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Research Article

DOA Estimation of Noncircular Signals Using Quaternions

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The two-dimensional (2D) direction-of-arrival (DOA) estimation problem for noncircular signals using quaternions is considered in this paper. In the framework of quaternions, we reconstruct the conjugate augmented output vector which reduces the dimension of covariance matrix. Compared with existing methods, the proposed one has two main advantages. Firstly, the estimation accuracy is higher since quaternions have stronger orthogonality. Secondly, the dimension of covariance matrix is reduced by half which decreases the computational complexity. Simulation results are presented verifying the efficacy of the algorithm.

1. Introduction

In the wake of developments in array signal processing, noncircular signals have been widely used in modern communication systems, such as amplitude modulation (AM) and binary phase shift keying (BPSK) signals. By exploiting the noncircular properties, both the second-order characteristics and the conjugate relation characteristics can be used. Thus, the resolution is improved [1].

In recent years, considerable alternatives have been proposed to take care of noncircular signals, for example, [2, 3]. In [4], the author proposed the extended 2q-MUSIC method for noncircular sources. Subsequently, Chen et al. presented a method to deal with the mixed noncircular and circular signals in [5]. However, most existing algorithms are based on the conjugate augmented output vector. Recently, a few methods for direction-of-arrival (DOA) estimation were presented based on the hypercomplex framework [6–8]. In [9, 10], Gou et al. used biquaternion-based algorithms to estimate the DOAs of noncircular signals. Instead of concatenating the data recorded by different sensors to a long vector, hypercomplex can link the data to different imaginary parts. And it has been proved that the hypercomplex vector orthogonality provides a more accurate estimation of the signal subspace than the long vector orthogonality constraint [6]. Nevertheless, compared with biquaternions, quaternion-based methods will be more attractive for decreasing the computational complexity.

In this paper, the problem of DOA estimation for noncircular signals within the quaternion framework is considered. We reconstruct the conjugate augmented output vector which can lead to the dimension reduction of covariance matrix. Thus, the computational load required by eigendecomposition is decreased. Additionally, since quaternions have stronger orthogonality compared with complex number, the proposed algorithm exhibits better performance in accuracy. It is also worthwhile to note that most existing algorithms using hypercomplex frames are based on vector sensor. Motivated by this fact, we want to break the platform limitations and extend it to a wider usage.

The rest of this paper is organized as follows. In Section 2, we briefly introduce some notations about quaternions, and on this basis, we use it to construct the data model for noncircular signals. Section 3 analyzes the computational complexity. Some numerical examples to illustrate the performances of the proposed algorithm are given in Section 4, followed by concluding remarks.

2. The Proposed Algorithm

2.1. Some Notations about Quaternions. A quaternion $q \in H$ is a four-dimensional (4D) hypercomplex number [11] and has a Cartesian form given by

$$q = a + ib + jc + kd, \tag{1}$$

where $a, b, c, d \in R$ are called its components. In view of its widespread usage in subsequent sections, it is worthwhile to review some notations before proceeding to the physical problems of interest. The three imaginary units i, j, k are square roots of -1 and are related through the famous relations:

$$\begin{aligned} ij &= -ji = k \\ ki &= -ik = j \\ jk &= -kj = i \\ i^2 &= j^2 = k^2 = ijk = -1. \end{aligned} \tag{2}$$

In what follows, we will list some of the properties of quaternions that will be used throughout this correspondence.

(a) The product of quaternions is associative: $(qp)r = q(pr)$.

(b) The product of two quaternions is not commutative: $qp \neq pq$.

(c) Just as with the complex numbers, the conjugate of a quaternion q is obtained by negating its imaginary part and is defined as $\bar{q} = a - ib - jc - kd$.

(d) The norm of a quaternion q is defined as $\|q\| = a^2 + b^2 + c^2 + d^2$; in addition, it also equals the product of a quaternion and its conjugate; that is, $\|q\| = q\bar{q}$.

(e) A quaternion $q \in H$ with $\|q\| = 1$ is said to be a unit quaternion.

(f) Quaternion vector orthogonality provides a more accurate estimation of the signal subspace than the long vector orthogonality constraint.

