mode and Imaging

Geometry of MIMO SAR

The MIMO SAR utilizes an across-track array to gain the third dimension resolution. The transmitter and receiver antennas are usually distributed nonuniformly along the linear across-track array. Assume a MIMO SAR platform flies at the altitude of $H$ along the $X$-axis with velocity $v$. The MIMO array with $M$ transmit elements and $N$ receive elements is linearly laid out along the $Y$-axis. Signal sorting would be operated after the collecting of data, and $MN$ valuable signals can be saved for the imaging process during each pulse. The ideal geometry of MIMO SAR 3D imaging is indicated in Figure 1.

Ignoring two-way antenna characteristics and propagation attenuation, after demodulation and range compression, the echo signal transmitted by the $m$th transmitter $T_m$ and reflected by a generic point target $P(x_0, y_0, z_0)$ and received by the $n$th receiver $R_n$ is given by

$$e(\tau, t) = \rho \cdot s\left(\tau - \frac{2\tau(t)}{c}\right) \exp\left(-j4\pi\frac{r(t)}{\lambda}\right), \quad (1)$$

where $s(\tau)$ is the transmitted signal, $\tau$ is the fast time, $t$ is the slow time, $\rho$ is the reflectivity of $P$, $c$ is the time speed, and $\lambda$ is the timespeed, and $\lambda$.

As described in [16, 17], the image is formed in three steps, namely, the compression in the range direction, the focusing in the azimuth direction using the SAR principle, and the beam forming operation to focus the data in the across-track direction.

3. Imaging Geometry of MIMO SAR with Motion Errors

In practice, due to the presence of atmospheric turbulence that produces sensor track deviations from an ideal straight track, motion errors need to be accurately compensated. There have been numerous good methods dealing with the translational motion errors but ignoring the rotational motion errors [10-12]. These methods are adequate for single-channel SAR, because the sensor is usually placed at the centroid of the carrier aircraft and the rotational motion errors can be compensated through beam control. However, for MIMO SAR system, the rotational motion errors would change the baseline position and decrease the quality of the final image eventually.

Before the introduction of our method, we first discuss the core element that causes the defocusing of SAR images, that is, the range migration error.

As shown in Figure 2, a blue dashed arrow identifies the offset from the ideal position of the carrier plane to its true position. Then the translational motion error is $\Delta r = [\Delta x_1, \Delta y_1, \Delta z_1]$. The subscript $t$ means time-variant. In Figure 3, the rotational motion error of the carrier aircraft is characterized by three angles $[\theta_x, \theta_y, \theta_z]$ with $\theta_x$ denoting the pitch angle, $\theta_y$ denoting the roll angle, and $\theta_z$ denoting the yaw angle. Among them, $\theta_z$ does not affect the array position. The rotation matrixes of $\theta_x$ and $\theta_y$ can be written as $T_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{bmatrix}$ and $T_y = \begin{bmatrix} \cos \theta_y & -\sin \theta_y & 0 \\ \sin \theta_y & \cos \theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ respectively. Then, the real position of the APC can be denoted as

$$A = \begin{bmatrix} x_t, 0, H \end{bmatrix}^T + T_x T_y \begin{bmatrix} 0, y, 0 \end{bmatrix}^T + \Delta r^T = \begin{bmatrix} x_t - y \sin \phi_x + \Delta x, y \cos \phi_x \cos \phi_z + \Delta y, \\ H + y \cos \phi_x \sin \phi_x + \Delta z \end{bmatrix}. \quad (3)$$

As shown in Figure 1, the ideal geometry of MIMO SAR 3D imaging.
The slant range from \( A \) to the point target \( P \) can now be written as

\[
\begin{align*}
    r_{\text{real}}(t) & = \left( (x_t - y \sin \phi_t + \Delta x_t - x_0)^2 
    + (y \cos \phi_t \cos \psi_t + \Delta y_t - y_0)^2 
    + (H + y \cos \phi_t \sin \phi_t + \Delta z_t - z_0)^2 \right)^{1/2} \\
    & = \sqrt{r(t)^2 + r_e^2},
\end{align*}
\]

where

\[
\begin{align*}
    r_e & = \left( 2 \left( x_t + \Delta x_t - x_0 \right) (-y \sin \phi_t) + \Delta x_t^2 \right) \\
    & + 2 \Delta x_t \left( x_t - x_0 \right) + 2 \left( -\Delta y_t y_0 - \Delta z_t z_0 \right) \\
    & + 2 y \left( y_0 \left( 1 - \cos \phi_t \cos \psi_t \right) - z_0 \cos \phi_t \sin \phi_t \right. \\
    & \left. + \Delta y_t \cos \phi_t \cos \psi_t + \Delta z_t \cos \phi_t \sin \phi_t \right) \right)^{1/2}
\]

is the range error caused by the translational and rotational motion errors and needs to be compensated. In the next section, the JMC-MOCO method is discussed in detail to estimate and compensate the range error.

