

## Research Article

# A New Approximate Method for Lightning-Radiated ELF/VLF Ground Wave Propagation over Intermediate Ranges

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A new approximate method for lightning-radiated extremely low-frequency (ELF) and very low-frequency (VLF) ground wave propagation over intermediate ranges is presented in this paper. In our approximate method, the original field attenuation function is divided into two factors in frequency domain representing the propagation effect of the ground conductivity and Earth's curvature, and both of them have clearer formulations and can more easily be calculated rather than solving a complex differential equation related to Airy functions. The comparison results show that our new approximate method can predict the lightning-radiated field peak value over the intermediate range with a satisfactory accuracy within maximum errors of 0.0%, -3.3%, and -8.7% for the earth conductivity of 4 S/m, 0.01 S/m, and 0.001 S/m, respectively. We also find that Earth's curvature has much more effect on the field propagation at the intermediate ranges than the finite ground conductivity, and the lightning-radiated ELF/VLF electric field peak value (V/m) at the intermediate ranges yields a propagation distance  $d$  (km) dependence of  $d^{-1.32}$ .

## 1. Introduction

Lightning-radiated very low-frequency (VLF) (3–30 kHz) and low-frequency (LF) (30–300 kHz) signals can propagate along a spherical earth with finite conductivity at an intermediate distance of hundreds to a couple of thousand kilometers, and the observed lightning-radiated electromagnetic field signal would be significantly attenuated and distorted due to the propagation effects (e.g., Wait [1–4], Shao and Jacobson [5], and Zhang et al. [6–9]). In the intermediate range (within 1500 km), a planar earth assumption will bring some obvious errors, and the propagation effect of a spherical earth should be considered.

The general problem of the radiation from an antenna and the propagation of ground wave over homogeneous earth has been studied since the 1910s (e.g., Watson [10, 11], Sommerfeld [12], Van der Pol [13], and Norton [14]) and

has been discussed in detail by Wait [1–4]. In addition, the ground wave propagation attenuation function for spherical inhomogeneous earth has been derived by Wait [3, 4]. However, the formula of attenuation function involves the roots of a complex differential equation that is related to the highly oscillatory Airy functions, and the computation of the propagation attenuation function is very difficult and time consuming. In 1980, Hill and Wait [15] generalized the previous computation methods of the attenuation function and presented an approximate method to calculate the ground wave attenuation function for arbitrary surface impedance along the spherical earth surface. In their method, different approximate formulas were used according to the phases of normalized surface impedance and a fourth-order Runge-Kutta formula had to be used to obtain the roots of differential equations. In 1993, Maclean and Wu [16] presented a Taylor series method to compute the attenuation function, and

the roots of the complex differential equation were approximated by a Taylor series in their method.

In 2009, in order to study the propagation effect of lightning-radiated VLF/LF field along the spherical earth with finite conductivity, based on the formulas presented by Wait [3, 4], Shao and Jacobson [5] made some modifications for their study. In their method, a complex Newton–Raphson’s root-finding method was used to solve the complex differential equation related to the highly oscillatory Airy functions. Although this method has high accuracy and strong applicability, the iteration process may be somewhat complicated and time consuming.

Therefore, in this paper, we will present an improved new approximate method for computing the propagation effect of lightning-produced extremely low-frequency (ELF) and very low-frequency (VLF) ground wave propagation over intermediate ranges. In our method, we will divide the ground wave attenuation function presented by Wait [3, 4] into two attenuation factors representing the propagation effects of the finite ground conductivity and Earth’s curvature, and these two attenuation factors have more clear and simple expressions in the frequency domain, which can be easily calculated by multiplying them rather than solving a complex differential equation related to Airy functions.

## 2. Method Introduction

*2.1. General Equations.* For a vertical electric dipole source located on the surface of a smooth spherical earth, the vertical electric field strength  $E$  at a great circle distance  $d$  can be expressed as [3, 4]

$$E = E_0 W, \quad (1)$$

where  $E_0$  is the vertical electric field of the same dipole source located on a flat and perfectly conducting ground and  $W$  is the attenuation function accounting for the Earth’s curvature ( $\rho$ ) and finite ground conductivity ( $\sigma$ ). For an electric dipole with current moment  $I_m$ , the produced vertical electric field  $E_0$  on the earth surface at a long distance can be approximately calculated by the following formula, which is derived from Uman [17].

$$E_0 = -\frac{1}{2\pi\epsilon_0 c^2 d} \frac{\partial I_m}{\partial t}, \quad (2)$$

where  $I_m$  is the current moment,  $d$  is the great circle distance between the source and the receiver,  $c$  is the light speed, and  $\epsilon_0$  is the dielectric constant.

