Research Article

Outage Analysis of Multihop Wireless Backhaul Using Millimeter Wave under Blockage Effects

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We consider multihop millimeter-wave (mm-Wave) wireless backhaul communications, by which small cell base station (SBS) clusters can connect to a macrocell base station (MBS). Assuming the mm-Wave wireless backhaul links suffer from outage caused by obstacles that block the line-of-sight (LoS) paths, we derive the statistics of a perhop distance based on the blockage model using stochastic geometry and random shape theory and analyze the multihop outage probability using the statistics of a perhop distance. We also provide an optimal number of hops to minimize the end-to-end outage performance between the MBS and the destination SBS cluster when the end-to-end distance is given.

1. Introduction

Dense deployment of small cells over traditional macrocell is considered as a key enabling technique for the emerging fifth-generation (5G) cellular networks [1–4]. For the small cell network deployment, millimeter-wave (mm-Wave) wireless backhaul is a cost-effective and scalable solution with large contiguous bandwidths. Moreover, the line-of-sight (LoS) nature of mm-Wave communication with directional antenna arrays can help to control the signal interference [5].

However, mm-Wave signals are more sensitive to blockage effects than signals in lower frequency bands, as indicated by the measurement data in [6–8]. Thus, the performances of the mm-Wave cellular systems are vastly affected by blockage effects [9]. There are two widely used approaches to incorporate the impact of blockages into signal propagation: ray tracing and stochastic modeling. In the ray tracing approach, blockages are characterized explicitly by their sizes, locations, and shapes. Therefore, this method is appropriate for environment-specific simulations based on electromagnetic simulation tools, which perform complex numerical calculations for ray tracing [10, 11]. On the other

hand, in the stochastic models, the statistics of blockages are characterized with smaller number of parameters compared to ray tracing. Therefore, the stochastic models are used to analyze general networks with acceptable accuracy.

In [12], the authors propose mm-Wave channel model incorporating the blockage effects based on stochastic geometry and random shape theory. Stochastic geometry has been a powerful technique to evaluate system performance in the conventional cellular networks [13], which reveals the impacts of multiple system parameters such as base station density, transmit power, and path-loss exponent. The key idea in [12] is to model random obstacles (e.g., buildings) as rectangles with random sizes and orientations whose centers form a Poisson point process (PPP) on 2-dimensional space. The model proposed in [12] can capture distance-dependent characteristics of the blockage effects, which is more realistic compared to the conventional shadowing model using log-normal distribution.

Using this mm-Wave propagation model in [10], the authors in [14] present a framework to derive signal-to-interference-plus-noise ratio (SINR) distributions, which can be used to analyze coverage and rate performances. Also, the

outage probability of a macrodiversity system with multiple base stations (BSs) that are connected by wire to each other is analyzed in [15], where an outage occurs when there is no LoS path from all the base stations to the user. The analysis in [15] presents that this macrodiversity coming from the unblocked BS selection can be exploited to mitigate blockage in mm-Wave cellular systems.

In [16], the outage performance of mm-Wave wireless backhaul links between a macrocell base station (MBS) and small cell base stations (SBSs) is also studied using the channel model in [10]. Specifically, in [16], the multiple SBSs in a cluster have wired connections to each other, and the wireless backhaul link between the MBS and the SBS cluster is assumed to be reliable as long as there exists one or more SBSs that have blockage-free LoS paths from the MBS. In other words, the MBS can selectively choose an unblocked SBS to construct a wireless backhaul link by beam steering, which is equivalent to the macrodiversity in [15].

However, while [16] only considers the one-hop communication between the MBS and a single SBS cluster, the 5G cellular networks may have multiple SBS clusters, which require multihop transmissions to improve the cell coverage [1, 2]. Considering the distance dependence of the blockage effects (i.e., the likelihood of a blockage event increases as the distance increases) at mm-Wave, multihop communication can be an effective solution to build mm-Wave wireless backhaul systems. In this context, motivated by the limitation in [16], we extend the single-hop wireless backhaul system in [16] to a multihop scenario with multiple SBS clusters. Moreover, it is noteworthy that the wireless backhaul links between two SBS clusters studied in this paper are distinct from the system model in [16] because we consider multiple-points-to-multiple-points (SBSs-to-SBSs) links, while [16] is focused on single-point-to-multiple-points (MBS-to-SBSs) links. Therefore, with different distance statistics from [16], the intercluster SBS-to-SBS communication can benefit from higher order of spatial diversity compared with the MBS-to-SBS communication in [16].