As stated above, since the product of two quaternions is not commutative, there are two kinds of quaternion eigenvalues, that is, right eigenvalue and left eigenvalue. In this paper, quaternion eigenvalue and eigenvector refer to right eigenvalue and right eigenvector whenever there is no possibility of confusion.

2.2. Modeling Noncircular Signals Using Quaternions. Without loss of generality, we consider an array with N ($N = 8$) identical antenna elements as shown in Figure 1. The elements are uniformly distributed around a circle with radius R in the xOy plane. The circular array is not the conventional uniform circular array (UCA) for the dipoles point towards different directions. It is essentially the polarization sensitive array. Moreover, we assume that each dipole in the array is a short dipole whose output voltage is proportional to the electric field along the dipole.

The first element is taken as the reference with respect to other elements. The general expression for the element location is $\mathbf{r}_n = (R\cos(w_n), R\sin(w_n), 0)^T$, where $w_n = 2\pi(n-1)/N$ and $(\cdot)^T$ represent the angle from the $\mathbf{x}$-axis and the transpose operator, respectively.

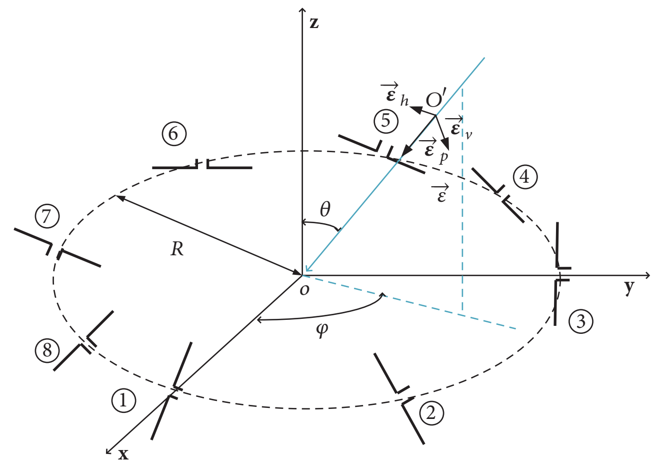

Figure 1: The geometry of the circular array.

Provided that $\mathbf{a}_m$ is the signal steering vector and there are M incident narrowband signals impinging on the array from distinct directions, then the array output can be expressed as

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{a}_m s_m(t) + \mathbf{n}(t), \tag{3}$$

where $s_m(t)$ is the signal and $\mathbf{n}(t)$ is assumed to be zero mean, complex Gaussian processes statistically independent of each other, with covariance $\sigma_n{}^2$. The mth signal has an elevation angle θ_m and an azimuth angle φ_m. And the signals are assumed to be in the far-field with respect to the sensor location.

As a side note, for circular signals, the data model in (3) is qualified and the traditional MUSIC algorithm [12] can handle them. The processing procedure involves constructing the covariance matrix of $\mathbf{x}(t)$ and carrying out the eigenvalue decomposition to separate the signal subspace from the noise subspace. By using the orthogonality of these two subspaces, the DOA estimation can be performed. However, for noncircular sources, the above model is not satisfied. The conventional solution is to create the conjugate augmented output vector by exploiting noncircular properties; that is,

$$\begin{aligned} \mathbf{y}(t) &= \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}*(t) \end{bmatrix} = \sum_{m=1}^{M} \begin{bmatrix} \mathbf{a}_m s_m(t) \\ \mathbf{a}_m^* s_m^*(t) \end{bmatrix} + \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{n}*(t) \end{bmatrix} \\ &= \sum_{m=1}^{M} \begin{bmatrix} \mathbf{a}_m & \\ & \mathbf{a}_m^* \end{bmatrix} \begin{bmatrix} s_m(t) \\ s_m^*(t) \end{bmatrix} + \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{n}*(t) \end{bmatrix} \\ &= \sum_{m=1}^{M} \begin{bmatrix} \mathbf{a}_m & \\ & \mathbf{a}_m^* \end{bmatrix} \begin{bmatrix} 1 & \\ & e^{-j\varpi_m} \end{bmatrix} \begin{bmatrix} s_m(t) \\ s_m(t) \end{bmatrix} + \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{n}*(t) \end{bmatrix} \\ &= \sum_{m=1}^{M} \underbrace{\begin{bmatrix} \mathbf{a}_m \\ \mathbf{a}_m^* e^{-j\varpi_m} \end{bmatrix}}_{\mathbf{a}_{cm}} s_m(t) + \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{n}*(t) \end{bmatrix}, \end{aligned} \tag{4}$$