For the electromagnetic field computation of the cloud-ground lightning, both the source and the receiver are assumed to be on the earth surface and the attenuation function  $W$  in (1) can be approximately expressed as

$$W_{\text{Wait}} = e^{-j\pi/4} \sqrt{\pi x} \sum_{s=1}^{\infty} \frac{e^{-jxt_s}}{t_s - q^2}, \quad (3)$$

where  $x = (k_0 R/2)^{1/3} (d/R)$ ,  $q = -j(k_0 R/2)^{1/3} \Delta$ , the normalized earth surface impedance  $\Delta = k_0/k\sqrt{1 - (k_0/k)^2}$ , and the

wavenumber in the earth  $k = \omega\sqrt{\epsilon_r\epsilon_0\mu_0 - j\sigma\mu_0/\omega}$ .  $R$  is the Earth’s radius, and  $\omega$  is the angular frequency.  $\epsilon_0$  and  $\mu_0$  are the dielectric constant and magnetic permeability of free space, respectively.  $\epsilon_r$  and  $\sigma$  are the Earth’s relative dielectric constant and conductivity, respectively.  $t_s$  are the roots of the complex differential equation

$$w_1'(t) - qw_1(t) = 0, \quad (4)$$

and  $w_1(t)$  is expressed as

$$w_1(t) = \sqrt{\pi}(Bi(t) - jAi(t)), \quad (5)$$

where  $Ai(t)$  and  $Bi(t)$  are the Airy functions defined by Miller [18].

In order to obtain the roots  $t_s$ , Shao and Jacobson [5] used the Newton–Raphson root-finding method, and the initial guess for the root is facilitated with the mean of the two roots from  $w_1(t) = 0$  and  $w_1'(t) = 0$  corresponding to  $|q| = \infty$  and  $|q| = 0$ , respectively. This method is very complicated because the Airy functions are highly oscillatory in the entire complex plane, and we have to iterate many times to obtain roots for different angular frequencies and different earth conductivities. However, because the Newton–Raphson root-finding method has higher accuracy, we will regard the results of this method as a “true value” to evaluate the accuracy of our proposed approximation algorithm.

*2.2. Our Approximate Method.* The ground conductivity and Earth’s curvature are the two factors affecting the propagation of electromagnetic wave over the earth surface; in our approximate method, the propagation attenuation function  $W$  in (1) is divided into two factors

$$W_{\text{our}} = W_{\sigma} W_{\rho}, \quad (6)$$

where  $W_{\sigma}$  and  $W_{\rho}$  represent the propagation attenuation factors caused by the ground conductivity ( $\sigma$ ) and Earth’s curvature ( $\rho$ ), respectively.

The attenuation function  $W_{\sigma}$  just describes the propagation effect of planar earth with finite conductivity, without considering the curvature of the earth, and it can be expressed as below (e.g., Wait [19], Cooray [20], Zhang et al. [8, 9]):

$$W_{\sigma} = 1 - j\sqrt{\pi p} \exp(-p) \operatorname{erfc}(j\sqrt{p}), \quad (7)$$

where  $p = -j\omega d\Delta^2/2c$ ,  $\operatorname{erfc}$  is the complementary error function,  $c$  is the light speed, and other symbols have the same expression and meaning as shown in (3).

Attenuation function  $W_{\rho}$  describes the effect of Earth’s curvature; it corresponds to a perfectly conducting but spherical ground, which means  $|q| = 0$  in (3). Then (3) and (4) can be simplified as

$$W_{\rho} = e^{-j\pi/4} \sqrt{\pi x} \sum_{s=1}^{\infty} \frac{e^{-jxt_s}}{t_s}, \quad (8)$$

$$w_1'(t) = 0. \quad (9)$$

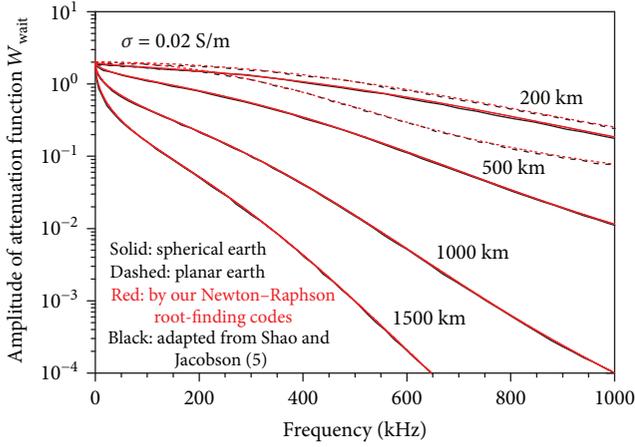


FIGURE 1: Comparison of the simulated amplitude of attenuation functions  $W_{\text{wait}}$  in the frequency domain calculated by our Newton–Raphson root-finding codes and that presented by Shao and Jacobson [5]. The solid lines are computed for a spherical earth with an assumed conductivity of 0.02 S/m. The dashed lines are for a planar earth with the same conductivity, and only 200 and 500 km path lengths are presented for the planar earth. Note that the attenuation function value presented here is twice as much as the calculated value by (3), because Shao and Jacobson [5] included the mirror effect of perfectly conducting ground in the attenuation function.