The contributions of this paper are fourfold. First, we derive the probability distribution, mean, and variance of a perhop distance (i.e., the distance between two randomly located SBSs in two adjacent SBS clusters), considering the spatial diversity. Second, we analyze the perhop outage performance of mm-Wave wireless backhaul links between two adjacent SBS clusters. Third, the outage analysis is extended from the perhop link to multihop systems. Lastly, we present an optimal and suboptimal hop count to minimize the end-to-end outage performance between the MBS and the destination SBS cluster for a given end-to-end distance, where the suboptimal hop count is derived based on only the perhop outage performance.

This paper is organized as follows. We introduce the system model in Section 2. We derive the probability distribution, mean, and variance of the distance between two SBS clusters in Section 3. The outage performances of perhop and multihop cases are analyzed in Section 4. In Section 5, we show optimal hop distance and hop count to minimize the outage performance and propose suboptimal hop distance and hop count that give close enough outage probabilities

FIGURE 1: Illustration of a 5G-network with mm-Wave wireless backhaul.

FIGURE 2: Multihop wireless backhaul.

to the optimal ones. Finally, conclusions are provided in Section 6.

2. System Model

We consider a 5G network with a single MBS and multiple SBS clusters as an example illustration in Figure 1. In the figure, the macrocell is indicated by the gray hexagon, and the SBS clusters are represented by the sky blue circles. Each SBS cluster has multiple SBSs, which are connected to each other through wires to enable spatial diversity in SBS cluster [15, 16]. The MBS is connected to the core network, and SBSs can access the core network via the MBS using wireless backhaul, which is indicated by the dotted blue lines in the figure. Moreover, some SBS clusters, which are far from the MBS, trigger multihop transmissions for the backhaul links to overcome path loss and blockage effects. For example, in Figure 1, SBS Cluster 2 communicates with the MBS in two hops over Cluster 1.

As shown In Figure 2 focusing on the multihop wireless backhaul links, we assume that SBSs in each cluster are distributed over a circle. In the figure, the areas of the multihop clusters are denoted by Areas $\mathcal{S}_1, \mathcal{S}_2, \dots$, and $\mathcal{S}_K$ with radii of $R_1, R_2, \dots, R_K$, respectively. Also, the hop distances are denoted by L_k , where $k \in \{1, 2, \dots, K\}$. As in [16], we assume that in k -th cluster multiple SBSs are uniformly distributed with intensity λ_k according to a homogeneous point process (PPP) [17], which is a widely used model for various types of

FIGURE 3: Intercluster mm-Wave wireless backhaul links.

cellular networks [18]. For the end-to-end wireless backhaul link with hops, the outage probability can be expressed as

$$P_{\text{out}} = 1 - \prod_{k=1}^K (1 - P_{\text{out};k}), \quad (1)$$

where $P_{\text{out};k}$ is the outage probability of k -th hop [19]. Assuming the outage events in different hops are mutually independent, the outage analysis over multiple hops can be decoupled into multiple intercluster links, while the first-hop outage $P_{\text{out};1}$ can be obtained based on the analysis in [16]. When we consider the multihop performance P_{out} , we assume uniform SBS clusters and equidistant hops for the mathematical tractability which we will explain in detail in Section 4.2. In general, however, the densities of the SBS clusters are dissimilar, which means variable $P_{\text{out};k}$ and hop distances for different k . Even in this case, the perhop analysis in this paper still can be used to optimize the end-to-end outage P_{out} , which is a function of the hop distances and the corresponding end-to-end hop count K .

