Research Article

Fast Estimation Method of Space-Time Two-Dimensional Positioning Parameters Based on Hadamard Product

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The estimation speed of positioning parameters determines the effectiveness of the positioning system. The time of arrival (TOA) and direction of arrival (DOA) parameters can be estimated by the space-time two-dimensional multiple signal classification (2D-MUSIC) algorithm for array antenna. However, this algorithm needs much time to complete the two-dimensional pseudo spectral peak search, which makes it difficult to apply in practice. Aiming at solving this problem, a fast estimation method of space-time two-dimensional positioning parameters based on Hadamard product is proposed in orthogonal frequency division multiplexing (OFDM) system, and the Cramer-Rao bound (CRB) is also presented. Firstly, according to the channel frequency domain response vector of each array, the channel frequency domain estimation vector is constructed using the Hadamard product form containing location information. Then, the autocorrelation matrix of the channel response vector for the extended array element in frequency domain and the noise subspace are calculated successively. Finally, by combining the closed-form solution and parameter pairing, the fast joint estimation for time delay and arrival direction is accomplished. The theoretical analysis and simulation results show that the proposed algorithm can significantly reduce the computational complexity and guarantee that the estimation accuracy is not only better than estimating signal parameters via rotational invariance techniques (ESPRIT) algorithm and 2D matrix pencil (MP) algorithm but also close to 2D-MUSIC algorithm. Moreover, the proposed algorithm also has certain adaptability to multipath environment and effectively improves the ability of fast acquisition of location parameters.

1. Introduction

Wireless target localization has found widespread application requirements in military and civilian fields. Parameter estimation is an important prerequisite for positioning algorithm, which determines the overall effectiveness of the positioning system. The location parameters include time of arrival (TOA), direction of arrival (DOA), time difference of arrival (TDOA), and received signal strength. The key technique of location finding based on TOA method is to estimate the propagation delay of the radio

signal arriving from the direct line-of-sight (DLOS) propagation path accurately. The existing TOA methods mainly include cross-correlation method [1], multiple signal classification (MUSIC) algorithm [2], delay estimation algorithm based on Markov Monte Carlo [3], propagator algorithm [4], and matrix pencil algorithm [5]. If the array antenna can be used to obtain the signal DOA for joint estimation [6, 7], then it will effectively reduce the number of nodes and system overhead of the localization system. In theory, a reference node can lock the location of the target source.

OFDM technology is a multicarrier modulation and high-speed transmission technology, which has been widely used in underwater acoustic communication system, IEEE 802.11a/g/n/ac [8], OFDM new system radar [9], WiMAX system [10], 3GPP LTE/LTE-Advanced [11], 5G mobile communication systems, and so on. However, the demand for target location information of OFDM technology is also increasing, especially in indoor, underground places, and other environments. Therefore, using OFDM array antenna receiving system to achieve accurate positioning of the target signal has become a hot spot of current research.

Due to the limitation of bandwidth, sampling rate, and the attributes of narrowband signals, OFDM signals cannot achieve joint estimation of TOA and DOA employing the triangle geometry relation as in [12, 13]. Ni et al. [14] introduces a TOA estimation algorithm for OFDM wireless signal, which combines coarse estimation with fine estimation. The algorithm needs to use the technology-guided detection to suppress multipath propagation, but the positioning accuracy is limited. Three methods including peak detection, modified maximum peak leak rate detection, and channel frequency domain response reconstruction are proposed in [15]. In the multipath environment of WINNER A1 LOS channel, the channel frequency response reconstruction method can provide the best performance, but this method is mainly based on the received preamble sequence. In the environment of indoor multipath, the two-dimensional matrix pencil (2D MP) algorithm is presented in [16] to achieve the joint estimation of time delay and arrival angle, which has low computational complexity, but sacrifices the effective array element and bandwidth. Zhou et al. [17] provide a joint estimation algorithm of delay and DOA, which uses the frequency domain data of single antenna for MUSIC time delay estimation. This algorithm uses the array data for MUSIC DOA estimation, but it fails to give a matching method of time delay and arrival angle, and its computational complexity is high. Ba et al. [18] present a joint estimation of delay and angle using 2-MUSIC for two-dimensional searching, which solves the pairing problem, but its computational complexity is very high, which makes it hard to apply in practice.

In this paper, a fast estimation method of space-time two-dimensional positioning parameters based on Hadamard product is presented, which can offer the closed-form solution of parameter estimation directly without pseudo spectral peak search. This method can implement parameter pairing and improves the efficiency significantly. The computational complexity analysis, CRB derivation, and simulation are also presented. The simulation results show that the proposed algorithm has higher precision of parameter estimation close to the 2D-MUSIC algorithm, but the computational complexity is greatly reduced, which effectively improves the ability of fast acquisition of the positioning parameters.

The rest of the paper is organized as follows: Section 2 introduces the data model. Section 3 presents the method of fast estimation and the computational complexity analysis in detail. The CRB for the method is derived in Section 4. Simulation and experimental results are presented in Section 5. Conclusions are drawn in Section 6.