The parameter  $t_s$  in (8) is the roots of (9) and can be approximately expressed as below according to Sollfrey [21]:

$$t_s = e^{-j\pi/3} \pi \left( \frac{3\pi v_s}{2} \right)^{2/3}, \quad (10)$$

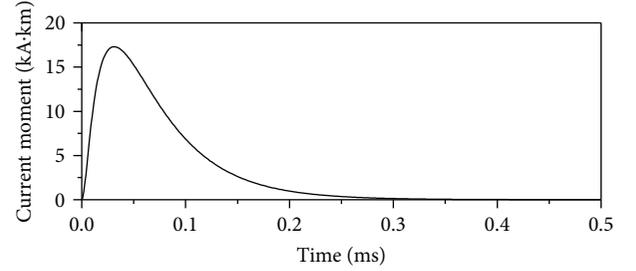
$$v_s = s - \frac{3}{4} - \frac{0.00795}{s - 3/4}, \quad s = 1, 2, \dots \quad (11)$$

The error of this approximate expression (10) is 0 for  $s = 1$ , 0.0024 for  $s = 2$ , 0.0012 for  $s = 3$ , and less than 0.0005 for all higher values of  $s$  [21]. Note that we have slightly modified the original formulas in Sollfrey [21] in order to let the value of  $s$  in (3), (8), (10), and (11) both start at 1.

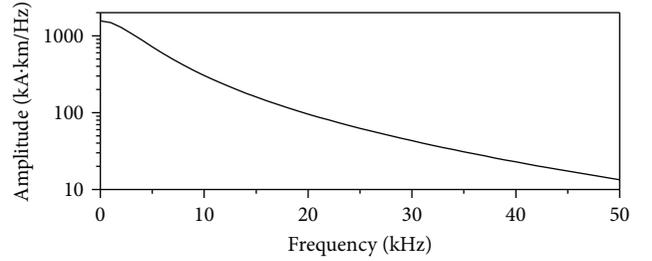
### 3. Results and Analysis

**3.1. Validation of Our Approximate Method.** Firstly, according to (3), we compute Wait’s original formula by using the Newton–Raphson root-finding method and compare our result with that presented by Shao and Jacobson [5], as shown in Figure 1. The attenuation functions in the frequency domain ranges from 0 to 1000 kHz. The ground conductivity is assumed to be 0.02 S/m, and both source and observed sites are located on the ground. It is found that the calculated attenuation function  $W_{\text{wait}}$  by our Newton–Raphson root-finding codes is nearly the same as that presented in [5], which means that our Newton–Raphson root-finding codes are correct.

In the following section, in order to validate the accuracy of our approximate method for predicting the lightning-



(a)



(b)

FIGURE 2: The current moment waveshape (a) in the time domain and (b) in the frequency domain used for computing the lightning VLF radiation at an intermediate distance.

radiated field in intermediate range, we adopt a commonly used current moment for lightning electromagnetic radiation at a significant distance (e.g., Cummer [22], Hu and Cummer [23]). This current moment model was originally developed by Bruce and Golde [24] and summarized by Jones [25] and is expressed as

$$I_m(t) = I_0 \frac{v_0}{\gamma} \left[ e^{-at} - e^{-bt} \right] \left[ 1 - e^{-\gamma t} \right], \quad (12)$$

where  $I_0 = 20$  kA,  $v_0 = 8 \times 10^7$  m/s,  $\gamma = 3 \times 10^4$ /s,  $a = 2 \times 10^4$ /s, and  $b = 2 \times 10^5$ /s. Figure 2 shows the waveform and spectrum of this current moment.

Figure 3 shows the comparison results of our approximate formula in (6) with Wait’s original formula in (3) for the ocean propagation path considering the curvature of the earth. It is found that our approximate method ( $E_{\text{our}}$ ) has the same results as Wait’s method ( $E_{\text{wait}}$ ). For the smooth ocean path with the conductivity of 4 S/m, the finite conductivity has no effect on the lightning-radiated electromagnetic field of very low frequency with tens to hundreds of kHz, and further analysis shows that the smooth ocean path has no effect on the lightning-radiated field in high frequency of tens of MHz. However, compared with the effect of the finite conductivity, Earth’s curvature has much more attenuation effect on the propagation of lightning field, and the attenuation increases with propagation distance. Within about 200 km, the effect caused by Earth’s curvature can be nearly ignored and the ocean surface approximately can be assumed to be flat. However, Earth’s curvature causes an attenuation of about 22% and 52% at the observed distance of 500 km and 1000 km, respectively.