To analyze the outage performances of the following hops, we consider the intercluster outage $P_{\text{out};k}$ for $k \geq 2$ as shown in Figure 3. Clusters $k-1$ and k have M and N SBSs, which are uniformly distributed over $\mathcal{S}_{k-1}$ and $\mathcal{S}_{k-2}$ with intensities λ_{k-1} and λ_k , respectively. These intensities correspond to the average numbers of SBSs per unit area (i.e., SBSs/m²). In other words, the numbers of the SBSs in the two clusters are assumed to be M and N , which are independent random variables following Poisson probability distributions as

$$\begin{aligned} \Pr[M = m] &= \frac{(\lambda_{k-1} \pi R_{k-1}^2)^m}{m!} e^{-\lambda_{k-1} \pi R_{k-1}^2}, \\ \Pr[N = n] &= \frac{(\lambda_k \pi R_k^2)^n}{n!} e^{-\lambda_k \pi R_k^2}, \end{aligned} \quad (2)$$

where $\Pr[X = x]$ denotes the probability that $X = x$ and x is a nonnegative integer. Therefore, the average numbers of the SBSs in the two clusters are given by $\mathbb{E}\{M\} = \lambda_{k-1} \pi R_{k-1}^2$ and $\mathbb{E}\{N\} = \lambda_k \pi R_k^2$, where $\mathbb{E}\{\cdot\}$ implies the expectation operator. We assume the centers of the two clusters are separated by L_k

with fixed ratios to the two radii as $L_k = a_k R_{k-1} = b_k R_k$, where $a_k > 2$ and $b_k > 2$ (i.e., there is no intersection between $\mathcal{S}_{k-1}$ and $\mathcal{S}_k$) (we assume the dense deployment of SBSs with wired connections in each cluster for coverage extension, especially to solve the *hotspots* problem, and the geographically disjoint SBS clusters. In addition, considering the construction cost, we assume that the SBSs in different clusters communicate with each other through the multihop wireless backhaul.).

Let $d_{m,n}$ be the distance between the mm-Wave SBSs m and n in $\mathcal{S}_{k-1}$ and $\mathcal{S}_k$, respectively, as shown in the figure. Also, as in [12, 14–16], the probability of no blockage in an individual SBS-to-SBS (S2S) path is $P_{\text{LoS}(m,n)} = e^{-\beta d_{m,n}}$, where β is the parameter that captures density and size of obstacles. The greater β means obstacles with higher density and larger sizes, which results in lower $P_{\text{LoS}(m,n)}$. In this paper, we assume $P_{\text{LoS}(m,n)}$ can be interpreted as the probability that the communication link between the SBSs m and n in the two clusters is reliable.

As in [16], we assume that the multiple SBSs in the same cluster are connected to each other through wires and thus the wireless backhaul link between the two clusters is reliable if one (or more) of the individual S2S links between the two clusters is reliable. An outage event of the intercluster mm-Wave wireless backhaul link is caused by blockage of LoS path. Before data transmission, with beam steering, each SBS in Cluster $k-1$ can selectively connect to one of the SBSs in Cluster k whose LoS path is blockage-free. Also, once one SBS finds an unobstructed LoS path, the SBS notifies the other SBSs in the same cluster via the wired intracluster connections.

3. Probability Distribution of S2S Distance $d_{m,n}$ between Two SBS Clusters

In order to investigate the performance of the mm-Wave wireless backhaul system, we need to consider probability distribution of the S2S distance $d_{m,n}$ between the two SBS clusters, since $P_{\text{LoS}(m,n)}$ is a function of $d_{m,n}$, which is a random variable. As shown in Figure 4(a), suppose the distance between the center of the left SBS cluster O_{k-1} and SBS n in the right SBS cluster is v_n . To derive the probability distribution of

FIGURE 4: An example illustration to derive the PDF of $d_{m,n}$.

$d_{m,n}$, we first find the probability distribution function (PDF) of v_n using an example illustration in Figure 4 (because v_n is the distance between a random location in Area $\mathcal{S}_k$ and the fixed point O_{k-1} , the PDF of v_n is identical to the PDF of the distance of the single-hop wireless backhaul in [16].), and then conditioned on v_n we will obtain the PDF of $d_{m,n}$. In Figure 4(b), the shaded area (Area $\mathcal{T}$) is the feasible region of v_n in the circle (Area $\mathcal{S}_k$). Consequently, the cumulative density function (CDF) of v_n is expressed as the ratio between Areas $\mathcal{T}$ and $\mathcal{S}_k$ as

$$F_{v_n}(x) = \Pr[v_n \leq x] = \frac{\psi R_k^2 + \xi x^2 - R_k L_k \sin \psi}{\pi R_k^2}, \quad (3)$$

