It should be noted that the peak sidelobe level (PSLL) significantly influences the performance of the multibeam imaging sonar. Although a great amount of work has been done to suppress the PSLL of the array, one can verify that these methods do not provide optimal results when applied to the case of multiple patterns. In order to suppress the PSLL for multibeam imaging sonar array, a hybrid algorithm of binary particle swarm optimization (BPSO) and convex optimization is proposed in this paper. In this algorithm, the PSLL of multiple patterns is taken as the optimization objective. BPSO is considered as a global optimization algorithm to determine best common elements’ positions and convex optimization is considered as a local optimization algorithm to optimize elements’ weights, which guarantees the complete match of the two factors. At last, simulations are carried out to illustrate the effectiveness of the proposed algorithm in this paper. Results show that, for a sparse semicircular array with multiple patterns, the hybrid algorithm can obtain a lower PSLL compared with existing methods and it consumes less calculation time in comparison with other hybrid algorithms.

1. Introduction

The multibeam imaging sonar is becoming more and more popular for its high resolution in the applications of underwater detection [1]. The system may comprise an array of 100–200 channels each associated with a hydrophone, amplifier, filter, and analog-to-digital converter (ADC), which results in considerable hardware complexity and computational load [2]. Consequently, it is meaningful to design a sparse array by selectively removing some elements, while keeping the same spatial aperture of the array. Once the number of elements and the spatial aperture have been fixed, the beam pattern of the sparse array, as compared with that of an equally spaced array, exhibits about the same mainlobe width but is characterized by much higher sidelobe level [3]. Thus beam performance optimization under multiple-pattern conditions is a priority. Here the design of the optimization model involves the peak sidelobe level (PSLL) suppression of multiple patterns for a sparse array, with the constraint of fixed number of elements and spatial aperture. Since the height of PSLL depends on the element parameters (i.e., positions and weight coefficients), it is important to select both the best common elements’ positions for multiple patterns and the best elements’ weights for each single pattern of an unequally spaced array.

This is a high-dimensional nonlinear problem which has received more and more attention in the past thirty years [4]. A range of techniques have been proposed for sidelobe level suppression with single-pattern requirement, such as simulated annealing (SA) algorithm, genetic algorithm (GA), and particle swarm optimization (PSO) [5]. Some methods only determine optimal sidelobe-minimizing positions assuming the weights were constant [6, 7]. In [8], the authors attempted to simultaneously optimize the weights and the positions of a linear array using SA algorithm. Recently, hybrid evolutionary algorithms were inspired by combining two algorithms together, which could suppress sidelobe level more effectively [9–11]. The work in [12] optimized elements’ positions firstly and weight coefficients secondly, which resulted in a mismatch between the two factors. The work in
[13] studied pattern synthesis of unequally spaced array based on the hybrid algorithm of GA and convex optimization, but GA has a very slow convergence speed in high-dimensional problems. What is more, these methods can not be easily extended to the case of multiple patterns since best elements’ positions usually change with different patterns; thus the problem of minimizing the sidelobe level in the multibeam imaging sonar array has not been addressed yet.

PSO attracts more and more attention nowadays since it is easy to implement compared with other global optimization techniques. Binary particle swarm optimization (BPSO) [14] is used to solve discrete optimization problems in the binary search spaces. However, both PSO and BPSO may easily get trapped in local optima and show slow convergence rate when solving complex and high-dimensional multimodal objective functions. Motivated by the work in [13], a hybrid optimization algorithm of BPSO and convex optimization [15] is proposed in this paper, where convex optimization has strong local optimization capability. In our study, BPSO is used to determine optimal common elements’ positions for multiple patterns globally, while convex optimization is used to optimize elements’ weights for each pattern locally, which guarantees the complete match of the two factors and gets better optimization performance by reducing the dimension of particles in BPSO. Simulations demonstrate that, with the constraint of fixed number of elements, the proposed algorithm can effectively suppress the PSLL of sparse sonar array for multiple-pattern case while consuming less calculation time compared with other hybrid algorithms.

