Research Article

Design of Ring-Focus Elliptical Beam Reflector Antenna

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A new method for the design of elliptical beam reflector antenna is presented in this paper. By means of the basic principles of ring-focus antenna, a circularly symmetric feed and two specially shaped reflectors are used to form an elliptical beam antenna. Firstly, the design process of this ring-focus elliptical beam antenna is studied in detail. Transition function is defined and used in the design process. Then, combining the needs of practical engineering, a ring-focus elliptical beam reflector antenna is manufactured and tested. The gain at center frequency (12GHz) is 37.7dBi with an aperture efficiency of 74.6%. 3dB beam-width in \( \phi = 0^\circ \) and \( \phi = 90^\circ \) plane is 2.6\(^\circ\) and 1.4\(^\circ\), respectively. Ratio of the elliptical beam (ratio of 3dB beam-width in \( \phi = 0^\circ \) and \( \phi = 90^\circ \) plane) is 2.6/1.4 = 1.85, substantially equal to designed ratio 2. Simulating and testing results match well, which testify the effectiveness of this design method.

1. Introduction

In some special applications of radio engineering, the space for antenna carrier, instead of being circular, has some strict constraints. For example, on most occasions of vehicular and ship-borne satellite communication, the aperture of antenna is required to be rectangular or elliptical. In this situation, conventional rotationally symmetric reflector antenna is no longer suitable, while elliptical aperture antenna is a good choice. On the other hand, the antenna beam is also required to be elliptical on some occasions. For example, in many satellite scenarios the desired coverage on the ground is elliptical, so elliptical beam antenna is needed [1]; what is more, in vehicle satellite communication, if the pattern of vehicle antenna is elliptical, when the antenna has a narrow beam in the azimuth plane and a broad beam in the vertical plane, it can track the azimuth plane and no longer track the vertical plane, so the tracking system is largely simplified [2]. All in all, elliptical beam reflector antenna is being more and more widely used in radio engineering and worthy of a further study.

Basically, there are three methods for the design of elliptical beam reflector antenna. First is simply to cut the parabolic antenna into ellipse or rectangular. By using this method, the energy leakage is very large, so the antenna efficiency is low. Second is to use elliptical feed in order to get a high efficiency [3]. This design method brings the disadvantage of poor polarization performance and is hard to be used on the occasions of requiring a polarization rotation; what is more, it is forbidden to be used on the occasions of circular polarization. Third is to use offset shaped dual-reflector antenna in order to form an elliptical beam [4–6]. The projection of main reflector is an ellipse, while the projection of subreflector is circular (in the plane vertical to the main beam direction). The advantages of this design method are as follows: the radiation pattern of the feed is circularly symmetric, so it is easy to design and manufacture and achieve good circular polarization characteristic; the antenna aperture has an equivalent taper level, so it can achieve elliptical beam with a high antenna efficiency; additionally, due to the use of offset dual-reflector structure, the high side lobe level in linear polarization and beam squint in circular polarization can be reduced by properly deploying the main and subreflector. However, offset dual-reflector structure also increases the longitudinal and lateral dimensions of the antenna, not conducive to vehicular and ship-borne satellite communication.

In this paper, we propose a new method for the design of elliptical beam reflector antenna. By means of the basic
principles of ring-focus antenna, we use a circularly symmetric feed and two specially shaped reflectors to form an elliptical beam antenna. Firstly, shaped subreflector is designed in order to transform the circular beam of feed into elliptical beam; then, axial symmetric shaped main reflector is designed in order to make sure that the antenna satisfies the law of reflection and aplanatic condition. Compared to the offset dual-reflector structure, this design method can reduce the antenna's longitudinal height effectively, making it especially suitable for vehicular and ship-borne satellite communication.