4. Joint Multichannel MOCO Method for MIMO SAR 3D Imaging

To compensate the motion errors, the conventional method is used to estimate and compensate the motion errors for each channel. However, the inevitable estimating errors of each channel may cause the incoherence among channels which would cause defocusing and false targets. Combining the linear properties of the across-track array, in this section we present a new method which can compensate both the translational motion errors and the rotational motion errors and preserve the coherence among different channels at the same time.

To avoid the incoherence after motion error compensation, the motion error estimating and compensating need to be jointed among channels. From (4) and (5) we can see that, due to the rotational motion errors, the range error differs along the linear across-track array. To clearly uncover the relationship between the range error and the array, we expand (4) according to Taylor expansion as follows:

\[
\begin{align*}
    r_{\text{real}}(t) & = r_B + \frac{(x_t - x_0)^2}{2r_0} + \alpha_t + \beta_t \cdot y + \frac{y^2}{2r_0},
\end{align*}
\]

where \( r_0 = \sqrt{y_0^2 + (H - z_0)^2} \) is the ideal zero-Doppler range from the center APC to the point target, \( (x_t - x_0)^2/2r_0 \) is the ideal range migration, and

\[
\begin{align*}
    \alpha_t & = \frac{\Delta x_t (x_t - x_0)}{r_0} - \Delta y_t y_0 + \Delta z_t \frac{H - z_0}{r_0} \\
    & + \frac{\Delta x_t^2 + \Delta y_t^2 + \Delta z_t^2}{2r_0},
\end{align*}
\]
\[ J(\hat{A},\hat{B}) = \sum_{k=1}^{K} (\hat{\gamma}_k - \gamma_0 - \hat{A} - \hat{B}y_k)^2, \]  \\
\text{(13)}

where \( \hat{\gamma}_k \) denotes the Doppler rate of the \( k \)-th channel. In order to estimate the two unknown parameters \( A \) and \( B \) in (9), a cost function is designed as follows:

\[ J(A, B) = \sum_{k=1}^{K} (\hat{\gamma}_k - \gamma_0 - A - By_k)^2, \]

\text{(13)}
where \( y_k \) is the position of the \( k \)th channel. The aim is to find \((A, B)\) that minimizes \( J \), that is, the stagnation point. Let the partial derivatives of \( J \) be equal to 0; we have

\[
\frac{\partial J}{\partial A} = -2 \sum_{k=1}^{K} \tilde{y}_k + 2KY_0 + 2KA + 2B \sum_{k=1}^{K} y_k = 0,
\]

\[
\frac{\partial J}{\partial B} = -2 \sum_{k=1}^{K} \tilde{y}_k y_k + 2Y_0 \sum_{k=1}^{K} y_k
\]

\[+ 2A \sum_{k=1}^{K} y_k + 2B \sum_{k=1}^{K} y_k^2 = 0.\]

By solving (14), \( A \) and \( B \) can be written as

\[A = \frac{\sum_{k=1}^{K} \tilde{y}_k \times \sum_{k=1}^{K} y_k^2 - \sum_{k=1}^{K} (\tilde{y}_k y_k) \times \sum_{k=1}^{K} y_k}{N \sum_{k=1}^{K} y_k^2 - (\sum_{k=1}^{K} y_k)^2} - Y_0,\]

\[B = \frac{K \sum_{k=1}^{K} (\tilde{y}_k y_k) - \sum_{k=1}^{K} y_k^2 - \sum_{k=1}^{K} y_k y_k}{K \sum_{k=1}^{K} y_k^2 - (\sum_{k=1}^{K} y_k)^2}.\]

This method combines all the estimating results of each channel to obtain the linear coefficient and then derives each Doppler rate. Comparing to the conventional SC-MOCO method, this method has three advantages: (1) the estimating accuracy is dramatically improved; (2) the influence from occasional bad estimating values can be suppressed; and (3) the multichannels maintain coherence after motion estimation. This will be demonstrated in Section 5 through simulations.

