Research Article

Research on Side Lobe Suppression of Time-Modulated Sparse Linear Array Based on Particle Swarm Optimization

Lei Liang , Jie Sun, Hailin Li , Jialing Liu, Yachao Jiang , and Jianjiang Zhou

College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, No. 29 Jiangjun Avenue, 211100 Nanjing, China

Correspondence should be addressed to Lei Liang; yushan_ll@163.com and Hailin Li; nuaahlhs@nuaa.edu.cn

Received 18 January 2019; Revised 15 May 2019; Accepted 15 July 2019; Published 7 August 2019

Academic Editor: Rodolfo Araneo

Copyright © 2019 Lei Liang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

An efficient pattern synthesis approach is proposed for the synthesis of a time-modulated sparse linear array (TMSLA) in this paper. Due to the introduction of time modulation, the low/ultralow side lobe level can be obtained with a low amplitude dynamic range ratio. Besides, it helps reduce the difficulty of antenna feeding system effectively. Based on particle swarm optimization (PSO) and convex (CVX) optimization, this paper proposes a hybrid optimization method to suppress the grating lobes of the sparse arrays, peak side lobe level (PSLL), and peak sideband level (PSBL). Firstly, the paper utilizes the CVX optimization as a local optimization algorithm to optimize the elements' switch-on duration time, which reduces the side lobe of the array. Secondly, with the PSBL as the objective function, the paper adopts the PSO as a global optimization algorithm to optimize the elements' positions and switch-on time instant, which helps reduce the loss of sideband power caused by time modulation. With respect to the time modulation model, variable aperture sizes (VAS) and more flexible pulse-shifting (PS) schemes are used in this paper. Owing to the introduction of time modulation and CVX optimization, the proposed method is much more feasible and efficient than conventional approaches. Furthermore, it has better array pattern synthesis performance. Numerical examples of the TMSLA and comparisons with the reference are presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Because of its low side lobe level and strong radiation directivity, the antenna array is easy to realize beam scanning and shaping and has been widely used in radars, wireless communication, and other fields. However, a large number of elements in the antenna array increase the structural complexity and feeding difficulty of the antenna system. Therefore, the sparse array with random array elements can achieve beam shaping, low mutual coupling, economical efficiency, and high resolution [1–3]. But the problem of a high side lobe level and gating lobe caused by the sparse array are yet to be solved. At the same time, with the development of radar system and the improvement of anti-jamming capability, in order to achieve a lower side lobe level, the antenna element requires a large dynamic

amplitude ratio, which is larger for the sparse array. This makes the design of the feed network very difficult. Literature studies on the current sparse array pattern synthesis research show that the synthesis of sparse array antennas with a low side lobe and small feed dynamic amplitude ratio has become an important research direction. There are two main directions in the current sparse array pattern synthesis research. One is to optimize the position of the array element only, and the other is to optimize the excitation weight and position of the array element jointly. Combining these two directions, many sparse array optimization algorithms have been derived.

A family of position mutated hierarchical particle swarm optimization algorithms with time-varying acceleration coefficients has been presented in [4]. Its advantage is that the algorithm did not require any controlling parameter. It

simulated a 28-element linear array, and the lowest side lobe level is -26.851 dB with a 14.022 wavelength array aperture. So, each element was adjusted near the position of the 0.5 wavelength, which is not a real sparse array, and the excitation amplitude is not optimized yet. In [5], a type of sparse linear array reconstruction technique based on the matrix pencil method (MPM) and the forward-backward matrix pencil method (FBMPM) is presented, and it gets multiple patterns which consist of a pencil-beam, a flat-top beam, and a cosecant beam. The algorithm not only reduces the array elements but also ensures the accuracy and robustness of the system. In [6, 7], Bayesian algorithm is used to synthesize linear and planar arrays. By optimizing the position and excitation amplitude of array elements, the number of array elements is effectively reduced and the side lobe level is lower.