2. Design Process of Ring-Focus Elliptical Beam Antenna

A specially shaped subreflector is used in order to transform the circularly symmetric beam of the feed into an elliptical beam; then the main reflector is designed to make sure that the optical path is equal. The detailed design process of this ring-focus elliptical beam reflector antenna is as follows: firstly, determine the long and short axis plane of the main and subreflector; secondly, carefully design the transition function in order to determine the entire shaped subreflector; thirdly, according to the law of reflection and aplanatic condition, determine the shaped main reflector; lastly, verify the design, as it is important to check whether the four constraint conditions are satisfied.

2.1. Long and Short Axis of the Reflector. The coordinate of ring-focus elliptical beam reflector antenna is shown in Figure 1, where the reference plane of the coordinate system and $r$, $\theta$, $\varphi$ constituting orthogonal spherical coordinates. $o$ is the origin point of the coordinate system.

In these two planes, the normal is still in the plane and parameters can be designed by the principles of ring-focus antenna, but for arbitrary $\varphi = \varphi_0$ plane, the normal is out of the plane and parameters should be designed using transition function. The parameters in $\varphi = \varphi_0$ and $\varphi = 90^\circ$ plane are shown in Figure 2. Point $o$ is the phase center of the feed and also one focus of the subreflector ellipse; $f_1$ and $f_2$ are the other focus of the ellipse in $\varphi = \varphi_0$ and $\varphi = 90^\circ$ plane, respectively. At the same time, they are also the focus of main reflector. The main parameters are as follows: main reflector diameter $D_m$, subreflector diameter $D_s$, focus of diameter ratio $\eta$, and distance between vertex of the subreflector and phase center of the feed $oP = a_0$. Parameters like focal length of the main paraboloid $F$, opening angle of main reflector $\psi$, opening angle of subreflector $\theta_m$, focus length of the subreflector ellipse $2c$, long axis of the subreflector ellipse $2a$, and eccentricity of the subreflector ellipse $e$ can be calculated by basic principles of ring-focus antenna. In this letter, subscript 2 represents parameters in $\varphi = 90^\circ$ plane, subscript 1 represents parameters in $\varphi = \varphi_0$ plane, and subscript without 1 or 2 represents common parameters of the two planes.

Firstly, determine the geometric parameters of $\varphi = 90^\circ$ plane, that is, the long axis plane. The parameters can be calculated as follows [7]:

$$F_2 = \eta (D_{m2} - D_{s2}),$$
$$\psi_2 = 2 \tan^{-1} \left( \frac{D_{m2} - D_{s2}}{4F} \right) = 2 \tan^{-1} \left( \frac{1}{4\eta} \right),$$
$$\beta_2 = \tan^{-1} \left( \frac{D_{s2}}{2a_0 - D_{s2} \cot \psi_2} \right),$$
$$2c_2 = a_0 f_2 = \frac{D_{s2}}{2 \sin \beta_2},$$
$$\theta_m = \sin^{-1} \left[ \frac{D_{s2} - (2a_0 + (D_{s2}/2) \tan (\psi_2/2))}{D_{s2}^2/4 + (2a_0 + (D_{s2}/2) \tan (\psi_2/2))^2} \right],$$
$$e_2 = \frac{D_{s2}/2 \sin \beta_2}{a_0 + D_{s2}/2 \sin \psi_0}.$$

Then, the geometric parameters of $\varphi = 0^\circ$ plane, that is, the short axis plane, can be determined according to the aplanatic condition (all optical paths are equal). The optical
path, $C_k$, in $\varphi = 90^\circ$ plane can be written as (according to the reference plane $xoy$)

$$C_k = \frac{D_{m2}/2}{\sin \psi_2} + a_0 - \left(a_0 - \frac{D_{m2} \cos \psi_2}{2 \sin \psi_2}\right)$$

(2)