2. Data Model

Considering a node equipped with an equidistant line array composed of M antenna elements, the distance between the signal source and the antenna array is far enough, and θ_i and τ_i represent DOA and propagation delay of the multipath signal, respectively. $s(t)$ is the OFDM symbol transmitted with N subcarriers. On the account of the wireless multipath propagation, the received signal at the m th antenna element can be formulated as

$$r_m(t) = \sum_{i=0}^{L_p-1} a_i s(t - \tau_{i,m}) + n_m(t), \quad (1)$$

where L_p is the number of the multipath components and $a_i = |a_i|e^{j\varphi_i}$ and $\tau_{i,m}$ are the complex attenuation of the i th path and propagation delay of the i th path in m th antenna, respectively. The phase of the complex attenuation φ_i is normally assumed random from one snapshot to another with a uniform probability density function $U(0, 2\pi)$ [19]. $n_m(t)$ is additive Gauss white noise. The time delay in each antenna element depends not only on the propagation delay but also on the direction of arrival. In particular, for a uniform linear array, the propagation delay associated to be the i th arriving path in m th antenna is given by $\tau_{i,m} = \tau_i + md \sin \theta_i / c$, $0 \leq m \leq M-1$, with d being the distance between adjacent array elements in the array and c the speed of light. After the Fourier transform of the received signal $r_m(t)$, the sampled discrete frequency domain channel response of the k th subcarrier at the m th antenna element is given by

$$\begin{aligned} x_{m,k} &= H_{m,k} + n_{m,k} \\ &= \sum_{i=0}^{L_p-1} \alpha_i e^{-j2\pi f_c \tau_i} e^{-j2\pi (k/T)(\tau_i + (md \sin \theta_i / c))} e^{-j2\pi f_c (md \sin \theta_i / c)} \\ &\quad + n_{m,k}, \end{aligned} \quad (2)$$

where $0 \leq k \leq N-1$. In (2), it is clear that the effect of time delay $md \sin \theta_i / c$ coming

from the antenna array is negligible compared with τ_i , so this formula can be further simplified as

$$\begin{aligned} x_{m,k} &= H_{m,k} + n_{m,k} \\ &= \sum_{i=0}^{L_p-1} \alpha_i e^{-j2\pi (f_c + (k/T))\tau_i} e^{-j2\pi (md \sin \theta_i / \lambda)} + n_{m,k}, \end{aligned} \quad (3)$$

where $n_{m,k}$ denotes additive Gauss white noise with mean zero and variance σ^2 at the k th subcarrier and the m th antenna element, f_c is the carrier frequency of OFDM signal, $T = NT_s$ is the net data length of OFDM symbol, and T_s is the sampling interval. We can then write this formula (3) in a vector form.

$$\mathbf{X}_m = \mathbf{H}_m + \mathbf{n}_m = \mathbf{D}_m(\tau, \theta)\alpha + \mathbf{n}_m, \quad (4)$$

where

$$\mathbf{D}_m(\tau, \theta) = \begin{bmatrix} d_{m,0}(\tau_0, \theta_0) & d_{m,0}(\tau_1, \theta_1) & \cdots & d_{m,0}(\tau_{L_p-1}, \theta_{L_p-1}) \\ d_{m,1}(\tau_0, \theta_0) & d_{m,1}(\tau_1, \theta_1) & \cdots & d_{m,1}(\tau_{L_p-1}, \theta_{L_p-1}) \\ \vdots & \vdots & \ddots & \vdots \\ d_{m,N-1}(\tau_0, \theta_0) & d_{m,N-1}(\tau_1, \theta_1) & \cdots & d_{m,N-1}(\tau_{L_p-1}, \theta_{L_p-1}) \end{bmatrix}_{N \times L_p}. \quad (5)$$

Moreover, $d_{m,k}(\tau_i, \theta_i) = e^{-j2\pi(f_c + (k/T))\tau_i} e^{-j2\pi(md \sin \theta_i/\lambda)}$, set $\mathbf{d}_m(\tau_i, \theta_i) = \mathbf{s}(\tau_i) \oplus \mathbf{a}_m(\theta_i)$, and $\oplus$ denotes Hadamard product. Then,

$$\begin{aligned} \mathbf{s}(\tau_i) &= [e^{-j2\pi(f_c + (0/T))\tau_i} \quad e^{-j2\pi(f_c + (1/T))\tau_i} \quad \cdots \quad e^{-j2\pi(f_c + (K-1/T))\tau_i}]^T, \\ \mathbf{a}_m(\theta_i) &= [e^{-j2\pi(md \sin \theta_i/\lambda)} \quad e^{-j2\pi(md \sin \theta_i/\lambda)} \quad \cdots \quad e^{-j2\pi(md \sin \theta_i/\lambda)}]^T. \end{aligned} \quad (6)$$

So, the frequency domain response vector of M extended array elements can be expressed as

$$\mathbf{X} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{M-1} \end{bmatrix} + \mathbf{n} = \begin{bmatrix} \mathbf{D}_0(\tau, \theta) \\ \mathbf{D}_1(\tau, \theta) \\ \vdots \\ \mathbf{D}_{M-1}(\tau, \theta) \end{bmatrix} \alpha + \mathbf{n} = \mathbf{D}(\tau, \theta) \alpha + \mathbf{n}, \quad (7)$$

where

$$\begin{aligned} \mathbf{D}(\tau, \theta) &= [\mathbf{d}(\tau_0, \theta_0) \quad \mathbf{d}(\tau_1, \theta_1) \quad \cdots \quad \mathbf{d}(\tau_{L_p-1}, \theta_{L_p-1})]_{MN \times L_p}, \\ \mathbf{d}(\tau_i, \theta_i) &= [\mathbf{d}_0^T(\tau_i, \theta_i) \quad \mathbf{d}_1^T(\tau_i, \theta_i) \quad \cdots \quad \mathbf{d}_{M-1}^T(\tau_i, \theta_i)]^T. \end{aligned} \quad (8)$$