where $(\cdot)^*$ and ϖ_m denote the conjugation and the noncircular phase, respectively. In [13], the author constructed the data covariance matrix through (4) and, on this basis, estimated the DOAs of noncircular signals. However, the dimension of the covariance matrix was increased to $2N \times 2N$. In view of this, we can reconstruct the conjugate augmented vector and transform it to a quaternion vector:

$$\begin{aligned}\mathbf{y}_Q(t) &= \mathbf{x}(t) + i\mathbf{x}*(t) \\ &= \sum_{m=1}^{M}\left(\mathbf{a}_m s_m(t) + i\mathbf{a}_m^* e^{-j\varpi_m} s_m(t)\right) + \mathbf{n}(t) + i\mathbf{n} \\ &\quad *(t).\end{aligned} \tag{5}$$

This reconstructed model is reasonable since eigenstructure methods are based on the decomposition of the vector space spanned by the observation vector $\mathbf{x}$ in orthogonal subspaces using energy criteria. Thus, the eigenvalue and its corresponding eigenvector derived from the quaternion covariance matrix are also satisfied the subspace condition.

Then, the covariance matrix can be represented as follows:

$$\mathbf{R}_{yy} = E\left\{\mathbf{y}_Q(t)\,\overline{\overline{\mathbf{y}}}_Q(t)\right\}, \tag{6}$$

where $\overline{\overline{(\cdot)}}$ represents the conjugate-transpose operator in quaternion field. And the dimension of (6) is $N\times N$. However, under this circumstance, using spectral estimation to find the peak is expensive for the covariance matrix involving four parameters, that is, θ, φ, γ, and η. The parameters γ and η, the auxiliary polarization angle and the polarization phase difference [14], are used to depict the polarization state. The multidimensional search costs a lot and the efficiency is not high. In fact, the polarization parameters can be separated from the covariance matrix. Towards this purpose, it is necessary to obtain the analytical expression of the steering vector.

We first introduce the array element spatial phase matrix of the mth signal as follows:

$$\Upsilon_m = \Upsilon_{\theta_m,\varphi_m} = \begin{bmatrix} u_{m,1} & & \\ & \ddots & \\ & & u_{m,N}\end{bmatrix}. \tag{7}$$

Υ_m is the $N \times N$ diagonal matrix and it describes the spatial coherent structure of the output [15].

The kth diagonal element denotes the space phase factor

$$u_{m,k} = e^{-j2\pi(\boldsymbol{\varepsilon}_p^T(\theta_m,\varphi_m)\mathbf{r}_k)/\lambda_m}, \tag{8}$$

where

$$\boldsymbol{\varepsilon}_p(\theta_m,\varphi_m) = -\left[\sin\theta_m\cos\varphi_m, \sin\theta_m\sin\varphi_m, \cos\theta_m\right]^T \tag{9}$$

with λ_m representing the wavelength of the mth signal.

We assume that g represents the matched gain, and the generalized polarization sensitive matrix of the array can be expressed as follows:

$$\Gamma = g\begin{bmatrix} \sin\beta_1\cos\alpha_1 & \sin\beta_1\sin\alpha_1 & \cos\beta_1 \\ \vdots & \vdots & \vdots \\ \sin\beta_n\cos\alpha_n & \sin\beta_n\sin\alpha_n & \cos\beta_n \\ \vdots & \vdots & \vdots \\ \sin\beta_N\cos\alpha_N & \sin\beta_N\sin\alpha_N & \cos\beta_N\end{bmatrix}. \tag{10}$$

The dimension of Γ is $N \times 3$. The pair of variables (α_n, β_n) represents the direction of the nth dipole in the array.