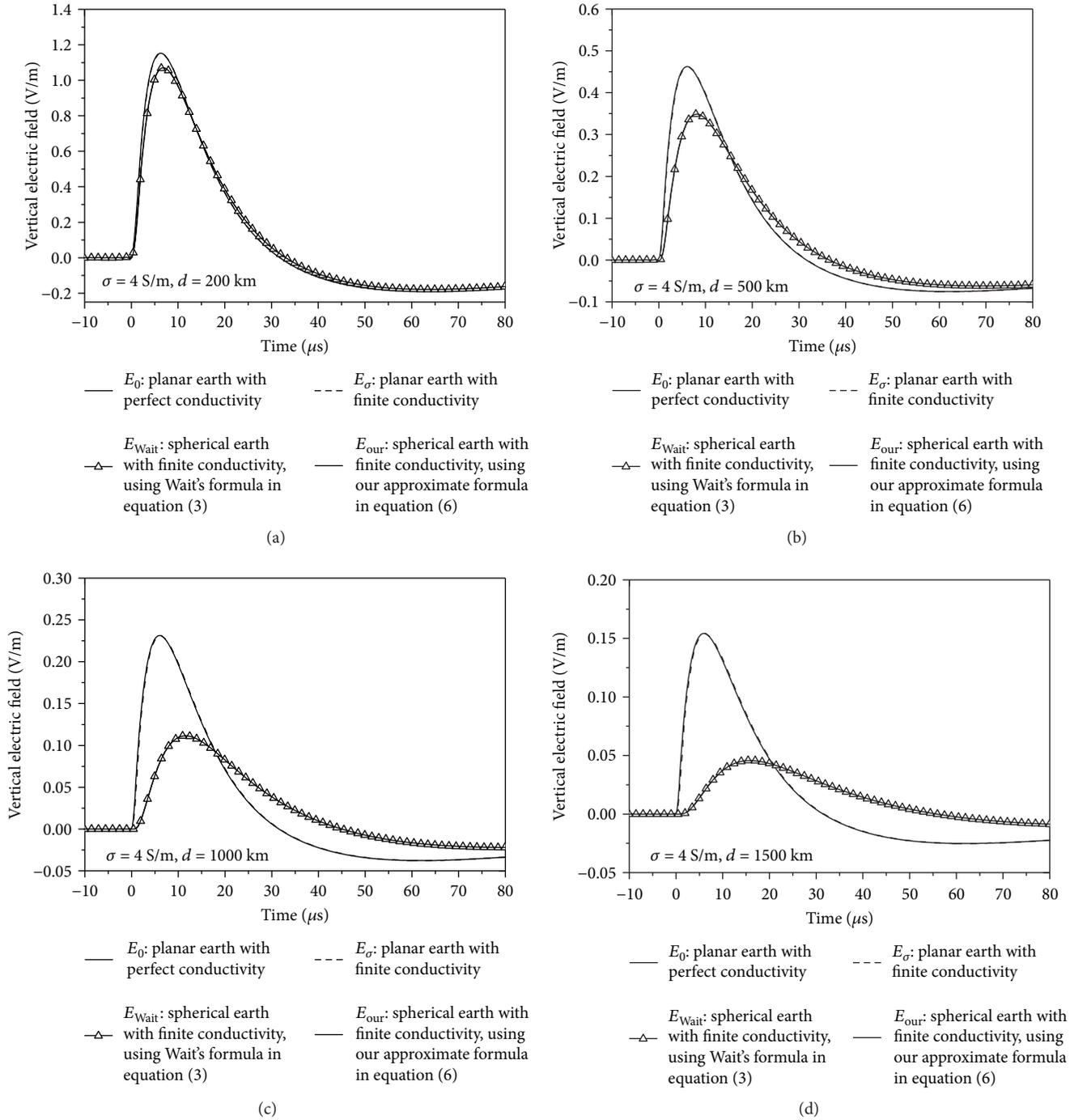


FIGURE 3: Comparison of the vertical electric field calculated by our approximate formula in (6) and Wait's original formula in (3) for the ocean propagation path considering the earth curvature at different distances of (a) 200 km, (b) 500 km, (c) 1000 km, and (d) 1500 km. Wait's original formula is solved by the Newton-Raphson root-finding method. The polarity of a vertical electric field pointing downward is defined to be positive.

Figures 4 and 5 further show the comparison results of the vertical electric field calculated by our approximate formula and Wait's formula for the propagation path with the conductivity of 0.01 S/m and 0.001 S/m, respectively, considering the effect of Earth's curvature. Table 1 and Table 2 further show the detail relative errors of our approximate method for the computing of field peak ( $E_p$ ) and

waveform rise time (RT). It is found that our approximate method can also nearly produce the same result as that calculated by Wait's formula, including of the field peak and waveform rise time, which means that our approximate method is valid and can predict the lightning-radiated field waveform over the intermediate range. Our approximate method can predict the lightning-radiated