2. Model of Optimization

Let us consider the semicircular array of multibeam imaging sonar with $N$ elements, which are uniformly distributed shown in Figure 1. Assuming that the incident signal impinges upon the array with the arriving angle $\theta$ from the far field, the steering vector of the array is represented by

$$
G(\theta) = \begin{bmatrix}
\exp\left(-\frac{j2\pi R \cos(\theta - \varphi_1)}{\lambda}\right) \\
\exp\left(-\frac{j2\pi R \cos(\theta - \varphi_2)}{\lambda}\right) \\
\vdots \\
\exp\left(-\frac{j2\pi R \cos(\theta - \varphi_N)}{\lambda}\right)
\end{bmatrix},
$$

(1)

where $\varphi_k = (k - 1)\pi/(N - 1)$ for $k = 1, 2, \ldots, N$. In addition, $R$ is the radius of the array and $\lambda$ is the wavelength of the incident signal. Therefore, the response of the array is then given by

$$
P^{(l)}(\theta) = \sum_{k=1}^{N} w_k^{(l)} \exp\left(-\frac{j2\pi R \cos(\theta - \varphi_k)}{\lambda}\right),
$$

(2)

where $l = 1, 2, \ldots, L$, $L$ denotes the total number of the desired patterns and $w_k^{(l)}$ denotes the weight coefficient of the $k$th element for the $l$th pattern. We denote $\mathbf{X}$ as the weight coefficient vector of the $l$th pattern

$$
\mathbf{X}^{(l)} = [w_1^{(l)}, w_2^{(l)}, \ldots, w_N^{(l)}],
$$

and then the equation in (2) can be rewritten as

$$
P^{(l)}(\theta) = \mathbf{W}^{(l)} \mathbf{G}(\theta).
$$

(3)

The problem considered here is to minimize the sidelobe level with multiple desired patterns in the sparse sonar array. The patterns include pencil beams with different mainlobe directions. For different patterns, the elements have optimized common positions but with probably different weight coefficients. The elements’ positions can be expressed as the binary vector $\mathbf{X}_1 = (x_1, x_2, \ldots, x_N)^T$. In order to keep the array aperture fixed, we must guarantee $x_1 = 1$ and $x_N = 1$. The remaining components in $\mathbf{X}_1$ are either “1” (preserved) or “0” (removed). The real and the imaginary part of the weights can be expressed as $\mathbf{X}_2 = (x_{N+1}, x_{N+2}, \ldots, x_{2N})^T$, $\mathbf{X}_3 = (x_{2N+1}, x_{2N+2}, \ldots, x_{3N})^T$, respectively, where each component ranges from $-1$ to $1$. Thus the joint optimization variable is $\mathbf{X} = [\mathbf{X}_1^T, \mathbf{X}_2^T, \mathbf{X}_3^T]^T$ and the problem can be formulated as

$$\min_{\mathbf{X}} \text{PSLL}$$

subject to

$$\begin{align}
p^{(l)}(\theta_{d_i}) &= 1, \quad l = 1, 2, \ldots, L \\
x_1 &= 1, x_N = 1 \\
x_2, x_3, \ldots, x_{N-1} &= 0 \text{ or } 1 \\
-1 &\leq x_{N+1}, x_{N+2}, \ldots, x_{3N} \leq 1
\end{align}
$$

(4)

Here, “min” represents the abbreviation of “minimize” and “s.t.” represents the abbreviation of “subject to.” The first line of (4) involves the objective function needed to be minimized and the remaining four lines involve the constraints. In addition, $\theta_{d_i}$ denotes the mainlobe direction for the $l$th pattern and PSLL denotes the peak sidelobe level for multiple patterns which is expressed as

$$\text{PSLL} = \max_{l=1,2,\ldots,L} \left\{ \max_{\theta_{d_i} \in I_l} \left\{ \left\| \frac{p^{(l)}(\theta_{d_i})}{p^{(l)}(\theta_{d_i})} \right\| \right\} \right\},
$$

(5)

where $\theta_{d_i}, i = 1, 2, \ldots, I_l$ are the sampling angles of the sidelobe region $\Omega_i$ for the $l$th pattern.
3. Proposed Algorithm

3.1. Binary Particle Swarm Optimization. Particle swarm optimization (PSO) is a population-based stochastic optimization algorithm proposed by Eberhart and Kennedy in 1995. It is inspired by social behavior of birds, which can effectively solve complicated optimization problems in the real search space.