Then, the signal steering vector is obtained

$$\mathbf{a}_m = \mathbf{a}_{\theta_m,\varphi_m,\gamma_m,\eta_m} = \underbrace{\Upsilon_m\Gamma\Psi_m}_{\Lambda_m}\mathbf{h}_m, \tag{11}$$

where

$$\begin{aligned}\mathbf{h}_m &= \left[\cos\gamma_m \quad \sin\gamma_m e^{j\eta_m}\right]^T \\ \Psi_m &= \begin{bmatrix} -\sin\varphi_m & \cos\theta_m\cos\varphi_m \\ \cos\varphi_m & \cos\theta_m\sin\varphi_m \\ 0 & \sin\theta_m\end{bmatrix}.\end{aligned} \tag{12}$$

Up to this point, we have found that the polarization information only exists in $\mathbf{h}_m$. In this case, we rewrite $\mathbf{a}_{cm}$ in (4):

$$\begin{aligned}\mathbf{a}_{cm} &= \begin{bmatrix}\mathbf{a}_m \\ \mathbf{a}_m^* e^{-j\varpi_m}\end{bmatrix} = \begin{bmatrix}\Lambda_m\mathbf{h}_m \\ \Lambda_m^*\mathbf{h}_m^* e^{-j\varpi_m}\end{bmatrix} \\ &= \underbrace{\begin{bmatrix}\Lambda_m & \\ & \Lambda_m^*\end{bmatrix}}_{\Lambda_{cm}}\underbrace{\begin{bmatrix}\mathbf{h}_m \\ \mathbf{h}_m^* e^{-j\varpi_m}\end{bmatrix}}_{\mathbf{h}_{cm}}\end{aligned} \tag{13}$$

Then, Λ_{cm} has nothing to do with polarization and is successfully separated from the array manifold. The dimension of Λ_{cm} is $2N \times 4$. So, we only need to traverse the angle parameters to determine the DOAs which largely reduces the amount of computations. Please note that Λ_{cm} derived from (13) corresponds to the conjugate augmented vector. To apply it to the quaternion case, we transform it as follows:

$$\Lambda_{qm} = \Lambda_m + i\Lambda_m^*. \tag{14}$$

From (14), it appears that Λ_{qm} is a complex number. In fact, Λ_m has the exponential part which makes Λ_{qm} a quaternion. Thus, the spatial spectra function can be expressed as

$$P(\theta,\varphi) = \frac{1}{\overline{\overline{\Lambda}}_{qm}G_q\overline{\overline{G}}_q\Lambda_{qm}}, \tag{15}$$

where G_q is $N\times(N-M)$ quaternion matrix composed of the $(N-M)$ eigenvectors corresponding to the $(N-M)$ minimum eigenvalues of $\mathbf{R}_{yy}$. Then, the DOAs can be obtained by varying (θ,φ) within a given domain with a chosen step.

3. Complexity Analysis

This section will evaluate the computational complexity of the proposed algorithm. To better demonstrate the superiority in computation, the conventional conjugate augmented vector (CAV) method is included for comparison. We focus on the estimation of the covariance matrix and evaluate it in terms of memory requirements and arithmetical operations (i.e., floating additions and multiplications).

As stated above, the dimension of the CAV method is $2N \times 2N$ in complex field. And it requires $4N^2$ complex memory units, corresponding to $8N^2$ floating memory units. However, the dimension of the proposed method is $N \times N$ in quaternion field. Thus, it takes up N^2 quaternion memory units, which equals $4N^2$ floating memory units. In this case, the proposed algorithm reduces by half the memory units.

As for the second aspect, that is, the arithmetical operations, we know that the multiplication between two quaternions contains 16 floating multiplications and 12 floating additions. And for the complex case, it involves 4 floating multiplications and 3 floating additions. Thus, constructing the covariance matrix costs $(16 + 12) \times N^2$ operations for the proposed method and $(4 + 3) \times 4N^2$ operations for the CAV method. Next, the computation in eigenvalue decomposition will be considered. As is known to all, the complexity for the decomposition operation is $O(N^3)$. So, the values for CAV method and the proposed one are $O(8N^3)$ in complex field and $O(N^3)$ in quaternion field, respectively. The above values will become $O(32N^3)$ and $O(16N^3)$, respectively, in terms of floating point number.