In $\varphi = 0^\circ$ plane,

$$C_k = \frac{D_{m1}}{2 \tan (\psi_1/2)},$$

(3)

where $\tau$ is the ratio of elliptical beam (in this letter, $\tau$ is equal to 2). So parameters in $\varphi = 0^\circ$ plane can be calculated as follows:

$$\psi_1 = 2 \tan^{-1} \left[\tau \frac{\psi_2}{2}\right],$$

$$\beta_1 = \tan^{-1} \left[\frac{2 \tan (\psi_1/2) \tan (\theta_m/2)}{\tan (\psi_1/2) - \tan (\theta_m/2)}\right],$$

$$2c_1 = a_0 \frac{\sin \psi_1}{\sin (\psi_1 + \beta_1)},$$

$$e_1 = \frac{\sin \psi_1}{\sin (\beta_1 + \psi_1) + \sin \beta_1},$$

$$\frac{D_{a1}}{2} = a_0 \frac{\sin \psi_1 \sin \beta_1}{\sin (\beta_1 + \psi_1)}.$$  

2.2. Design of Subreflector. Once the long and short axis plane of the reflector are carefully designed, the next problem is how to determine the parameters in arbitrary $\varphi$ plane. The basic idea is to define a transition function; that is, a parameter changes with $\varphi$ from $\varphi = 0^\circ$ plane to $\varphi = 90^\circ$ plane. Parameters $\beta$, $r$, and $s$ can be chosen as transition function. In this letter, distance between the edge of the subreflector and $z$-axis in $\varphi$ plane $s(\varphi)$ is chosen, as shown in Figure 3. Transition function should have the following two characteristics.

(1) Satisfy the boundary conditions; that is to say, in $\varphi = 0^\circ$ and $\varphi = 90^\circ$ plane, $s(\varphi)$ should be equal to $s_1$ and $s_2$, respectively,

$$s(0) = s_1 = \frac{D_{s1}}{2},$$

$$s\left(\frac{\pi}{2}\right) = s_2 = \frac{D_{s2}}{2}.$$  

(5)

(2) $s(\varphi)$ should change slowly in $\varphi = 0^\circ$ and $\varphi = 90^\circ$ plane, in order to make sure that the normal of the subreflector in these two planes is still in the corresponding plane; that is, $(ds(\varphi)/d\varphi)|_{\varphi=0,90^\circ} = 0$.

All transition functions should satisfy the above two conditions. A common choice is polynomial function, such as $s(\varphi) = A + B\varphi + C\varphi^2 + D\varphi^3$. According to the above two conditions, the following can be derived:

$$s(0) = A = s_1,$$

$$s\left(\frac{\pi}{2}\right) = A + B\cdot \frac{\pi}{2} + C\cdot \left(\frac{\pi}{2}\right)^2 + D\cdot \left(\frac{\pi}{2}\right)^3 = s_2,$$

$$\frac{ds(\varphi)}{d\varphi} \bigg|_{\varphi=0} = B = 0,$$

$$\frac{ds(\varphi)}{d\varphi} \bigg|_{\varphi=\pi/2} = B + 2C\cdot \frac{\pi}{2} + 3D\cdot \left(\frac{\pi}{2}\right)^2 = 0.$$  

(6)

Then $s(\varphi)$ is designed. According to the author’s design experience, a better choice of transition function is trigonometric function, such as

$$s(\varphi) = \frac{D_{s1}}{2} + \frac{1}{2} \left(D_{s2} - D_{s1}\right) \sin^2 \varphi.$$  

(7)

This transition function satisfies the above two conditions and is used in this paper.

Other parameters can be calculated using the geometrical relationship shown in Figure 3,

$$r(\theta_m) = \frac{s(\varphi)}{\sin \theta_m},$$

$$a_0 - \frac{s(\varphi)}{\tan \psi(\varphi)} = 2c(\varphi) \cos \beta(\varphi),$$