By eigenvalue decomposition of $\mathbf{R}_{\mathbf{X}\mathbf{X}} = E[\mathbf{X}\mathbf{X}^H]$, we can obtain mutually orthogonal subspace $\mathbf{U}_S$ and noise subspace $\mathbf{U}_N$ based on Hadamard product-extended signal. Using space-time pseudo spectral search can estimate τ_i and θ_i , but this method is very time-consuming, which costs large amount of calculation, and is hard to practical application. Here, the fast estimation method of two-dimensional positioning parameters based on Hadamard product is discussed.

3. Fast Estimation Method of Two-Dimensional Parameters

3.1. Algorithm Description. According to (7), the frequency domain estimation vector of the reference antenna element is obtained as

$$\mathbf{X}_{0,:} = \mathbf{H}_{0,:} + \mathbf{n}_{0,:} = \mathbf{S}(\tau) \mathbf{a}' + \mathbf{n}_{0,:}, \quad (9)$$

where $\mathbf{X}_{0,:} = [x_{0,:}, x_{0,:}, \dots, x_{0,:}]^T$ and $\mathbf{H}_{0,:} = [H_{0,:}, H_{0,:}, \dots, H_{0,:}]^T$ are the estimation vectors and response vectors of the channel in frequency domain, respectively, $\mathbf{S}(\tau) = [\mathbf{s}(\tau_0), \mathbf{s}(\tau_1), \dots, \mathbf{s}(\tau_{L_p-1})]$ is Vandermonde matrix of the N frequency points at the reference antenna element, and $\mathbf{s}(\tau_i) =$

$[1, e^{-j2\pi\tau_i/T}, \dots, e^{-j2\pi(N-1)\tau_i/T}]^T$, $\alpha' = [\alpha'_0, \alpha'_1, \dots, \alpha'_{L_p-1}]^T$, $\alpha'_i = \alpha_i e^{-j2\pi f_c \tau_i}$, and $\mathbf{n}_{0,:} = [n_{0,:}, n_{0,:}, \dots, n_{0,:}]^T$ are the additive Gauss white noise vector.

For the reference antenna element, the eigendecomposition of the covariance matrix with P snapshot measurement data can be expressed as

$$\hat{\mathbf{R}}_{\mathbf{X}_{0,:} \mathbf{X}_{0,:}} = \frac{1}{P} \sum_{j=1}^P \mathbf{X}_{0,:}^{(j)} \mathbf{X}_{0,:}^{(j)H} = \hat{\mathbf{U}}_S \hat{\mathbf{\Lambda}}_S \hat{\mathbf{U}}_S^H + \hat{\mathbf{U}}_N \hat{\mathbf{\Lambda}}_N \hat{\mathbf{U}}_N^H, \quad (10)$$

where $\hat{\mathbf{U}}_S$ and $\hat{\mathbf{U}}_N$ are, respectively, the signal subspace and noise subspace and $\hat{\mathbf{\Lambda}}_S$ and $\hat{\mathbf{\Lambda}}_N$ are the signal eigenvalue diagonal matrix and noise eigenvalue diagonal matrix. By using the noise eigenvector based on Hadamard product to extract space parameter information, MUSIC-type algorithm function can be constructed as

$$P'_{\text{MUSIC}}(\tau) = \mathbf{s}^H(\tau) \hat{\mathbf{U}}_N \hat{\mathbf{U}}_N^H \mathbf{s}(\tau). \quad (11)$$

The zero value of the function gives the estimation of the time delay, which is equivalent to the value obtained by pseudo spectral peak search. Let $z_\tau = e^{-j2\pi\tau/T}$ and $\mathbf{s}(z_\tau) = [1, z_\tau, \dots, z_\tau^{N-1}]^T$ be the power term of z_τ , and (11) is not the polynomials of z_τ . Then, $\mathbf{s}^H(z_\tau) = \mathbf{b}^T(z_\tau^{-1}) = [1, z_\tau^{-1}, \dots, z_\tau^{-(N-1)}]$, the polynomials of Root-MUSIC for TOA, can be expressed as

$$\hat{f}(z_\tau) = z_\tau^{N-1} \mathbf{b}^H(z_\tau) \hat{\mathbf{U}}_N \hat{\mathbf{U}}_N^H \mathbf{b}(z_\tau). \quad (12)$$

Then, by utilizing the sampled N frequency points at the reference element, the corresponding delay estimation can be obtained as

$$\hat{\tau}_i = -\text{angle}(\hat{z}_{\tau_i}) \left(\frac{T}{2\pi} \right), \quad i = 0, 1, \dots, L_p - 1, \quad (13)$$

where $\hat{z}_{\tau_0}, \hat{z}_{\tau_1}, \dots, \hat{z}_{\tau_{L_p-1}}$ are L_p roots which are the closest to the unit circle in (12).