Where $L_k - R_k \leq x \leq L_k + R_k$, $\psi = \arccos((R_k^2 + L_k^2 - x^2)/2R_k L_k)$, and $\xi = \arccos((x^2 + L_k^2 - R_k^2)/2x L_k)$. As a result, the PDF of v_n is

$$f_{v_n}(x) = \frac{x}{\pi R_k^2} \left(\frac{R_k/L_k}{\sqrt{1 - (L_k^2 - x^2 + R_k^2)/4L_k^2 R_k^2}} - \frac{(x/L_k - (L_k^2 + x^2 - R_k^2)/2L_k x)}{\sqrt{1 - (L_k^2 + x^2 - R_k^2)/4L_k^2 x^2}} - \frac{(L_k^2 - x^2 + R_k^2)/2L_k R_k}{\sqrt{1 - (L_k^2 - x^2 + R_k^2)^2/4L_k^2 R_k^2}} + 2 \arccos\left(\frac{L_k^2 + x^2 - L_k^2}{2L_k x}\right) \right), \quad (4)$$

where $L_k - R_k \leq x \leq L_k + R_k$. As shown in [16], when $L_k > 2R_k$, we can derive an approximate PDF by treating the area

enclosed by the path $\overline{ABCD}$ as the region bounded by the path $\overline{AB'CDA}$ in Figure 4(b):

$$f_{v_n}^*(x) = \frac{2\sqrt{R_k^2 - (L_k - x)^2}}{\pi R_k^2}, \quad (5)$$

where $L_k - R_k \leq x \leq L_k + R_k$. With this approximated PDF, we can obtain $\mathbb{E}\{v_n\} = L_k$ and $\mathbb{V}\{v_n\} = R_k^2/4$.

Then, going back to Figure 4(a), conditioned on v_n , the PDF of $d_{m,n}$ can be derived as the ratio of the area within a closed path $\overline{ABCD}$ to the entire circle $\mathcal{S}_{k-1}$, which is given by

$$F_{d_{m,n}|v_n}(x | v_n) = \frac{\theta_n R_{k-1}^2 + \phi_n x^2 - R_{k-1} v_n \sin \theta_n}{\pi R_{k-1}^2}, \quad (6)$$

where $v_n - R_{k-1} \leq x \leq v_n + R_{k-1}$, $\theta_n = \arccos((R_{k-1}^2 + L_k^2 - x^2)/2R_{k-1} v_n)$, and $\phi_n = \arccos((x^2 + v_n^2 - R_{k-1}^2)/2x v_n)$. Hence, the corresponding conditional PDF of $d_{m,n}$ is given by

$$f_{d_{m,n}|v_n}(x | v_n) = \frac{dF_{d_{m,n}|v_n}(x | v_n)}{dx} = \frac{x}{\pi R_{k-1}^2} \left(\frac{R_{k-1}/v_n - (v_n^2 - x^2 + R_{k-1}^2)/2v_n R_{k-1}}{\sqrt{1 - (v_n^2 - x^2 + R_{k-1}^2)^2/4v_n^2 R_{k-1}^2}} - \frac{x/v_n - (v_n^2 + x^2 - R_{k-1}^2)/2v_n x}{\sqrt{1 - (v_n^2 + x^2 - R_{k-1}^2)^2/4v_n^2 x^2}} + 2 \arccos\left(\frac{v_n^2 + x^2 - R_{k-1}^2}{2v_n x}\right) \right). \quad (7)$$

As the PDF of v_n , for a large enough L_k relative to R_k and R_{k-1} , this conditional PDF has an approximate form

$$f_{d_{m,n}|v_n}^*(x | v_n) = \frac{2\sqrt{R_{k-1}^2 - (v_n - x)^2}}{\pi R_{k-1}^2}, \quad (8)$$

where $v_n - R_{k-1} \leq x \leq v_n + R_{k-1}$. Hence, the PDF of $d_{m,n}$ is given by

$$\begin{aligned} f_{d_{m,n}}^*(x) &= \int_{L-R_k}^{L+R_k} f_{d_{m,n}|v_n}^*(x | v_n = y) f_{v_n}^*(y) dy \\ &= \int_{L-R_k}^{L+R_k} \frac{2\sqrt{R_{k-1}^2 - (y - x)^2}}{\pi R_{k-1}^2} \frac{2\sqrt{R_k^2 - (L - y)^2}}{\pi R_k^2} dy, \end{aligned} \quad (9)$$