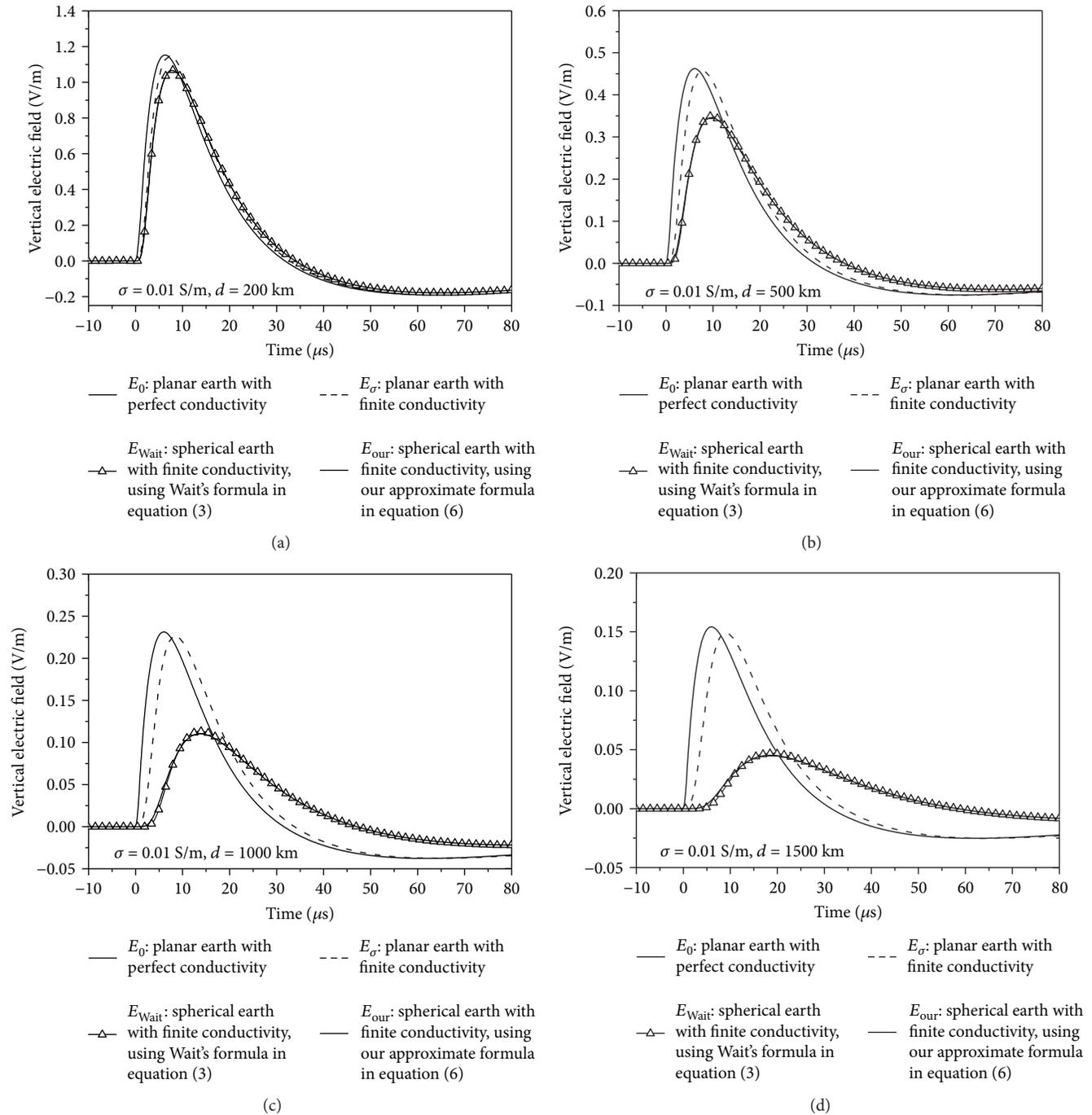


FIGURE 4: Similar to that of Figure 3 but for finite conductivity of 0.01 S/m at the different distances of (a) 200 km, (b) 500 km, (c) 1000 km, and (d) 1500 km.

field peak value over the intermediate range with a satisfactory accuracy within maximum errors ranging from of 0.2%, 3.3%, and 8.7% for the earth conductivity of 4 S/m, 0.01 S/m, and 0.001 S/m, respectively.

Also, it is shown that, for the planar earth, the field peak value decreases with the decrease of the ground conductivity, and the waveform rise times increase with the decrease of the ground conductivity, which is because the high frequencies are selectively attenuated by the finitely conducting ground, causing the amplitude of the

electromagnetic fields to decrease and the rise time to increase (e.g., Cooray et al. [26], Cooray [27, 28], Delfino et al. [29, 30]). For instance, for the planar earth, at the distance of 1500 km, the peak values of the vertical electric field are 0.15 V/m and 0.11 V/m for the conductivity of 4 S/m and 0.001 S/m, respectively. However, for the spherical earth, at the same observation distance, the peak values are almost the same for different earth conductivity, which implicates that for the propagation of a lightning-radiated ELF/VLF electromagnetic wave over intermediate