Similarly, the DOA estimation of the frequency response of the channel at the first frequency point of the M antenna elements can be obtained by the above algorithm.

$$\hat{\theta}_i = \arcsin \left[-\text{angle}(\hat{z}_{\theta_i}) \left(\frac{\lambda}{2\pi d} \right) \right], \quad i = 0, 1, \dots, L_p - 1. \quad (14)$$

Since the delay and DOA are estimated separately, the corresponding parameter pairing is required.

3.2. Parameter Pairing. By using the frequency domain channel responses at the reference array element and the next element, the matrix $\mathbf{Y}$ is constructed as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{X}_{0,:} \\ \mathbf{X}_{1,:} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_0(\tau, \theta) \\ \mathbf{D}_1(\tau, \theta) \end{bmatrix} \alpha + \begin{bmatrix} n_{0,:} \\ n_{1,:} \end{bmatrix} = \mathbf{D}'(\tau, \theta) \alpha + \mathbf{n}', \quad (15)$$

where $\mathbf{D}' = [\mathbf{d}'(\tau_0, \theta_0), \mathbf{d}'(\tau_1, \theta_1), \dots, \mathbf{d}'(\tau_{L_p-1}, \theta_{L_p-1})]$,

$$\mathbf{d}'(\tau_i, \theta_i) = \begin{bmatrix} 1, e^{-j2\pi\tau_i/T}, \dots, e^{-j2\pi(N-1)\tau_i/T}, \\ e^{-j2\pi d \sin \theta_i/\lambda}, \dots, e^{-j2\pi d \sin \theta_i/\lambda} e^{-j2\pi(N-1)\tau_i/T} \end{bmatrix}^T. \quad (16)$$

The covariance matrix of $\mathbf{Y}$ can be an eigenvalue decomposed as $\hat{\mathbf{R}}_{\mathbf{Y}\mathbf{Y}} = 1/P \sum_{j=1}^P \mathbf{Y}\mathbf{Y}^H = \hat{\mathbf{V}}_S \hat{\mathbf{\Phi}}_S \hat{\mathbf{V}}_S^H + \hat{\mathbf{V}}_N \hat{\mathbf{\Phi}}_N \hat{\mathbf{V}}_N^H$, where $\hat{\mathbf{V}}_S \in \mathbb{C}^{2MN \times L_p}$ and $\hat{\mathbf{V}}_N \in \mathbb{C}^{2MN \times (2MN - L_p)}$ are, respectively, the signal subspace and the noise subspace; obviously, the column vector $\mathbf{d}'(\tau, \theta)$ and noise subspace $\hat{\mathbf{V}}_N$ are orthogonal, which make $\mathbf{d}'(\tau, \theta) \cdot \hat{\mathbf{V}}_N = 0$. In order to achieve the pairing of delay and arrival angle, the cost function can be obtained by

$$h(\tau, \theta) = \mathbf{d}'^H(\tau, \theta) \hat{\mathbf{V}}_N \hat{\mathbf{V}}_N^H \mathbf{d}'(\tau, \theta). \quad (17)$$

So the pairing of the estimated parameters is to minimize the cost function

$$\{\hat{\tau}_i, \hat{\theta}_i\} = \min_{i,j} h(\hat{\tau}_i, \hat{\theta}_j), \quad i, j = 0, 1, \dots, L_p - 1. \quad (18)$$

Firstly, select a fixed $\hat{\tau}_i$ from $\{\hat{\tau}_i\}_{i=0}^{L_p-1}$, and let $\hat{\theta}_i$ traverse $\{\hat{\theta}_i\}_{i=0}^{L_p-1}$, then the $\{\hat{\tau}_i, \hat{\theta}_i\}$ corresponding to the minimum value of the cost function is the correct pairing value.

3.3. Computational Complexity Analysis. The computational complexity of the algorithm in this paper consists of the following four aspects: the estimation of the covariance matrix $\hat{\mathbf{R}}_{\mathbf{X}_{:,0}\mathbf{X}_{:,0}}$ and $\hat{\mathbf{R}}_{\mathbf{X}_{0,:}\mathbf{X}_{0,:}}$, the computational complexity is $O(M^2P)$ and $O(N^2P)$, respectively; the eigenvalue decomposition of the covariance matrix $\hat{\mathbf{R}}_{\mathbf{X}_{:,0}\mathbf{X}_{:,0}}$ and $\hat{\mathbf{R}}_{\mathbf{X}_{0,:}\mathbf{X}_{0,:}}$, the computational complexity is $O(M^3)$ and $O(N^3)$; the complexity of constructed polynomial is $O(M^2 - ML_p)$ and $O(N^2 - NL_p)$; the computational complexity of parameter pairing is $O((2N)^2P + (2N)^3 + (2N)^2L_p! + 4NL_p! - 2NL_pL_p!)$. So the total complexity of the algorithm is $O(M^3 + 9N^3 + M^2(P+1) + N^2(5P+1+4L_p!) - ML_p - NL_p + 4NL_p! - 2NL_pL_p!)$.