where $L_k - R_{k-1} - R_k \leq x \leq L_k + R_{k-1} + R_k$. This approximate PDF $f_{d_{m,n}}^*(x)$ gives the mean and variance as follows:

$$\begin{aligned} \mathbb{E}\{d_{m,n}\} &= \mathbb{E}\{\mathbb{E}\{d_{m,n} | v_n\}\} = \mathbb{E}\{v_n\} = L_k, \\ \text{VAR}\{d_{m,n}\} &= \mathbb{E}\{\text{VAR}\{d_{m,n} | v_n\}\} \\ &\quad + \text{VAR}\{\mathbb{E}\{d_{m,n} | v_n\}\} \\ &= \frac{R_{k-1}^2}{4} + \text{VAR}\{v_n\} = \frac{R_{k-1}^2 + R_k^2}{4}. \end{aligned} \quad (10)$$

These two statistics will be used to derive the perhop outage probability in the next section.

4. Outage Analysis

4.1. Intercluster Outage Analysis. In this section, we derive the probability that there is no reliable S2S LoS link between the two SBS clusters $\mathcal{S}_{k-1}$ and $\mathcal{S}_k$. We define an outage, if all the LoS S2S paths are blocked by obstacles. Let $U_{m,n}$ be a Bernoulli random variable as

$$U_{m,n} = \begin{cases} 1, & \text{w.p. } P_{\text{LoS}(m,n)} = e^{-\beta d_{m,n}}, \\ 0, & \text{w.p. } 1 - P_{\text{LoS}(m,n)} = 1 - e^{-\beta d_{m,n}}, \end{cases} \quad (11)$$

where if $U_{m,n} = 1$, it means that the mm-Wave link between SBS m in $\mathcal{S}_{k-1}$ and SBS n in $\mathcal{S}_k$ is reliable. Assuming that blockage is impenetrable, as in [12, 14–16], an outage event of the mm-Wave SBS occurs, if all $U_{m,n}$'s are zeros. In other words, if there is at least one LoS mm-Wave link with $U_{m,n} = 1$ for any pair m and n , the mm-Wave communication between the two SBS clusters is reliable. Therefore, the outage in k th hop can be defined as

$$\begin{aligned} P_{\text{out};k} &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Pr[U_{i,j}'\text{'s are all zeros} | M = m, N = n] \\ &\quad \cdot \Pr[M = m, N = n] + \Pr[M = 0] \\ &\quad + \Pr[N = 0], \end{aligned} \quad (12)$$

where $1 \leq i \leq m$ and $1 \leq j \leq n$.

Following [12, 14–16], for analytical tractability, we assume the outage events indicated by $U_{i,j}$'s in (12) are mutually *independent* for all i and j . Strictly speaking, the blockage events on different links are not always independent. Therefore, as in [15, 16], the outage probability in this paper may be a lower bound as a reference of system design and analysis. However, we note that the numerical results in [12] show that the error caused by the *independent link assumption* is minor and acceptable in accuracy. With the assumption of the independent blockage events, we have

$$\begin{aligned} \Pr[U_{i,j}'\text{'s are all zeros} | M = m, N = n] \\ = \prod_{j=1}^n \prod_{i=1}^m \mathbb{E}[1 - e^{-\beta d_{ij}}] = (E[1 - e^{-\beta d_{ij}}])^{mn}, \end{aligned} \quad (13)$$

where the distance d_{ij} between SBSs i and j from $\mathcal{S}_{k-1}$ and $\mathcal{S}_k$ is a random variable with the approximate PDF in (9). Letting $A = \mathbb{E}[1 - e^{-\beta d_{ij}}]$, it can be approximated by Taylor expansion as

$$\begin{aligned} A &\approx 1 - \int_{L-R_{k-1}-R_k}^{L+R_{k-1}+R_k} e^{-\beta x} f_{d_{ij}}^*(x) dx \\ &\approx 1 - \left(e^{-\beta \mathbb{E}\{d_{i,j}\}} + \frac{\beta^2}{2} e^{-\beta \mathbb{E}\{d_{i,j}\}} \text{VAR}\{d_{i,j}\} \right) \\ &\approx 1 - \left(1 + \frac{\beta^2 (R_{k-1}^2 + R_k^2)}{8} \right) e^{-\beta L_k}, \end{aligned} \quad (14)$$

where $L = a_k R_{k-1} = b_k R_k$. Thus, the outage probability in (12) is given by

$$P_{\text{out};k} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A^{mn} \Pr[N = n] \Pr[M = m]. \quad (15)$$

Thus, we can interpret A in (14) as the expected $P_{\text{out};k}$, when the number of SBSs in the two clusters is given ($N = n$ and $M = m$). Unfortunately, it is difficult to obtain a closed form expression of (15), but based on (14) and (15), we can observe some important properties of the intercluster outage probability $P_{\text{out};k}$ as follows.