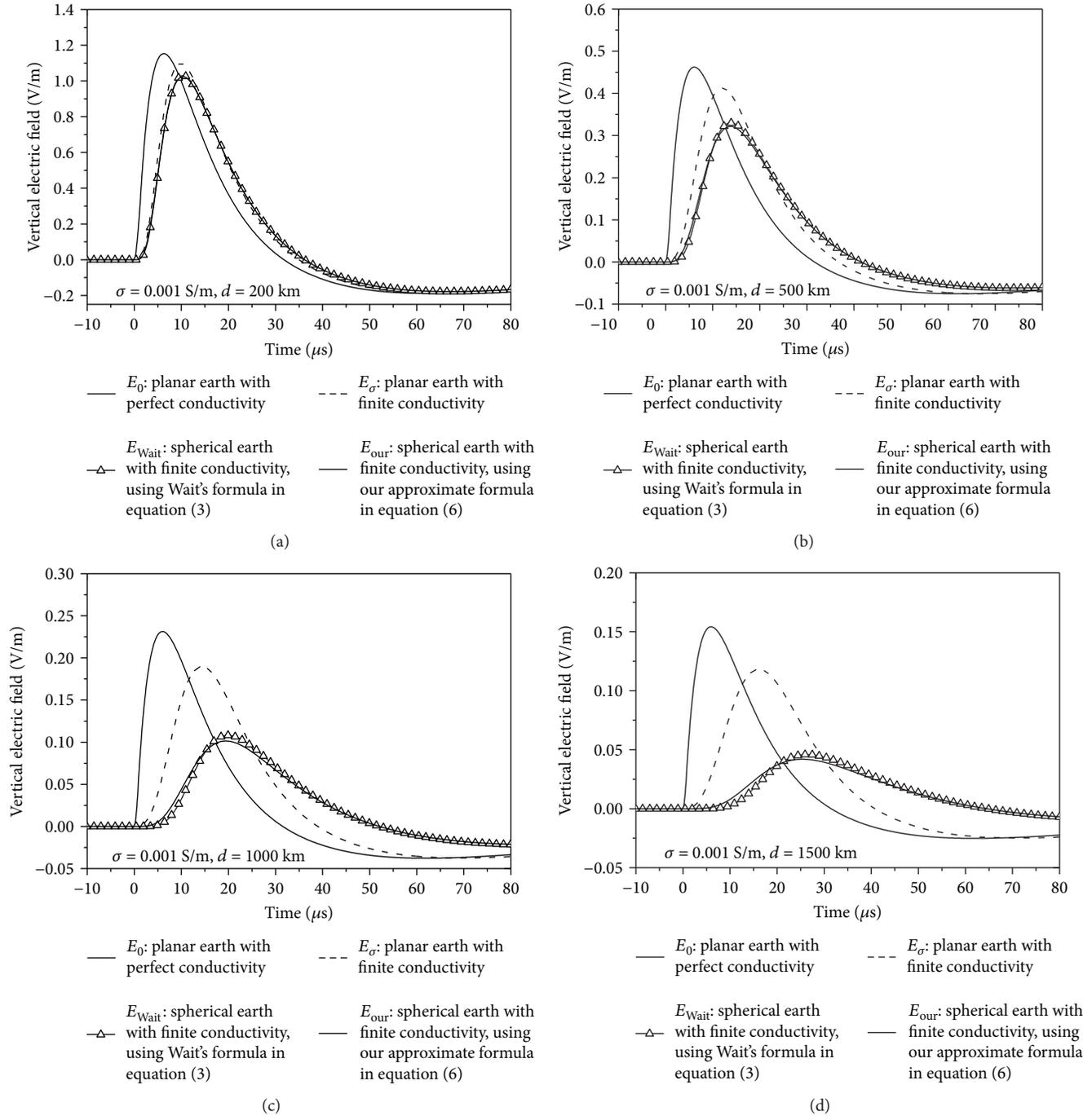


FIGURE 5: Similar to that of Figure 3 but for finite conductivity of 0.001 S/m at the different distances of (a) 200 km, (b) 500 km, (c) 1000 km, and (d) 1500 km.

ranges, Earth's curvature has more attenuation effect than the finite conductivity.

In order to further explain the validity of our approximate method, we compare the attenuation functions in the frequency domain calculated using our approximate formula in (6) and Wait's original formula in (3); the compared results are shown in Figure 6. Figure 6(a) shows the amplitude of the attenuation functions; the black line is the attenuation function proposed by Wait ( $W_{\text{Wait}}$ ) and the green

line is our approximate formula ( $W_{\text{our}}$ ). The red line is the attenuation function only accounting for Earth's curvature ( $W_\rho$ ), and the blue line is the attenuation function only accounting for the ground conductivity ( $W_\sigma$ ). It can be seen that the amplitude of the attenuation function  $W_\sigma$  is very different from that of  $W_{\text{Wait}}$ , and the total attenuation function is predominantly determined by the effect caused by Earth's curvature ( $W_\rho$ ). However, for the phase angle of attenuation function (see Figure 6(b)), although

TABLE 1: Relative errors of the peak value ( $E_p$ ) calculated by our approximate method for different earth conductivity ( $\sigma$ ) varying from 0.001 S/m to 4 S/m. The propagation distances ( $d$ ) between lightning and observation point ranges from 200 km to 1500 km.

$\sigma$ (S/m)	$d$ (km)	Peak value (V/m)		Error ( $(E_{p\text{our}} - E_{p\text{wait}})/E_{p\text{wait}}$ ) (%)
		Wait's formula ( $E_{p\text{wait}}$ )	Our approximate formula ( $E_{p\text{our}}$ )	
4	200	1.070	1.070	0.0
	500	0.349	0.349	0.0
	1000	0.111	0.111	0.0
	1500	0.046	0.046	0.0
0.01	200	1.068	1.066	-0.2
	500	0.350	0.346	-1.0
	1000	0.113	0.110	-2.4
	1500	0.047	0.045	-3.3
0.001	200	1.030	1.024	-0.6
	500	0.330	0.321	-2.6
	1000	0.108	0.101	-6.2
	1500	0.046	0.042	-8.7

TABLE 2: Errors of the waveform rise time (RT) of the vertical electric field calculated by our approximate method for different earth conductivity ( $\sigma$ ) and propagation distances ( $d$ ).