(8)

$$a_0 + \frac{s(\varphi)}{\sin \psi(\varphi)} = r(\theta_m) + \left(r(\theta_m) \cos \theta_m - 2c(\varphi) \cos \beta(\varphi)\right).$$
Then,
\[
\psi (\phi) = 2 \tan^{-1}\left( \frac{\cot \frac{\theta_m}{2} - \frac{2a_0}{s(\phi)}}{2} \right),
\]
\[
\beta (\phi) = \tan^{-1}\left( \frac{s(\phi) \tan \psi (\phi)}{a_0} - s(\phi) \right),
\]
\[
2c (\phi) = \frac{s(\phi)}{\sin \beta (\phi)},
\]
\[
e (\phi) = \frac{2c (\phi) \sin \psi (\phi)}{a_0 \sin \psi (\phi) + s(\phi)},
\]
\[
r (\theta) = a_0 \frac{1 - e (\phi) \cos \beta (\phi)}{1 + e (\phi) \cos \beta (\phi) - \theta},
\]
\[
x_s = r (\theta) \sin \theta \cos \phi,
\]
\[
y_s = r (\theta) \sin \theta \sin \phi,
\]
\[
z_s = r (\theta) \cos \theta,
\]
where \((x_s, y_s, z_s)\) is the coordinate of arbitrary point on subreflector.

2.3. Design of Main Reflector. The coordinate of main reflector can be calculated by the law of reflection and aplanatic condition. Suppose the point on main reflector corresponding to \((x_s, y_s, z_s)\) on subreflector is \((x_{m_s}, y_{m_s}, z_{m_s})\), the distance between \((x_s, y_s, z_s)\) and \((x_{m_s}, y_{m_s}, z_{m_s})\) is \(d_m\), and the unit vector of \((x_s, y_s, z_s)\) and \((x_{m_s}, y_{m_s}, z_{m_s})\) is \(\vec{m}; \vec{m}\) points to main reflector,
\[
\vec{m} = \vec{e}_{r(\theta)} - 2 [\vec{e}_{n_s} \cdot \vec{e}_{r(\theta)}] \vec{e}_{n_s},
\]
where \(\vec{e}_{n_s}\) is the unit vector of normal on subreflector. \(\vec{e}_{r(\theta)}\) is the unit vector of \(\vec{r}(\theta)\) and \(r(\theta)\) is the norm of vector \(\vec{r}(\theta)\). Corresponding point on the main reflector can be calculated as
\[
\vec{m} = m_x \vec{a}_x + m_y \vec{a}_y + m_z \vec{a}_z,
\]
\[
x_m = x_s + d_m m_x,
\]
\[
y_m = y_s + d_m m_y,
\]
\[
z_m = z_s + d_m m_z.
\]
In it,
\[
m_x = \frac{1}{r (\theta)} \left[ x_s - 2 \left( n_s x_s + n_s y_s + n_s z_s \right) n_s x_s \right],
\]
\[
m_y = \frac{1}{r (\theta)} \left[ y_s - 2 \left( n_s x_s + n_s y_s + n_s z_s \right) n_s y_s \right],
\]
\[
m_z = \frac{1}{r (\theta)} \left[ z_s - 2 \left( n_s x_s + n_s y_s + n_s z_s \right) n_s z_s \right],
\]
\[
d_m = \frac{C_h - r (\theta) + z_s}{1 - m_z},
\]
where \(C_h\) is constant of equivalent optical path.

Suppose the derivatives of vector \(\vec{r}(\theta)\) with respect to \(\theta\) and \(\phi\) are as follows:
\[
\frac{\partial \vec{r}(\theta)}{\partial \theta} = \frac{\partial x_s}{\partial \theta} \vec{a}_x + \frac{\partial y_s}{\partial \theta} \vec{a}_y + \frac{\partial z_s}{\partial \theta} \vec{a}_z,
\]
\[
\frac{\partial \vec{r}(\phi)}{\partial \phi} = \frac{\partial x_s}{\partial \phi} \vec{a}_x + \frac{\partial y_s}{\partial \phi} \vec{a}_y + \frac{\partial z_s}{\partial \phi} \vec{a}_z.
\]
Then,
\[
\vec{e}_{n_s} = \frac{\vec{r}_\theta \times \vec{r}_\phi}{|\vec{r}_\theta \times \vec{r}_\phi|},
\]
2.4. Verify the Design. The main and subreflector can be designed by the formulas given above. At last, it is important to verify the design, that is, whether the following four constraint conditions are satisfied.