A binary version of PSO, that is, binary particle swarm optimization (BPSO), was proposed to solve discrete optimization problems. What makes BPSO different from PSO optimization (PSO) is a population-based stochastic optimization algorithm proposed by Eberhart and Kennedy in 1995. It is inspired by social behavior of birds, which can effectively solve complicated optimization problems in the real search space.

Besides, PSO is often constrained within \([0,1]\), and \(S(x)\) denotes the Sigmoid function given by

\[
S(x) = \frac{1}{1 + e^{-x}}.
\]

Considering that the velocity of the particles is often constrained within \([-v_{\text{max}}, v_{\text{max}}]\), \(S(v_{\text{id}})\) is especially described as

\[
S\left(v_{\text{id}}\right) = \begin{cases} 
1 & v_{\text{id}} = -v_{\text{max}} \rightarrow -\infty \\
0.5 & v_{\text{id}} = 0 \\
1 & v_{\text{id}} = v_{\text{max}} \rightarrow \infty \\
\frac{1}{1 + e^{-v_{\text{id}}}} & \text{else}
\end{cases}
\]

Thus \(S(v_{\text{id}})\) varies between \([1/(1 + e^{v_{\text{max}}}), 1/(1 + e^{-v_{\text{max}}}]\), which is a subset of \([0,1]\).

In this paper, BPSO is adopted to determine the best elements’ positions of the sparse sonar array with elements placed at optional positions. The first position and last position are always occupied and the rest of the positions are uncertain. The fitness function has only one term that aims at minimizing the PSLL for multiple patterns according to (5).

3.2. Convex Optimization. Convex optimization refers to the minimization of a convex objective function subject to convex constraints. Since the joint optimization variable is very high-dimensional, the improved BPSO mixed with the convex optimization is proposed to get better optimization performance by reducing the dimension of particles. For each particle, the weights for each pattern can be optimized by convex optimization when computing fitness value, on the premise that the elements’ positions \(x_1, x_2, \ldots, x_N\) are selected by BPSO. For different patterns, the elements have optimized common positions but with probably different weight coefficients. Thus the optimization subproblem in the fitness function can be formulated as

\[
\text{for } l = 1 : L
\]

\[
\min \max_{\theta_{d,l} \in \Omega} \left| W^{(l)} G(\theta_{d,l}) \right|
\]

\[
\text{s.t. } W^{(l)} G(\theta_{d,l}) = 1
\]

\[
y = \max_{\theta_{d,l} \in \Omega} \left\{ W^{(l)} G(\theta_{d,l}) \right\}
\]

end

\[
\text{PSLL} = \max_{l = 1 \ldots L} (y),
\]

where \(\theta_{d,l} = 1, 2, \ldots, L\), are the sampling angles of the side-lobe region for the \(l\)th pattern and \(\theta_{d,l}\) denotes the mainlobe direction for the \(l\)th pattern.

As is known to all, formulation (9) is a convex problem that can be solved by SeDuMi or CVX of the Matlab toolbox effectively, which is helpful to save the calculation time of the hybrid algorithm. When the optimization subproblem is solved, the best weight coefficients for the \(l\)th pattern are \(W^{(l)} = [w_{1}^{(l)}, w_{2}^{(l)}, \ldots, w_{N}^{(l)}], l = 1, 2, \ldots, L\). Then the peak side-lobe level for multiple patterns is taken as the return value of the fitness function for each particle according to (5).