$\sigma$ (S/m)	$d$ (km)	Waveform rise time ( $\mu\text{s}$ )		Error ( $(\text{RT}_{\text{our}} - \text{RT}_{\text{wait}})/\text{RT}_{\text{wait}}$ ) (%)
		Wait's formula ( $\text{RT}_{\text{wait}}$ )	Our approximate formula ( $\text{RT}_{\text{our}}$ )	
4	200	6.7	6.7	0.0
	500	8.2	8.1	-1.2
	1000	11.4	11.5	0.9
	1500	15.7	15.7	0.0
0.01	200	7.7	7.8	1.3
	500	9.7	9.7	0.0
	1000	13.7	13.7	0.0
	1500	18.6	18.3	-1.6
0.001	200	10.3	10.3	0.0
	500	13.8	13.7	-0.7
	1000	19.8	19.3	-3.0
	1500	26.2	25.2	-3.8

the phase angles of  $W_\sigma$  and  $W_\rho$  are much different from  $W_{\text{wait}}$ , the value of  $W_\sigma W_\rho$  (our approximate formula) is very similar to that of  $W_{\text{wait}}$ . Therefore, in the frequency domain, we also see that our approximate formula is valid, and its amplitude and phase are both similar to those of  $W_{\text{wait}}$ .

*3.2. Dependence of the Lightning-Radiated ELF/VLF Electric Field Value on the Propagation Distance.* Figure 7 shows the field peak values within the field propagation distance of 1500 km for different ground conductivities ranging from 0.001 S/m to 4 S/m. The solid curves in Figure 7 show the fitted dependences of ground wave peak value on the propagation distance for the planar earth and the spherical earth. For the planar earth, within the distance of hundreds to thousands of kilometers from the lightning strike point, the field peak value ( $E_p$ , V/m) nearly yields

a propagation distance ( $d$ , km) dependence of  $d^{-1}$ , which is similar to that along the perfectly conducting ground or free space. However, it is worth noting that the field value propagating along the finitely conducting ground is still smaller than that for the perfectly conducting ground, especially when the ground conductivity is lower than 0.01 S/m. For the spherical earth, the field peak value ( $E_p$ , V/m) yields a propagation distance ( $d$ , km) dependence of  $d^{-1.32}$ . It is clear that the propagation attenuation is larger for spherical earth than for planar earth. This is mainly because the electromagnetic waves are prevented by the bulge of the earth from passing directly to the observation point, and the waves must reach the observation point by a process of bending around the curved surface of the earth, which will cause some extra field attenuation [31]. Therefore, in the lightning-radiated VLF field propagation at intermediate ranges of hundreds to thousands of

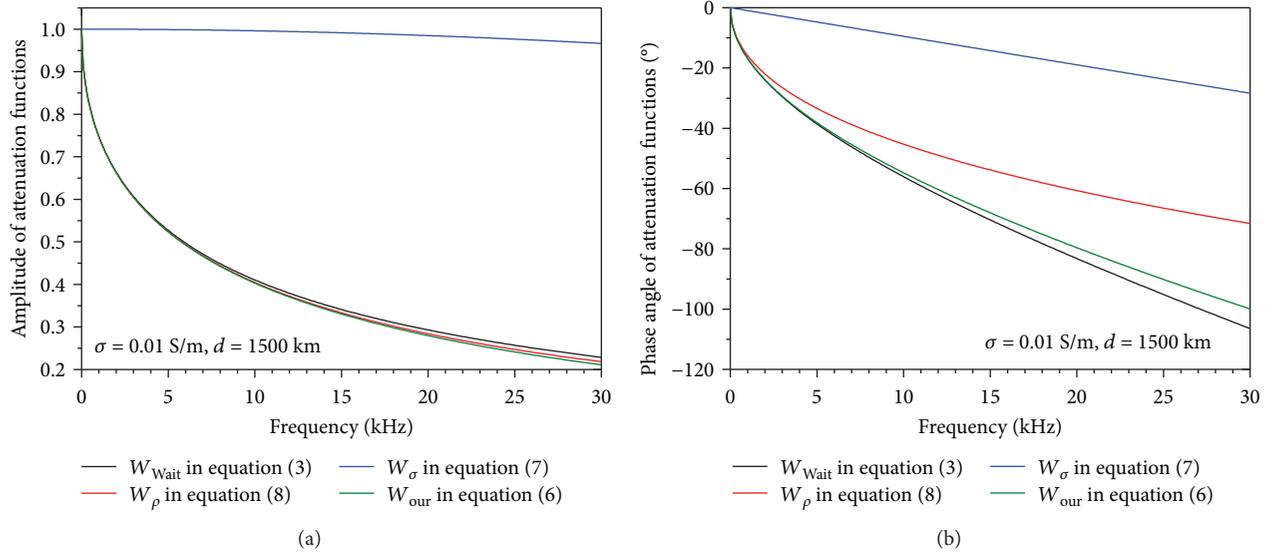


FIGURE 6: The (a) amplitude and (b) phase angle of attenuation functions ( $\sigma = 0.01$  S/m and  $d = 1500$  km).  $W_{\text{wait}}$  is the attenuation function proposed by Wait [3, 4], accounting for the effect of earth conductivity and curvature. Attenuation function  $W_{\rho}$  only accounts the effect of the earth curvature and  $W_{\sigma}$  only accounts the effect of the earth conductivity.  $W_{\text{our}} = W_{\sigma} W_{\rho}$  is the attenuation function calculated using our approximate method.