1. The opening angle of subreflector edge point to phase center in arbitrary \(\phi\) plane is equal to \(\theta_m\), in order to make sure that the radiation level of subreflector is equal.

2. The vertex of subreflector is the public point of all subreflector curves; otherwise, the subreflector has no solution.

3. The curves of main and subreflector in \(\phi = 0^\circ\) and \(\phi = 90^\circ\) plane are conventional ring-focus structure; the ratio of main reflector dimension is equal to the ratio of elliptical beam.

4. For subreflector point \((x_s, y_s, z_s)\) in \(\phi = 0^\circ \sim 90^\circ\) space, corresponding main reflector point \((x_{m_s}, y_{m_s}, z_{m_s})\) should also be designed in the space of \(\phi = 0^\circ \sim 90^\circ\); otherwise, the solution of main reflector is not unique.

The previous three constraint conditions are easy to satisfy, but the fourth constraint condition should be carefully treated. A good transition function can reduce the number of main reflector points out of \(\phi = 0^\circ \sim 90^\circ\) space.

3. Modeling of Ring-Focus Elliptical Beam Antenna

This ring-focus elliptical beam antenna is simulated by CST Microwave Studio. It should be noted that none of the existing full wave analysis tools (such as FEKO or HFSS or CST) can achieve the modeling of this antenna for the reason of complicated modeling. Both the main and subreflector are irregular shapes; that is, their Cartesian coordinates \((x, y, z)\) change continuously by \(\theta\) and \(\phi\), and it is impossible to express \(x, y, z\) by \(\theta\) and \(\phi\) in an analytical expression. A possible way to overcome this problem is to use NURBS modeling.

Nonuniform rational spline (NURBS) is a mathematical model commonly used in computer graphics for generating and representing curves and surfaces. NURBS curves and surfaces are generalizations of both B-splines and Bezier curves and surfaces, the primary difference being the weighting of the control points, which makes NURBS curves rational. By using a two-dimensional grid of control points, NURBS surfaces including planar patches and sections of spheres can be created.
The modeling procedures done in this paper are as follows: firstly, by using numerical simulation software (such as Matlab), calculate the discrete Cartesian coordinates of the main and subreflector (changed by $\theta$ and $\phi$); secondly, import the discrete Cartesian coordinates into professional 3D modeling software (such as 3D Studio Max) and use them as the control points of NURBS surface, in order to build the NURBS model of the main and subreflector; lastly, import the 3D model into full wave analysis tools (such as CST), in order to finish the simulation.

For the purpose of verifying the effectiveness of NURBS modeling, we simulate a ring-focus antenna, for which modeled by CST and NURBS, respectively. The parameters of this ring-focus antenna are as follows: main reflector diameter $D_m = 1000$ mm, subreflector diameter $D_s = 100$ mm, focus to diameter ratio $\eta = 0.4$, opening angle of subreflector $\theta_m = 55^\circ$, with a $-17$ dB taper level. Combining the given parameters and the conventional ring-focus design formulas, we can finish the ring-focus antenna design. The antennas modeled by CST and NURBS are called antennas A and B here for simplification. The designed model given by Matlab is shown in Figure 4. Using the discrete points calculated by Matlab as the control points, the NURBS surface including planar patches and sections of spheres can be created, which is shown in Figure 5. The gains of antennas A and B are 40.5 dBi and 40.4 dBi, with an aperture efficiency of 71.1% and 69.4%, respectively. The simulating radiation pattern in $\varphi = 90^\circ$ plane is shown in Figure 6. The simulating radiation pattern in $\varphi = 0^\circ$ and another arbitrary $\varphi$ plane is similar for the reason of the symmetrical characteristic of a ring-focus antenna, so it is not presented here for simplification. It can be seen that the simulating radiation pattern of antennas A and B coincide with each other, especially their copolarization, which declares that NURBS model method has little effect to the performance of a reflector antenna.