Evolutionary algorithms are increasingly widely used in the optimization of sparse arrays. Generally, global optimization processes such as genetic algorithm (GA) [8, 9] and differential evolution (DE) [10, 11] are easy to fall into a dead loop; thus, the result of synthesis was a local optimal solution. Meanwhile, the computational complexity of these global optimization techniques increases exponentially with the increase of unknowns, so these processes cannot be guaranteed to reach the optimal solution in a reasonable time. A sparse array was achieved by time modulation to control the on-off of array elements [12]. At the same time, the array configuration suitable for radar application was selected by combining genetic algorithm. However, the sideband radiation caused by element switch operation was not effectively suppressed, which would reduce antenna radiation efficiency.

A modified differential evolution (DE) algorithm based on harmony search algorithm was proposed in [13]. It combines the good local search capability of the classic DE and the great search diversity of harmony search algorithm to optimize the position of partial arrays. In this paper, only partial elements of the array are optimized, and thus, more research studies can be done to make the side lobe level of the array further reduced by global optimization of the array elements. Constrained by the minimum element spacing of 0.5 wavelengths, a two-step method was proposed to optimize the position of 17 elements to achieve a peak side lobe level of -20.59 dB and an optimized array aperture of 9.744 wavelengths [14]. Compared with the uniform array with 0.5 wavelengths spacing, the optimized aperture increases by only 1.244 wavelengths, indicating that the optimization effect was limited. A dynamic constrained multiobjective algorithm was applied to the sparse array to achieve the goal of a low side lobe and small bandwidth [15]. The minimum normalized amplitude weighted by this algorithm was 0.3850 and the maximum was 0.91. The difference was as high as 52.5%, which will bring great trouble to hardware implementation. In [16], the PSO algorithm was used to optimize the position and excitation of the array and to reduce the side lobe level at a faster speed while ensuring the performance of the main beam, but the proposed algorithm did not optimize the location and excitation jointly. None of the above studies

avoided the problem that computational results can be easily trapped in local optimum solutions.

Since the optimal solution of convex optimization is global optimal solution, more scholars use convex optimization in sparse array synthesis. Fuchs used convex optimization to realize beamforming in sparse arrays. The convex problem solved iteratively was transformed into a standard second-order cone programming (SOCP) for calculation, which achieved simultaneous optimization of element position and excitation, thus suppressing the grating lobes and obtaining the desired beam [17]. You et al. also used alternating convex optimization to optimize the position of antenna elements and suppress the side lobe level in [18]. In these two papers, the array aperture formed by 22 elements in the literature was only 9.66 wavelengths, so the average spacing of array elements is 0.439 wavelengths, which increases the risk of mutual coupling among array elements and the complexity of the system. A flat-top beam pattern is also synthesized in [18]. These papers validated the effectiveness of convex optimization algorithm in side lobe suppression and beamforming. Perturbed compressive sampling (PCS) and convex optimization were proposed to realize sparse array synthesis [19]. Array position optimization was achieved by PCS, while convex optimization was used to optimize the excitation amplitude, which ensures the array gain and side lobe level and reduces the number of array elements. However, the paper failed to give a specific description of the incentive magnitude.

The algorithm mentioned above will increase the feed dynamic range ratio (DRR) of the antenna system. In order to reduce it, a time-modulated array with low complexity has been extensively studied in recent years. By optimizing the amplitude excitation of the sparse array in the variable aperture sizes mode, Poli et al. obtained a lower side lobe level and effectively suppressed the sideband level [20]. In [21], pulse-shifting time modulation was used to optimize the switch-on duration time of the array element. The sideband level was effectively suppressed, under side lobe constraints and fixed element positions. However, the influence of a random distribution of array elements on the sideband level was not discussed.