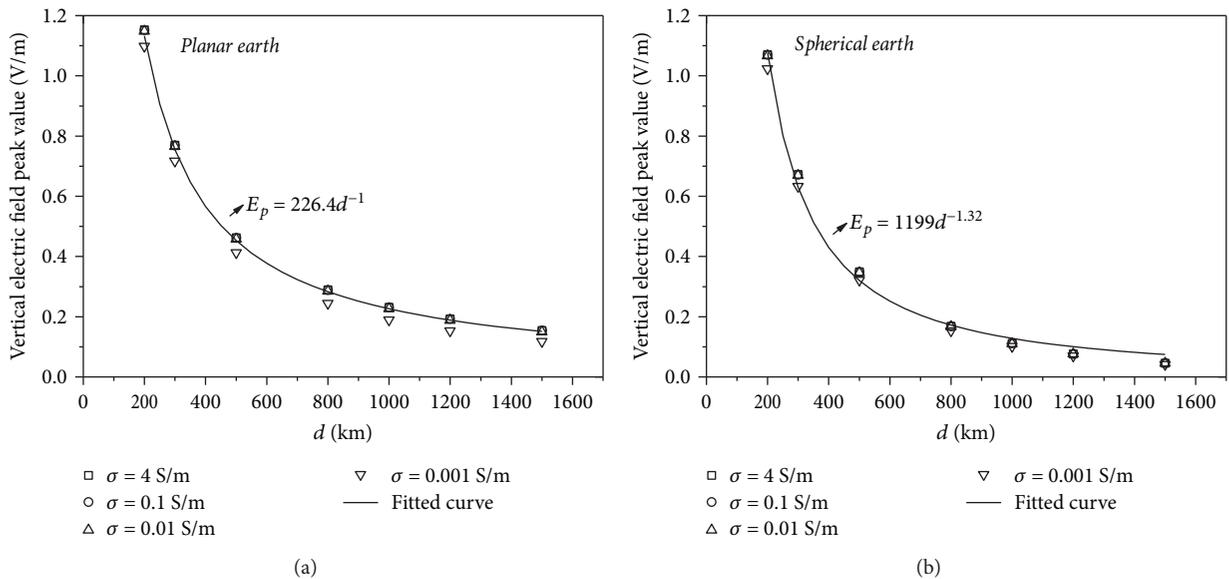


FIGURE 7: The lightning-radiated vertical electric field peak value as a function of propagation distance for (a) the planar earth and (b) the spherical earth. The lightning current source shown in Figure 2 is adopted. The solid curves are the fitted results of average field peak value for earth conductivity ranges from 0.001 S/m to 4 S/m.

kilometers, we should pay more attention to the effect of Earth's curvature.

#### 4. Conclusion and Discussion

Lightning discharges can radiate electromagnetic waves over a wide frequency range from a few Hz to many tens of MHz, but most of the electromagnetic energy is radiated in the ELF and VLF bands, and the higher frequency component is attenuated rapidly as the propagation distance increases

[22, 23, 32]. In this paper, we present a new approximate method for lightning-produced ELF/VLF ground wave propagation over intermediate ranges, which is validated by using Newton-Raphson root-finding method presented by Shao and Jacobson [5] for propagation path with different ground conductivities; we found our approximate method could predict the field peak and waveform rise time with satisfactory accuracy. The ground conductivity has little effect on the lightning-radiated ELF/VLF field peak at the intermediate ranges when it is larger than 0.01 S/m. However, Earth's

curvature has much more effect on the field propagation at the intermediate ranges than the finite conductivity. For example, the lightning-produced field peak of ELF/VLF frequency propagating over a spherical earth is just about 30%–40% of that propagating over a planar earth at a distance of 1500 km and 75%–80% for a distance of 500 km. For the planar earth, the field peak value ( $E_p$ , V/m) nearly yields a propagation distance ( $d$ , km) dependence of  $d^{-1}$ ; however, for spherical earth, the field peak value ( $E_p$ , V/m) yields a propagation distance ( $d$ , km) dependence of  $d^{-1.32}$ . Therefore, for the lightning-radiated VLF field propagation at intermediate ranges of hundreds to thousands of kilometers, we should pay more attention to the effect of Earth's curvature; for example, the current moment peak value predicted from the measured far field peak will be underestimated when using a far-field-current relationship at the assumed planar earth.

### Additional Points

**Key Points.** A new approximate method for lightning-radiated ELF/VLF ground wave propagation over intermediate ranges is presented. Lightning-radiated ELF/VLF electric field peak value at the intermediate ranges yields a propagation distance ( $d$ ) dependence of  $d^{-1.32}$ . We should pay more attention to the effect of earth curvature when determining the lightning discharge parameters.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

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