Thus, it is obvious that the computational burden is equivalent for these two methods in constructing the covariance matrix. However, the proposed method outperforms the CAV one in eigenvalue decomposition. To sum up, the use of quaternions in algorithms reduces both the computational complexity and memory requirements.

4. Simulation Results

In this section, Monte-Carlo simulation experiments are implemented to verify the effectiveness of the proposed algorithm. The array structure is shown in Figure 1. The root mean squared error (RMSE) is utilized as the performance measure. And 100 independent simulation experiments are carried out. The RMSE is defined as

$$\text{RMSE} = \sqrt{\frac{1}{100}\sum_{i=1}^{100}\left[\left(\widehat{\theta}_i - \theta_i\right)^2 + \left(\widehat{\varphi}_i - \varphi_i\right)^2\right]}, \qquad (16)$$

where $\{\widehat{\theta}_i, \widehat{\varphi}_i\}$ are the estimates of elevation angles and azimuth angles, respectively, at the ith run. In addition, the CAV method and Gou's method [10] are included for comparison.

We assume that there are three BPSK signals that can be received. The corresponding incident angles are $(15^\circ, 20^\circ)$, $(35^\circ, 40^\circ)$, and $(60^\circ, 65^\circ)$, respectively. The corresponding polarization auxiliary angle and the polarization phase difference are $(20^\circ, 25^\circ)$, $(55^\circ, 45^\circ)$, and $(65^\circ, 65^\circ)$. In addition, the

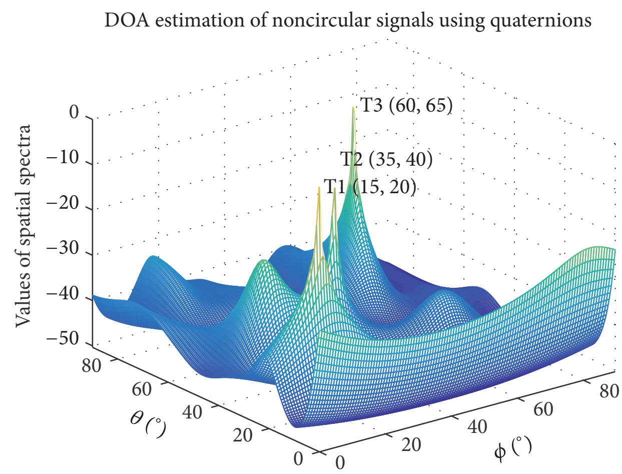

Figure 2: The spatial spectra of noncircular signals.

noncircular phases are $\pi/7$, $\pi/7$, and $2\pi/7$, respectively. The snapshot, K, is selected as 200 and the SNR is 10 dB. Figure 2 shows the simulation results of the proposed algorithm. The position of the spectra peak represents the corresponding DOA. Intuitively, the estimation accuracy of the proposed algorithm is high.

To better demonstrate the performance of the proposed method, we study the performance with a varying SNR from 0 dB to 30 dB. Without loss of generality, we select the aforementioned three sources as the targets to verify it. Figure 3 shows the RMSE versus SNR with the snapshots being 200. It can be seen that the proposed method outperforms the CAV method since quaternions have stronger orthogonality compared with complex number. And it have been proved that the stronger the orthogonality, the better the performance [6]. In addition, the proposed algorithm largely decreases the computational complexity, as stated in Section 3. It is worthwhile to note that the simulation curves in Figure 3 are not smooth since the statistical data have certain randomness. Compared with the biquaternion noncircular MUSIC (BNC) algorithm developed by Gou et al. [10], we can see that the RMSEs of the BNC method are close to those of the proposed one. The reason is that both quaternions and biquaternions impose the orthogonality constraint. And they only represent different mathematical languages. In [10], the author constructed the biquaternion data model from the standpoint of covariance. Nevertheless, we reconstructed the conjugate augmented output vector using quaternions in this correspondence. Therefore, in this case, both methods exhibit similar performance. However, from [10], we know that the complexity of eigenvalue decomposition using biquaternions is about $O(64N^3)$. And the corresponding value in this paper is about $O(16N^3)$. Then, using quaternions reduces the computational burden.