The total complexity of using 2D-MUSIC algorithm is $O(M^3N^3 + M^2N^2P + MN(MN - L_p)W_\tau W_\theta + (MN - L_p)W_\tau W_\theta)$, and W_τ and W_θ are, respectively, the number of grid of delay and angle of arrival search, that is, the

range of measurement search is divided by the search step. The computational complexity of ESPRIT algorithm is $O(2N^3 + 2M^3 + (4P+1)N^2 + L_p^2N)$. Table 1 compares the computational complexity of the three algorithms.

Assuming $M = 4$, $P = 100$, $W_\tau = 1000$, $W_\theta = 1000$, and $L_p = 3$, the computational complexity curves of the subcarrier number N in the three algorithms are plotted. It can be seen from Figure 1 that the computational complexity of this paper is slightly higher than that of the ESPRIT algorithm, but much lower than that of the 2D-MUSIC algorithm, which greatly reduces the amount of computation.

4. The Cramer-Rao Bound

The CRB is the lower bound of minimum variance which an unbiased estimation can achieve. The following is to derive the CRB of the model. Let the parameter vector be defined as $[\sigma^2 \eta^T]^T$, where $\eta = [\tau^T \theta^T]^T$. According to (7) and the relevant initial conditions, the joint probability density function of $\mathbf{X}(1), \mathbf{X}(2), \dots, \mathbf{X}(K)$ is

$$f\left(\frac{\mathbf{X}}{\eta}, \alpha\right) = \prod_{j=1}^K \frac{1}{(2\pi)^{MN} (\sigma^2/2)^{MN}} \exp\left(-\frac{1}{\sigma^2} \|\mathbf{X}(j) - \mathbf{D}\alpha(j)\|_2^2\right). \quad (19)$$

Thus, the log-likelihood function of (19), omitting the constant term, is

$$\ln f = -MNK \ln \sigma^2 - \frac{1}{\sigma^2} \sum_{j=1}^K [\mathbf{X}(j) - \mathbf{D}\alpha(j)]^H [\mathbf{X}(j) - \mathbf{D}\alpha(j)]. \quad (20)$$

To calculate the Fisher information matrix (FIM), the derivatives of (20) with respect to σ^2 , the real part $\text{Re } \alpha(j)$ of $\alpha(j)$, the imaginary part $\text{Im } \alpha(j)$ of $\alpha(j)$, and η can be obtained as

$$\begin{aligned} \frac{\partial \ln f}{\partial (\sigma^2)} &= -\frac{MNK}{\sigma^2} + \frac{1}{\sigma^4} \sum_{j=1}^K \mathbf{n}^H(j) \mathbf{n}(j), \\ \frac{\partial \ln f}{\partial \text{Re } \alpha(j)} &= \frac{2}{\sigma^2} \text{Re}[\mathbf{D}^H \mathbf{n}(j)], \\ \frac{\partial \ln f}{\partial \text{Im } \alpha(j)} &= \frac{2}{\sigma^2} \text{Im}[\mathbf{D}^H \mathbf{n}(j)]. \end{aligned} \quad (21)$$

Since η is a two-dimensional vector, the derivatives of (20) with respect to η are different from the ones in the angle-spread only model.

$$\begin{aligned} \frac{\partial \ln f}{\partial \theta_i} &= \frac{1}{\sigma^2} \sum_{j=1}^K \frac{\partial}{\partial \theta_i} \{2 \text{Re}[\alpha^H(j) \mathbf{D}^H \mathbf{X}(j)] - \alpha^H(j) \mathbf{D}^H \mathbf{D} \alpha(j)\} \\ &= \frac{2}{\sigma^2} \sum_{j=1}^K \text{Re}\left\{\alpha_i(j) \mathbf{g}_{\theta_i}^H \mathbf{n}(j)\right\}, \quad i = 0, \dots, L_p - 1, \end{aligned} \quad (22)$$

and similarly,

TABLE 1: Algorithm complexity comparison.

Algorithm type	Computational complexity
2D-MUSIC	$O(M^3N^3 + M^2N^2P + MN(MN - L_p)W_\tau W_\theta + (MN - L_p)W_\tau W_\theta)$
ESPRIT	$O(2N^3 + 2M^3 + (4P + 1)N^2 + L_p^2N)$
Proposed algorithm	$O(M^3 + 9N^3 + M^2(P + 1) + N^2(5P + 1 + 4L_p!) - ML_p - NL_p + 4NL_p! - 2NL_pL_p!)$

$$\frac{\partial \ln f}{\partial \tau_i} = \frac{2}{\sigma^2} \sum_{j=1}^N \text{Re} \left\{ \alpha_i(j) \mathbf{g}_{\tau_i}^H \mathbf{n}(j) \right\}, \quad i = 0, \dots, L_p - 1, \quad (23)$$

where $\mathbf{g}_{\theta_i}$ is the derivative with respect to θ_i of the i th column of $\mathbf{D}$. Similarly, $\mathbf{g}_{\tau_i}$ is the derivative with respect to τ_i of the i th column of $\mathbf{D}$. Due to $\mathbf{d}_m(\tau_i, \theta_i) = \mathbf{s}(\tau_i) \oplus \mathbf{a}_m(\theta_i)$, then

$$\mathbf{g}_{\theta_i} = \begin{bmatrix} \frac{\mathbf{s}(\tau_i) \oplus \partial \mathbf{a}_0 \theta_i}{\partial \theta_i} \\ \frac{\mathbf{s}(\tau_i) \oplus \partial \mathbf{a}_1 \theta_i}{\partial \theta_i} \\ \vdots \\ \frac{\mathbf{s}(\tau_i) \oplus \partial \mathbf{a}_{M-1} \theta_i}{\partial \theta_i} \end{bmatrix}, \quad (24)$$