Property 1. For given positive β , λ_{k-1} , λ_k , a_k , and b_k , $P_{\text{out};k} \rightarrow 1$ as $L_k \rightarrow 0$ (because $R_{k-1} = L_k/a_k \rightarrow 0$ and $R_k = L_k/b_k \rightarrow 0$) (as described in Section 2, we assume the two SBS clusters are disjoint by $L_k = a_k R_{k-1} = b_k R_k$ with $a_k > 2$ and $b_k > 2$). Thus, when $L_k \rightarrow 0$, the sizes of the two clusters also become zero, which triggers an outage since the clusters do not have any SBSs.). Moreover, as $L_k \rightarrow \infty$, $P_{\text{out};k} \rightarrow 1$. This property can be proved by $A \rightarrow 1$, as $L_k \rightarrow 0$ or ∞ .

Property 2. For fixed β , λ_{k-1} , λ_k , R_{k-1} , and R_k , as L_k increases (i.e., a_k and b_k increase), $P_{\text{out};k}$ increases, because A increases.

Property 3. When the other parameters are fixed, the greater β gives the higher $P_{\text{out};k}$.

Property 4. As λ_{k-1} and λ_k increase, $P_{\text{out};k}$ decreases because the expected numbers of the SBSs M and N increase.

FIGURE 5: Intercluster outage probability $P_{\text{out};k}$ versus L_k .

Figure 5 displays simulation results of the intercluster outage, where the horizontal axis indicates the hop distance L_k , while the vertical axis represents the intercluster outage probability $P_{\text{out};k}$. Assuming $R_k = R_{k-1}$, $\lambda_k = \lambda_{k-1}$, and $a_k = b_k$, the solid line is the baseline, while the other three graphs correspond to changes in β , λ_k , and b_k , respectively. We can observe all the four properties in Figure 5. To be specific, all the four graphs in the figure are convex functions of L , which corresponds to Property 1. Also, as described in Property 2, compared to the solid line (i.e., the baseline case), the dash-dot graph, which indicates the higher $a_k = b_k$, shows the higher outage rates. Property 3 can be observed by the comparison between the solid and the dotted curves. Lastly, Property 4 is found by the dashed line in the figure.

4.2. Multihop Outage Analysis with a Constant Hop Distance. In this section, we extend the outage analysis to the multihop wireless backhaul assuming the uniform SBS clusters (i.e., $L_k = L$, $R_k = R$, and $\lambda_k = \lambda$ for all k). With this assumption, the end-to-end outage probability in (1) between the MBS and the K -th SBS cluster is simplified as

$$P_{\text{out}} = 1 - (1 - P_{\text{out};1})(1 - P_{\text{out};k})^{K-1}, \quad (16)$$

where $P_{\text{out};1}$ is the outage probability for the first-hop link between the MBS and the first SBS cluster $\mathcal{S}_1$ presented in [16] and $P_{\text{out};k}$ is given in (15). Because the height of the MBS is typically much greater than that of SBS, based on [12], we need to use a smaller blockage parameter $\beta' = c\beta$ for the first hop, where $c < 1$. Thus, $P_{\text{out};1} < P_{\text{out};k}$. When the end-to-end distance between the MBS to the center of the K -th SBS cluster O_K is D , then the hop distance is $L = D/K$.