Figure 4 illustrates the RMSE versus the number of snapshots with the SNR fixed at 10 dB. Compared with Figure 3, we can draw similar conclusions. In particular, if we pick the points with snapshots being 400 and 800, respectively, we may find that the corresponding RMSEs

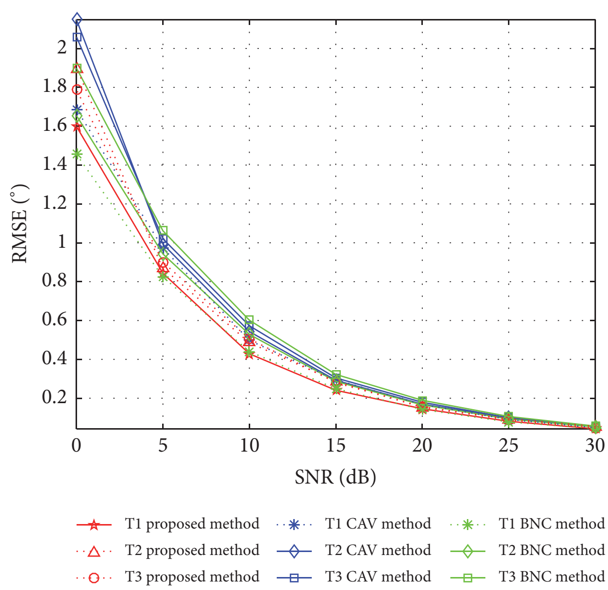

Figure 3: RMSE versus SNR with the snapshots being 200.

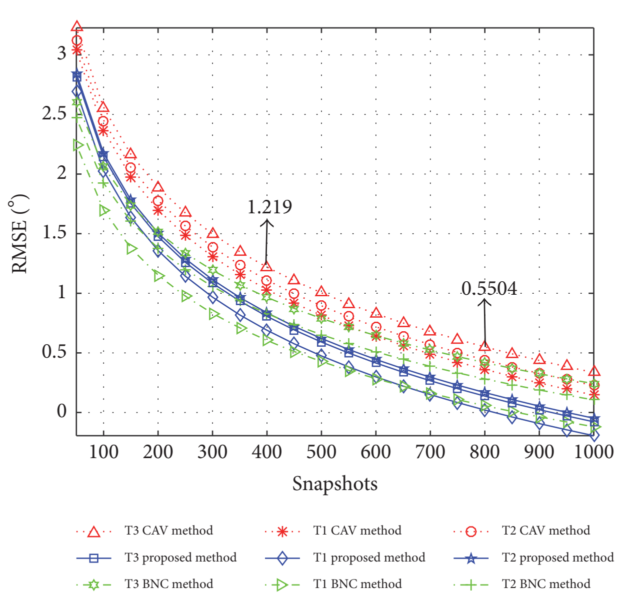

Figure 4: RMSE versus snapshots with the SNR fixed at 10 dB.

are 1.219 and 0.5504. This means that the former value is nearly twice as much as the latter one. In fact, these improvements can be predicted from the derivation of CRB. For the specific derivation process, one can refer to literature [16]. The number of snapshots can be extracted from the Fisher information matrix. Moreover, the CRB is found as the element of the inverse of that matrix. So, we can conclude that the RMSE is inversely proportional to K.

5. Conclusion

In this paper, we combine the quaternions with the conjugate augmented vector and present a DOA estimation algorithm for noncircular signals. Compared with existing methods, the proposed one has two main advantages. Firstly, it can give a more accurate estimation since quaternions have stronger orthogonality. Secondly, the dimension of the covariance matrix is reduced by half and, therefore, has a much lower computational complexity. In addition, the proposed method breaks the platform limitations of vector sensors and extend quaternions to a wider usage. The simulation results verify the effectiveness of the proposed method.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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