### 4. Testing Results

Combining the needs of practical engineering, we manufacture and test a ring-focus elliptical beam reflector antenna, as shown in Figure 7. This ring-focus elliptical beam reflector antenna has an aperture of $1 \times 0.5$ m and the center frequency is 12 GHz, in order to receive Ku-band satellite signal. The feed of this antenna is a RHCP corrugated conical horn antenna with a $-10$ dB beam-width of $80^\circ$, so the copolarization of this antenna is still RHCP after the reflection of two reflectors. The main parameters of this antenna in $\varphi = 90^\circ$ plane (the long axis plane) are inherited from the ring-focus antenna in Section 3. Combining the ratio of elliptical beam $\tau = 2$ and the formulas given above, we can finish the antenna design. The main parameters of this antenna are shown in Table 1.

<table>
<thead>
<tr>
<th>$D_m$ (mm)</th>
<th>$D_{12}$ (mm)</th>
<th>$D_{21}$ (mm)</th>
<th>$\eta$</th>
<th>$\tau$</th>
<th>$\beta_2$ (')</th>
<th>$\psi_2$ (')</th>
<th>$\beta_1$ (')</th>
<th>$\psi_1$ (')</th>
<th>$a_0$ (mm)</th>
<th>$C_e$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>100</td>
<td>500</td>
<td>80.6</td>
<td>0.4</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80.9</td>
<td>64</td>
<td>122.6</td>
<td>34.7</td>
<td>32.4</td>
<td>800</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Its simulating and testing radiation pattern in $\varphi = 0^\circ$, $\varphi = 45^\circ$, and $\varphi = 90^\circ$ plane is shown in Figures 8, 9, and 10, respectively. The gain at center frequency is 37.7 dBi with an aperture efficiency of 74.6%. The peak side lobe level
Figure 7: Photograph of ring-focus elliptical beam antenna.

Figure 8: Simulating and testing radiation pattern in $\varphi = 0^\circ$ plane.

Figure 9: Simulating and testing radiation pattern in $\varphi = 45^\circ$ plane.

Figure 10: Simulating and testing radiation pattern in $\varphi = 90^\circ$ plane.

Figure 11: Test VSWR of the feed and reflector.

(PSSL) in $\varphi = 90^\circ$ plane is $-10.3$ dB, relatively high when it compares with the $-14$ dB minimum requirement in satellite communication. This shortcoming may bring interference in signal reception and transmission, especially when the system has a relatively low G/T. Further works need to be done in order to decrease the PSLL. The cross-polarization level can be decreased by properly designing the axis ratio of the feed. The 3 dB beam-width in $\varphi = 0^\circ$, $\varphi = 45^\circ$, and $\varphi = 90^\circ$ plane is $2.6^\circ$, $1.7^\circ$, and $1.4^\circ$, respectively. Ratio of elliptical beam (ratio of 3 dB beam-width in $\varphi = 0^\circ$ and $\varphi = 90^\circ$ plane) is $2.6/1.4 = 1.85$, substantially equal to the designed ratio 2. The VSWR of the antenna is shown in Figure 11. The curve of feed test and reflector test is almost coincident, which declares that the reflector has little effect to the VSWR of the feed. Simulating and testing results match well, which testify the effectiveness of this design method.

The advantages of this design method are as follows: compared to the first design method, the aperture efficiency is improved effectively for the reason of equivalent taper level, which means less energy leakage; compared to the second design method, polarization rotation and circular polarization are improved with the use of axial symmetric feed and
reflector; compared to the third method, offset dual-reflector structure is replaced by symmetric dual-reflector structure; the antenna’s longitudinal height is reduced effectively, which makes it especially suitable for vehicular and ship-borne satellite communication.

5. Conclusion

A new method for the design of elliptical beam reflector antenna is presented in this paper. In order to testify the effectiveness of this design method, we manufacture and test a ring-focus elliptical beam antenna. By the use of axial symmetric structure; this antenna can reduce the antenna’s longitudinal height effectively and form an elliptical beam with a high efficiency. It can be a good candidate of many practical engineering.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References