According to the analysis of the above research, this paper proposes a joint optimization algorithm based on PSO and time modulation in order to solve the problems of high side lobe and gating lobe caused by the sparse arrays and high dynamic excitation amplitude ratio in the optimization process. The algorithm establishes the mathematical model of the time-modulated sparse array based on convex optimization theory, converts the amplitude and phase weights of the array elements into the optimal switch-on duration time, and reduces the dynamic amplitude ratio of the array. The PSO algorithm is used to optimize the position and switch-on time instant of the array elements; meanwhile, because the solution of convex optimization is the global optimal solution, the global optimal switch-on duration time can be obtained on the condition that the position of array elements and the switch-on time instant are fixed and the ultralow side lobe can be realized ultimately. At the same time, under the

constraints of side lobe level and bandwidth, the sideband level caused by time modulation is reduced.

2. Optimization Model

In this paper, an array of N isotropic elements of the sparse linear antenna array has been evaluated as shown in Figure 1. According to the mathematical formula mentioned in [22], the array factor without noise interference can be obtained as

$$F(\theta) = \sum_{n=1}^N \dot{I}_n e^{jkx_n \sin \theta}, \quad (1)$$

where $x_n (n = 1, 2, \dots, N)$ denotes the distance from each element to the origin of the coordinate, $k = 2\pi/\lambda$ is the propagation coefficient in free space, λ is the wavelength, and $\dot{I}_n = I_n e^{j\alpha_n}$, including amplitude excitation I_n and phase excitation α_n , is the static excitation of the n th element.

When the time modulation with a period of T_p is introduced into the linear array, the periodic switching function of n th element $U_n(t)$ is transformed from the time domain to the frequency domain by Fourier series expansion. The expanded formula can be written as

$$U_n(t) = \sum_{m=-\infty}^{+\infty} a_{mn} e^{j2\pi m f_p t}, \quad m = 0, \pm 1, \pm 2, \dots, \pm \infty, \quad (2)$$

where $f_p = 1/T_p$ is the time modulation frequency of the antenna array and $m (m = 0, \pm 1, \pm 2, \dots, \pm \infty)$ is the number of sideband. The m th sideband frequency is $f_m = f_0 + m f_p$, (at the center frequency $m = 0$). The coefficient of the switching function is $a_{mn} = 1/T_p \int_0^{T_p} U_n(t) e^{-j2\pi m f_p t} dt$. Under different time modulation modes, the switching functions $U_n(t)$ of the array elements are expressed in a different representation.

The far-field radiation electric field intensity at the center frequency f_0 and the sideband frequency f_m is given as follows [23]:

$$F_m(\theta, t) = e^{j2\pi(f_0+f_m)t} \sum_{n=1}^N w_{mn}^* e^{jkx_n \sin \theta} = W^H B_{\theta m}, \quad (3)$$

where $w_{mn} = \dot{I}_n a_{mn}$ is the complex excitation of the n th element in the m th sideband. $W_m = [w_{m1}, \dots, w_{mN}]^T$ is the complex excitation vector, $(\cdot)^T$ represents the transpose of matrix, and $B_{\theta m} = e^{j2\pi(f_0+f_m)t} [e^{jkx_1 \cos \theta}, e^{jkx_2 \cos \theta}, \dots, e^{jkx_N \cos \theta}]^T$ is the steering vector.

The peak side lobe level (PSLL) is defined as

$$\begin{aligned} \text{PSLL} &= \max_{\theta \in \Omega_s} \left\{ \frac{|F_0(\theta, t)|}{\max(|F_0(\theta_M, t)|)} \right\} \\ &= \max_{\theta \in \Omega_s} \left\{ \frac{|W^H B_{\theta 0}|}{\max(|W^H B_{\theta_M 0}|)} \right\}, \end{aligned} \quad (4)$$

where θ_M is the main radiation direction and Ω_s is the side lobe area of the antenna array pattern. In order to obtain low side lobe pattern synthesis, the optimized objective function can be expressed as

FIGURE 1: Diagram of a linear sparse array with N elements.

$$\begin{aligned} \min \quad & \max_{\theta \in \Omega_s} |W^H B_{\theta 0}|, \\ \text{s.t.} \quad & |W^H B_{\theta_M 0}| = 1, \\ & \lambda/2 < x_{n+1} - x_n \leq \lambda, \quad n = 1, \dots, N-1. \end{aligned} \quad (5)$$