### 5. Simulations

#### 5.1. Motion Error Estimation Comparison between the Proposed Method and the Conventional Method

In order to prove that the proposed method is of higher precision than the conventional method, a MIMO SAR system with a linear MIMO array is simulated. The key simulation parameters are listed in Table 1. The linear MIMO array contains 32 APCs. Nine points are set in a line along the track direction with spacing of 2 m. Common translational motion errors and rotational motion errors shown in Figure 4 are imported.
into the simulation. As shown in Figure 4, the yaw angle and the roll angle vary in a range of ±10°. The translational motion errors in the across-track direction and the elevation direction are within ±4 m. All the motion errors mentioned above vary frequently. The translational motion error in the track direction is, nonetheless, quite different due to the enormous inertia of the plane. It is hardly time-varying but accumulative. Hence, the added translational motion error in the track direction is around 20 m and changes slowly.

After range compression and migration correction, the echo data are processed separately by the proposed JMC-MOCO method and the conventional SC-MOCO method. For the center APC, the Doppler rate estimation errors (defined as Estimation error = \((\text{Estimated value} - \text{Real value})/\text{Real value}\) \times 100\%) of these two methods are shown in Figure 6. As can be seen from Figure 5, the Doppler rate estimation errors of the proposed method are small and less undulate, which means that the proposed method has significant improvement in estimation accuracy. Figure 6 shows the estimation results and the real value of the Doppler rate coefficients \(A\) and \(B\). Figures 5 and 6 prove that the proposed method is of high accuracy.

### 5.2. MIMO SAR 3D Imaging Simulation

Now we apply our method to simulated MIMO SAR 3D imaging and compare the imaging results with the conventional method. In the simulation, a complex 3D model containing 5 tall “buildings”

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**Table 1: Simulation parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>0.0313 m</td>
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<tr>
<td>Bandwidth</td>
<td>150 MHz</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>2 (\mu)s</td>
</tr>
<tr>
<td>PRF</td>
<td>180 Hz</td>
</tr>
<tr>
<td>Antenna width</td>
<td>2 m</td>
</tr>
<tr>
<td>Flying height</td>
<td>6 km</td>
</tr>
<tr>
<td>Flying speed</td>
<td>150 m/s</td>
</tr>
<tr>
<td>Incidence angle</td>
<td>60°</td>
</tr>
<tr>
<td>Baseline length</td>
<td>8 m</td>
</tr>
</tbody>
</table>

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**Figure 8:** The imaging results with (a) the SC-MOCO method and (b) the proposed JMC-MOCO method.

**Figure 9:** The magnified imaging results with (a) the SC-MOCO method and (b) the proposed JMC-MOCO method.
is constructed with 1020 points. The structure of the model is illustrated in Figure 7. The simulated linear MIMO array contains 512 equivalent APCs with spacing of \( \lambda / 2 \), where \( \lambda \) is the wavelength. The motion errors in Figure 4 are imported into the simulation. The parameters in Table I are adopted.

After MOCO with the conventional method and the proposed JMC-MOCO method, the 3D imaging results are shown in Figures 8(a) and 8(b), respectively. Only the points whose energy is stronger than −10 dB of the strongest points are pointed in Figure 8 in different colors according to their energy. As we can see, the 5 “buildings” in Figure 8(b) are finely focused, while, in Figure 8(a), there are some unexpected false points, as marked in circles. This is because the respective estimation and compensation of each channel break the incoherence. This problem is well fixed in the proposed JMC-MOCO method. The subscene marked in a box in Figure 8 is magnified in Figure 9 for better comparison. As can be seen, the 3D model is better focused with the proposed method. Hence, the proposed MOCO method can significantly improve the imaging quality.

6. Conclusions

This paper proposed a JMC-MOCO method to estimate and compensate both the translational motion errors and the rotational motion errors for the MIMO SAR 3D imaging. Instead of estimating and compensating motion errors channel by channel, the proposed MOCO method utilizes the linear properties of the linear MIMO array, combining all the channel data to improve the accuracy of motion error estimation. It is demonstrated through 3D scenario imaging simulation that this new method can significantly improve the image quality.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References