$$\mathbf{g}_{\tau_i} = \begin{bmatrix} \frac{\partial \mathbf{s}(\tau_i)}{\partial \tau_i \oplus \mathbf{a}_0(\theta_i)} \\ \frac{\partial \mathbf{s}(\tau_i)}{\partial \tau_i \oplus \mathbf{a}_1(\theta_i)} \\ \vdots \\ \frac{\partial \mathbf{s}(\tau_i)}{\partial \tau_i \oplus \mathbf{a}_{M-1}(\theta_i)} \end{bmatrix}.$$

Further, written more compactly

$$\frac{\partial \ln f}{\partial \theta} = \frac{2}{\sigma^2} \sum_{j=1}^K \text{Re} \left\{ (\text{diag}[\alpha(j)])^H \mathbf{G}_\theta^H \mathbf{n}(j) \right\}, \quad (25)$$

$$\frac{\partial \ln f}{\partial \tau} = \frac{2}{\sigma^2} \sum_{j=1}^K \text{Re} \left\{ (\text{diag}[\alpha(j)])^H \mathbf{G}_\tau^H \mathbf{n}(j) \right\},$$

where $\mathbf{G}_\theta = [\mathbf{g}_{\theta_0}, \dots, \mathbf{g}_{\theta_{L_p-1}}]$ and $\mathbf{G}_\tau = [\mathbf{g}_{\tau_0}, \dots, \mathbf{g}_{\tau_{L_p-1}}]$. Let $\mathbf{G} = [\mathbf{G}_\tau, \mathbf{G}_\theta]_{(MN \times 2L_p)}$ and $\mathbf{B}(j) = \mathbf{I}_2 \otimes \text{diag}[\alpha(j)]$, then we can obtain

$$\frac{\partial \ln f}{\partial \eta} = \frac{2}{\sigma^2} \sum_{j=1}^K \text{Re} \left\{ (\text{diag}[\alpha(j)])^H \mathbf{G}^H \mathbf{n}(j) \right\}. \quad (26)$$

By using the results proven in [20], the following expressions can be attained

FIGURE 1: Computational complexity of different algorithms.

$$E \left[\left[\frac{\partial \ln f}{\partial (\sigma^2)} \right]^2 \right] = \frac{MNK}{\sigma^4},$$

$$E \left[\left[\frac{\partial \ln f}{\partial \text{Re } \alpha(j)} \right] \left[\frac{\partial \ln f}{\partial \text{Re } \alpha(p)} \right]^T \right] = \frac{2}{\sigma^2} \text{Re} [\mathbf{D}^H \mathbf{D}] \delta_{j,p},$$

$$E \left[\left[\frac{\partial \ln f}{\partial \text{Re } \alpha(j)} \right] \left[\frac{\partial \ln f}{\partial \text{Im } \alpha(p)} \right]^T \right] = -\frac{2}{\sigma^2} \text{Im} [\mathbf{D}^H \mathbf{D}] \delta_{j,p},$$

$$E \left[\left[\frac{\partial \ln f}{\partial \text{Im } \alpha(j)} \right] \left[\frac{\partial \ln f}{\partial \text{Im } \alpha(p)} \right]^T \right] = \frac{2}{\sigma^2} \text{Re} [\mathbf{D}^H \mathbf{D}] \delta_{j,p},$$

$$E \left[\left[\frac{\partial \ln f}{\partial \text{Re } \alpha(j)} \right] \left[\frac{\partial \ln f}{\partial \eta} \right]^T \right] = \frac{2}{\sigma^2} \text{Re} [\mathbf{D}^H \mathbf{G} \mathbf{B}(j)],$$

$$E \left[\left[\frac{\partial \ln f}{\partial \text{Im } \alpha(j)} \right] \left[\frac{\partial \ln f}{\partial \eta} \right]^T \right] = \frac{2}{\sigma^2} \text{Im} [\mathbf{D}^H \mathbf{G} \mathbf{B}(j)],$$

$$E \left[\left[\frac{\partial \ln f}{\partial \eta} \right] \left[\frac{\partial \ln f}{\partial \eta} \right]^T \right] = \frac{2}{\sigma^2} \sum_{j=1}^K \text{Re} [\mathbf{B}^H(r) \mathbf{G}^H \mathbf{G} \mathbf{B}(j)]. \quad (27)$$

Finally, the FIM for the parameters is obtained. Using the results in [20], the CRB of the parameters can be shown as

FIGURE 2: The scatter plot of joint TOA and DOA estimation, SNR = 0 dB.

FIGURE 3: The scatter plot of joint TOA and DOA estimation, SNR = 5 dB.

$$\text{CRB}(\eta) = \frac{\sigma^2}{2} \left\{ \sum_{j=1}^K \text{Re} \left[\mathbf{B}^H(j) \mathbf{G}^H \left[\mathbf{I} - \mathbf{D}(\mathbf{D}^H \mathbf{D})^{-1} \mathbf{D}^H \right] \mathbf{G} \mathbf{B}(j) \right] \right\}^{-1}, \quad (28)$$

which completes the proof.