Figure 6 shows multihop simulation results with $c = 1/4$, where the x - and y -axes are the number of hops K and the

FIGURE 6: P_{out} versus K ($\beta = 0.02$, $\beta' = \beta/4$, $\lambda = 0.003$, and $b_k = 4$).

outage probability, respectively. The three different curves with the different markers (circle, $\times$ -marker, and triangle) represent the outage probability results for $D = 2, 3$, and 5 km, respectively. Also, the solid and dotted lines indicate the outage probabilities for the multihop (MH) and the perhop (PH) cases in (16) and (15), respectively. First, we can observe there exists an optimal hop count, denoted as K_{opt} , minimizes the outage probability for a given D . By the exhaustive search, we obtain $K_{\text{opt}} = 8, 12$, and 19 , for $D = 2, 3$, and 5 km, which means that the corresponding optimal hop distances $L_{\text{opt}} = D/K_{\text{opt}}$ are almost the same for the three values of D . Also, the outage curves for both the MH and PH have the same K_{opt} because of (i) almost negligible first-hop error (because $\beta' = \beta/4$) and (ii) the same convex curves of $P_{\text{out};k}$ for $k \geq 2$ as in Figure 6. Based on these observations, in the next section we propose the suboptimal hop count $\hat{K}_{\text{opt}}$ by mathematically deriving the suboptimal hop distance $\hat{L}_{\text{opt}}$, instead of the exhaustive search of K .

5. Suboptimal Hop Count $\hat{K}_{\text{opt}}$

As shown in Figure 5, there is an optimal hop distance L_k between two SBS clusters to minimize $P_{\text{out};k}$. Assuming a_k and b_k are fixed, we derive the suboptimal L_k by treating the intercluster S2S path as two-hop link between the two SBS clusters. For example, in Figure 3, suppose that we place an imaginary relay at O' on the straight line connecting O_{k-1} and O_k , assuming that the two clusters communicate with each other only through the relay. Suppose that the outage rates “from the left cluster to the relay” and “from the relay to the right cluster” are $P'_{\text{out};a}$ and $P'_{\text{out};b}$ with the distances l_a and l_b , respectively. Then, as finding the center of mass, the best location of the relay O' corresponds to $P'_{\text{out};a} = P'_{\text{out};b}$.

Using the outage probability between SBS clusters to the MBS in [16], $P'_{\text{out}:a}$ and $P'_{\text{out}:b}$ are expressed as

$$P'_{\text{out}:a} = \exp\left(-\lambda_{k-1}\pi R_{k-1}^2 \left(1 + \frac{\beta^2 R_{k-1}^2}{8}\right) e^{-\beta l_a}\right),$$

$$P'_{\text{out}:b} = \exp\left(-\lambda_k\pi R_k^2 \left(1 + \frac{\beta^2 R_k^2}{8}\right) e^{-\beta l_b}\right). \quad (17)$$

Therefore, when $P'_{\text{out}:a} = P'_{\text{out}:b}$, l_a and l_b are, respectively, obtained as

$$l_a = \frac{L_k}{2} + \frac{1}{2\beta} \ln \left[\frac{\lambda_{k-1} R_{k-1}^2 (1 + \beta R_{k-1}^2)}{\lambda_k R_k^2 (1 + \beta R_k^2)} \right], \quad (18)$$

$$l_b = \frac{L_k}{2} + \frac{1}{2\beta} \ln \left[\frac{\lambda_k R_k^2 (1 + \beta R_k^2)}{\lambda_{k-1} R_{k-1}^2 (1 + \beta R_{k-1}^2)} \right], \quad (19)$$

$$\hat{L}_{\text{opt}} = \frac{-6\sqrt[3]{4b_k^2} + 8\sqrt[3]{9b_k^2 + 3\sqrt{6b_k^4 - 39b_k^2 + 192b_k + 32}} + 2\sqrt[3]{2} \left(9b_k^2 + 3\sqrt{6b_k^4 - 39b_k^2 + 192b_k + 32} \right)^{2/3} + 16\sqrt[3]{4}}{3\beta\sqrt[3]{9b_k^2 + 3\sqrt{6b_k^4 - 39b_k^2 + 192b_k + 32}}}, \quad (20)$$