3.3. Procedure of the Hybrid Algorithm. In this paper, the hybrid algorithm of BPSO and convex optimization is proposed to optimize the beam performance, especially minimizing the side-lobe level of the sparse sonar array. BPSO is applied as global optimization algorithm to optimize the common elements’ positions and then convex optimization is applied as local optimization algorithm to optimize the weights when computing fitness value. All these guarantee that the particles can search the whole space with little possibility of falling into the local optimal solution. It is worth noting that the element number of the sparse array is fixed during the update of the particles’ positions; that is to say, the number of components that are “1” for each particle position should be unchangeable. If it varies, a few components will be randomly set to “1” or “0.” The procedure of the proposed algorithm is shown in Figure 2. Finally, the hybrid algorithm will converge to a set of common elements’ positions and multiple sets of weight coefficients with fixed number of elements.

4. Evaluations

For the multibeam imaging sonar, let us consider the semicircular array with 120 elements placed at 180 optional
positions, and the array is required to generate 538 beam patterns. The patterns include pencil beams whose mainlobe directions are uniformly distributed in the steering region, that is, $[45^\circ, 135^\circ]$. The radius of the array is 0.12 m and the wavelength of the incident signal is 0.0033 m.

In the BPSO, a population pool including 100 particles is initialized and the maximum number of runs is set equal to 100. The parameter values are chosen as $\omega = 1$, $c_1 = c_2 = 2$ for the search process and the maximum velocity is set as $v_{\text{max}} = 6.0$. Figure 3 shows the beam pattern that is optimized by the proposed algorithm in this paper. It can be seen that the PSLL of multiple patterns can be suppressed to $-22.1643$ dB. Figure 4 shows the elements’ distribution of the sparse array obtained by the proposed algorithm. The behavior of weight coefficients is significantly difficult to depict here because the algorithm obtains multiple sets of weights for multiple patterns.

For comparison, we apply the methods in [6, 12, 13] to multiple-pattern conditions and give the obtained results in Table 1. In the GA, the population size and the maximum number of runs are both set equal to 100. The crossover probability is set as 0.6 and the mutation probability is set as 0.04. The beam patterns are, respectively, visualized in Figures 5, 6, and 7. The average convergence curves of the proposed method and the other three methods discussed above are depicted in Figure 8. It demonstrates that the proposed

---

### Table 1: PSLL and calculation time of different methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>PSLL (dB)</th>
<th>Calculation time (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSO (only optimize positions)</td>
<td>$-7.4894$</td>
<td>—</td>
</tr>
<tr>
<td>BPSO with PSO (optimize positions firstly and weights secondly)</td>
<td>$-16.0519$</td>
<td>73.8</td>
</tr>
<tr>
<td>Hybrid algorithm of genetic algorithm and convex optimization</td>
<td>$-19.5392$</td>
<td>16.5</td>
</tr>
<tr>
<td>Proposed algorithm in the paper</td>
<td>$-22.1643$</td>
<td>7.5</td>
</tr>
</tbody>
</table>

---

**Figure 2**: Procedure of the proposed algorithm.

**Figure 3**: Beam pattern optimized by the proposed algorithm.

**Figure 4**: Elements’ distribution of the proposed algorithm.
algorithm has excellent global optimization performance and fast convergence speed.

In conclusion, the simulations demonstrate the effectiveness of the proposed optimization algorithm based on BPSO and convex optimization. Different from prior work in [6–8, 12, 13], the algorithm proposed in this paper considers both elements’ positions and weights during the PSLL suppression and keeps the complete match of the two factors. It is more important that the algorithm is applicable to the case of multiple patterns and can obtain a lower PSLL while consuming less calculation time compared with other methods based on the idea in prior literature.

5. Conclusion

In this paper, a hybrid optimization algorithm of BPSO and convex optimization is proposed to suppress the peak sidelobe level of sparse array for the multibeam imaging sonar. In this algorithm, BPSO is applied to determine optimal common elements’ positions for multiple patterns and convex optimization is to optimize the elements’ weights for each single pattern. The results show that, for the sparse semicircular array with 120 elements placed at 180 optional positions, the PSLL can be suppressed to $-22.1643$ dB by the proposed algorithm, which is lower than those reported in literatures, and the calculation time is significantly decreased. Thus the work here presents excellent optimization performance for multibeam imaging sonar.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.
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