5. Simulation Results

In this section, the performance of fast estimation of two-dimensional positioning parameters of TOA and DOA is studied. Monte Carlo simulation is utilized to verify the practicability and robustness of the proposed algorithm. The relevant parameters of narrowband array antenna in OFDM system are set as follows: the system bandwidth is $B = 20$ MHz, the FFT cycle is $T = 3.2 \mu\text{s}$, the number of subcarriers is $N = 64$, the carrier frequency is $f_c = 2.3$ GHz, the number of elements is $M = 12$, the array spacing is $d = \lambda/2$ meters, and so on.

5.1. Simulation 1. In the case of low SNR at 0 dB and 5 dB, assuming that the multipath number of the received signals is $L_p = 3$, the arrival time is $\tau_1 = 100$ ns, $\tau_2 = 140$ ns, and $\tau_3 = 180$ ns, respectively; the direction of arrival is $\theta_1 = 30^\circ$, $\theta_2 = 40^\circ$, and $\theta_3 = 60^\circ$, respectively. Simulation time of the algorithm is $S = 100$, the scatter diagrams of the joint estimation of TOA and DOA combined with three multipath components are obtained, Figures 2 and 3 show that the algorithm has a good performance for the joint estimation at low SNR conditions.

5.2. Simulation 2. Under different SNRs, other conditions are the same as simulation 1, and the scatter plots of TOA and DOA can be obtained. It can be seen from Figure 4 and Figure 5 that with the increase of the SNR, the dispersion of TOA and DOA gradually tends to a point, and the estimation precision is higher. At low SNR, the estimation accuracy amplitudes of TOA and DOA are all within a unit.

FIGURE 4: The scatter plot of TOA estimation under different SNRs.

5.3. Simulation 3. At low SNR of SNR = 5 dB, other conditions are the same as simulation 1, and the estimation error of TOA and DOA is defined as $\Delta\tau_{s,i} = \hat{\tau}_{s,i} - \tau_i$ and $\Delta\theta_{s,i} = \hat{\theta}_{s,i} - \theta_i$, respectively. Then, the scatter diagram of $\Delta\tau_{s,1}$, $\Delta\tau_{s,2}$, and $\Delta\tau_{s,3}$ corresponding to three multipath components is shown in Figure 6, and the corresponding $\Delta\theta_{s,1}$, $\Delta\theta_{s,2}$, and $\Delta\theta_{s,3}$ distribution are shown in Figure 7. It can be seen from Figures 6 and 7 that the algorithm at low SNR can almost achieve the unbiased estimation of TOA and DOA.

5.4. Simulation 4. Define the root mean square error (RMSE) as $\Omega_{\text{RMSE}} = \sqrt{(1/N) \sum_{n=0}^{N-1} (\hat{x}_n - x)^2}$, where $\hat{x}_n$ is the parameter estimated value obtained by the n th simulation and x is the true value of the corresponding parameter. The TOA and DOA estimation performances of the respective first path are shown in Figures 8 and 9, respectively. The number of multipath is $L_p = 2$, $L_p = 3$, and $L_p = 4$, respectively.

FIGURE 5: The scatter plot of DOA estimation under different SNRs.

FIGURE 6: The distribution of TOA estimation error, SNR = 5 dB.

The results are based on 100 Monte Carlo simulations. As expected, the TOA and DOA estimation performances of the first path decrease with the increase of the number of multipath. At low SNR of SNR=5 dB, the estimation accuracy under multipath conditions can still meet the positioning requirements, so the algorithm has certain adaptability to multipath environment.

5.5. Simulation 5. Under different SNRs, other conditions are the same as simulation 1. The proposed algorithm is compared with 2D-MUSIC algorithm, ESPRIT algorithm, 2D MP algorithm [16], and 1D MP algorithm [13]. The TOA and DOA RMSE curves of each algorithm's first path are shown in Figures 10 and 11, respectively, and compared with the CRB of this model. As shown in Figures 10 and 11, the performance of the proposed algorithm is obviously superior to that of ESPRIT algorithm and 2D MP

FIGURE 7: The distribution of DOA estimation error, SNR = 5 dB.

FIGURE 8: TOA estimation performance of the first path under different multipath numbers.

algorithm, and very close to 2D-MUSIC algorithm, and the accuracy of parameter estimation completely meets the positioning requirements.

Since the 1D MP algorithm is based on a single snapshot data, little information is available, and the 1D MP algorithm is the worst in low SNRs. However, the 2D MP algorithm extends the information, and the availability of the information is relatively increased so that the performance of 2D MP algorithm is better than that of 1D MP algorithm. In essence, the proposed algorithm is another form of MUSIC algorithm and has a similar estimation performance as 2D-MUSIC algorithm, of which the complex two-dimensional pseudo spectral peak search is avoided in

FIGURE 9: DOA estimation performance of the first path under different multipath numbers.

FIGURE 10: Performance comparison of different algorithms for TOA estimation.

the proposed algorithm. Therefore, it can realize the fast estimation of two-dimensional location parameters.