where $R_{k-1} = L_k/a_k$ and $R_k = L_k/b_k$. If plugging l_a and l_b back into (17), the overall outage rate of the two-hop link with the imaginary relay is $P'_{\text{out}:k} = 1 - (1 - P'_{\text{out}:a})(1 - P'_{\text{out}:b})$. The proposed suboptimal hop distance $\hat{L}_{\text{opt}}$ is L_k that minimizes $P'_{\text{out}:k}$. In other words, $\hat{L}_{\text{opt}}$ corresponds to the solution of $\partial P'_{\text{out}:k} / \partial L_k = 0$, while $\partial^2 P'_{\text{out}:k} / \partial L_k^2 > 0$. This suboptimal $\hat{L}_{\text{opt}}$ can be readily obtained by numerical calculation for given a_k , b_k , and β . If we assume the same SBS cluster sizes $R_k = R_{k-1}$ (i.e., $a_k = b_k$), we can obtain the suboptimal $\hat{L}_{\text{opt}}$ in (20). Therefore, if the hop distances are the same with the uniform PPP intensities (i.e., $\lambda_k = \lambda$ for all k), the suboptimal hop count to reach the distance D is $\hat{K}_{\text{opt}} = \|D/\hat{L}_{\text{opt}}\|$, where $\|x\|$ means the nearest integer to x .

Figures 7(a) and 7(b) show the simulation results of L_{opt} and K_{opt} , respectively. In Figure 7(a), the horizontal axis is the blockage factor β , while the vertical axis is the optimal and suboptimal hop distances L_{opt} and $\hat{L}_{\text{opt}}$. As shown in the figure, both L_{opt} and $\hat{L}_{\text{opt}}$, which are indicated by the solid and dotted lines, decrease, as β increases, because of the greater blockage effects. Also, comparing the two graphs, $\hat{L}_{\text{opt}} < L_{\text{opt}}$ with the small gap in the entire range of β , which validates the proposed suboptimal hop distance. In Figure 7(b), the x -axis represents the distance D from the MBS to the K -th SBS cluster (i.e., the destination SBS cluster), while the y -axis indicates the optimal and suboptimal hop counts K_{opt} and $\hat{K}_{\text{opt}}$, which are denoted by the solid and dotted curves. In the figure, we can observe that K_{opt} and $\hat{K}$ are almost linearly increasing, as D increases. Also, with the small difference, we observe that $\hat{K}_{\text{opt}} > K_{\text{opt}}$ because $\hat{L}_{\text{opt}} < L_{\text{opt}}$. Thus, based on $\hat{K}_{\text{opt}} = K_{\text{opt}} + \epsilon$ with a small ϵ , the suboptimal $\hat{K}_{\text{opt}}$ can be used to reduce the exhaustive search range of $\hat{K}_{\text{opt}}$.

Figure 8 shows the end-to-end outage probabilities versus β , where the solid and dotted lines correspond to the outage probabilities with K_{opt} and $\hat{K}_{\text{opt}}$, respectively. The markers

represent different λ and D . As shown in the figure, the gap between the outage probabilities with K_{opt} and $\hat{K}_{\text{opt}}$ becomes smaller as λ and D decrease. Especially, when $D = 1$ km, the outage performance with $\hat{K}_{\text{opt}}$ becomes extremely close to that with K_{opt} as β increases, which can be expected from the simulation results in Figure 7(b).

6. Conclusion

In this paper, we study the outage performance of the multihop wireless backhaul systems at mm-Wave in the presence of the blockage effects. Assuming that multiple SBSs are distributed in a cluster with the wired connection to each other and considering the spatial diversity in a cluster, the outage probability between two SBS clusters is derived. Through analysis and simulation, it is shown that there exists an optimal hop distance that minimizes the intercluster (perhop) outage probability. Then, assuming uniform SBS clusters we extend the perhop outage analysis to the multihop scenario, where we find that there exist optimal hop distance and hop count that minimize the end-to-end outage probability between the MBS and the destination SBS cluster when the end-to-end distance is given. Using the same trend of the perhop and multihop outage performances with respect to the perhop distance, we propose the suboptimal hop distance and corresponding suboptimal hop count. The suboptimal hop count shows a small gap to the optimal hop count, which means that the exhaustive search range of the optimal hop count can be significantly reduced. Potential extensions of this paper include addressing a wider scenario with dissimilar sizes and densities of the SBS clusters.

Competing Interests

The authors declare that they have no competing interests.

FIGURE 7: Optimal hop distance and hop count for $\lambda = 0.003$ and $b_k = 4$: (a) L_{opt} versus β and (b) K_{opt} versus D , when $\beta = 0.02$.

FIGURE 8: P_{out} versus β , when $b_k = 4$, $\lambda = \{0.0024, 0.003\}$, and $D = \{1 \text{ km}, 5 \text{ km}\}$.

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