6. Conclusion

The fast estimation of location parameter is a necessary condition for the quality of wireless positioning system. As the fact that 2D-MUSIC algorithm needs a long time two-dimensional pseudo spectral peak search, which makes it difficult to be applied in practice, a fast estimation method of two-dimensional positioning based on Hadamard product in OFDM system is proposed. The construction of the model, the theoretical analysis, and the process of CRB derivation

FIGURE 11: Performance comparison of different algorithms for DOA estimation.

are presented. The computation complexity of the proposed method by solving the closed-form solution and parameter pairing is much lower than that of 2D-MUSIC algorithm. The simulation results show that the parameter estimation performance of the proposed algorithm is not only better than that of ESPRIT algorithm and 2D matrix pencil algorithm, but also close to that of 2D-MUSIC algorithm, and it has certain adaptability to the multipath environment and certain practical value.

Notation

- T : Matrix transpose
- $*$: Matrix conjugate
- H : Matrix conjugate transpose
- $E[\bullet]$: Statistical expectation
- $\oplus$: Hadamard product
- $\otimes$: Kronecker product.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] A. Makki, A. Siddig, M. Saad, J. R. Cavallaro, and C. J. Bleakley, "Indoor localization using 802.11 time differences of arrival," *IEEE Transactions on Instrumentation and Measurement*, vol. 65, no. 3, pp. 614–623, 2016.

- [2] F.-X. Ge, D. Shen, Y. Peng, and V. O. K. Li, "Super-resolution time delay estimation in multipath environments," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 54, no. 9, pp. 1977–1986, 2007.
- [3] L. Jing, Z. Yong-Jun, and L. Dong-Hai, "Time delay estimation using Markov chain Monte Carlo method," *Acta Physica Sinica*, vol. 63, no. 13, article 130701, 2014.
- [4] F.-q. Wang and X.-f. Zhang, "Improved propagator method-based joint TOA and DOA estimation in impulse radio ultra wide band," *Journal of Electronics & Information Technology*, vol. 35, no. 12, pp. 2954–2959, 2013.
- [5] A. Gaber and A. Omar, "Utilization of multiple-antenna multicarrier systems and NLOS mitigation for accurate wireless indoor positioning," *IEEE Transactions on Wireless Communications*, vol. 15, no. 10, pp. 6570–6584, 2016.
- [6] A.-J. van der Veen, M. C. Vanderveen, and A. Paulraj, "Joint angle and delay estimation using shift-invariance techniques," *IEEE Transactions on Signal Processing*, vol. 46, no. 2, pp. 405–418, 1998.
- [7] R. Shafin, L. Liu, Y. Li, A. Wang, and J. Zhang, "Angle and delay estimation for 3-D massive MIMO/FD-MIMO systems based on parametric channel modeling," *IEEE Transactions on Wireless Communications*, vol. 16, no. 8, pp. 5370–5383, 2017.
- [8] "Part 11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications, amendment 4: enhancements for very high throughput for operation in bands below 6 GHz," in *IEEE 802.11ac Std*, 2013.
- [9] K. Huo and J. Zhao, "The development and prospect of the new OFDM radar," *Journal of Electronics & Information Technology*, vol. 37, no. 11, pp. 2776–2789, 2015.
- [10] "Advanced air interface, mobile WiMax (release 2)," in *IEEE Standard 802.16m*, 2011.
- [11] "Study on Channel Model for Frequency Spectrum Above 6 GHz (Release 14)," document TR 38.900, 3GPP, 2016.
- [12] F. Q. Wang, X. F. Zhang, and F. Wang, "Root-MUSIC-based joint TOA and DOA estimation in IR-UWB," *Journal on Communications*, vol. 35, no. 2, pp. 137–145, 2014.
- [13] R. Ding, Z. H. Qian, and X. Wang, "UWB positioning system based on joint TOA and DOA estimation," *Journal of Electronics & Information Technology*, vol. 32, no. 2, pp. 313–317, 2010.
- [14] H. Ni, G. Ren, and Y. Chang, "A TDOA location scheme in OFDM based WMANs," *IEEE Transactions on Consumer Electronics*, vol. 54, no. 3, pp. 1017–1021, 2008.
- [15] Z. He, Y. Ma, and R. Tafazolli, "Improved high resolution TOA estimation for OFDM-WLAN based indoor ranging," *IEEE Wireless Communications Letters*, vol. 2, no. 2, pp. 163–166, 2013.
- [16] A. Gaber and A. Omar, "A study of wireless indoor positioning based on joint TDOA and DOA estimation using 2-D matrix pencil algorithms and IEEE 802.11ac," *IEEE Transactions on Wireless Communications*, vol. 14, no. 5, pp. 2440–2454, 2015.
- [17] B. Zhou, C. Jing, and Y. Kim, "Joint TOA/AOA positioning scheme with IP-OFDM systems," *Wireless Personal Communications*, vol. 75, no. 1, pp. 261–271, 2014.
- [18] B. Ba, G. Liu, T. Li, Y. Lin, and Y. Wang, "Joint for time of arrival and direction of arrival estimation algorithm based on the subspace of extended hadamard product," *Acta Physica Sinica*, vol. 64, no. 7, pp. 0784031–0784039, 2015.
- [19] X. Li and K. Pahlavan, "Super-resolution TOA estimation with diversity for indoor geolocation," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 224–234, 2004.
- [20] P. Stoica and A. Nehorai, "Music, maximum likelihood, and Cramer-Rao bound," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 5, pp. 720–741, 1989.