The introduction of time modulation leads to a high sideband level, which reduces the radiation power. Therefore, it is necessary to minimize the sideband level on the premise of constraining the side lobe level. The sideband level can be defined as

$$\text{PSBL} = \max_{\theta, m} \left\{ \frac{|F_{\theta m}(\theta, t)|}{\max(|F_{\theta 0}(\theta, t)|)} \right\} = \max_{\theta, m} \left\{ \frac{|W^H B_{\theta m}|}{|W^H B_{\theta_M 0}|} \right\}. \quad (6)$$

Its objective function can be modified to

$$\begin{aligned} \min \quad & \max_{\theta, M} |W^H B_{\theta m}|, \\ \text{s.t.} \quad & |W^H B_{\theta_M 0}| = 1, \\ & |W^H B_{\theta 0}| \leq \sigma, \quad \theta \in \Omega_s, \\ & \lambda/2 < x_{n+1} - x_n \leq \lambda, \quad n = 1, \dots, N-1, \end{aligned} \quad (7)$$

where σ is the constraint value of PSLL, which depends on the requirements of different projects.

3. Optimization Algorithm

Because the space of the adjacent element is usually larger than half wavelength in the sparse array, optimizing the array element time sequences alone cannot effectively suppress the gate lobe and reduce the side lobe. Thus, based on the application of time modulation technology, the position of array elements and the switch-on duration time of array elements are taken as joint optimization variables to increase the dimension of optimization variables. However, with the introduction of an element position variable, the optimization problem will become a high-dimensional nonlinear nondeterministic polynomial hard (NP-hard) problem, which is no longer a standard convex optimization problem. Considering that the local optimal solution is the global optimal solution in convex optimization problems, the optimization model of the PSO-CVX hybrid algorithm is given by combining particle swarm optimization and convex optimization. The convex optimization algorithm is used to solve the optimal excitation and switch-on duration time

when the element position is determined. The PSO algorithm iteratively solves the optimal element position when the optimal excitation and switch-on duration time are given. Then, the optimization problem is transformed into a local convex optimization problem to optimize the antenna pattern of the sparse array.

3.1. Time Modulation Algorithm. Variable aperture size (VAS) modulation makes the aperture size of the uniform linear array change periodically, and the RF switch corresponding to each antenna unit closes simultaneously in the beginning of each timing cycle T_p . In the VAS mode, the complex excitation and the steering vector of the main frequency band and the sideband in formulas (5) and (7) are as follows:

$$\begin{aligned} W &= [I_1 \tau_1, \dots, I_N \tau_N]^T, \\ B_{\theta_0} &= (e^{jkx_1 \sin \theta}, \dots, e^{jkx_N \sin \theta})^T, \\ B_{\theta_m} &= \left(\frac{\sin(\pi m \tau_n)}{\pi m \tau_n} \cdot e^{j[kx_1 \sin \theta - \pi m \tau_1]}, \dots, \frac{\sin(\pi m \tau_n)}{\pi m \tau_n} \right. \\ &\quad \left. \cdot e^{j[kx_n \sin \theta - \pi m \tau_n]} \right)^T, \end{aligned} \quad (8)$$

where τ_n ($0 < \tau_n < 1$) is the normalized switch-on duration time.

Pulse shifting (PS) was proposed by Poli et al., an Italian scholar [21]. The turn-on time and duration of each antenna element are adjustable, which is no longer limited to the simultaneous turn-on state in the VAS mode. In the PS mode, the complex excitation and the steering vector of the main frequency band and the sideband in formulas (5) and (7) are as follows:

$$\begin{aligned} W &= [I_1 \tau_1, \dots, I_N \tau_N]^T, \\ B_{\theta_0} &= (e^{jkx_1 \sin \theta}, \dots, e^{jkx_N \sin \theta})^T, \\ B_{\theta_m} &= \left(\frac{\sin(\pi m \tau_n)}{\pi m \tau_n} \cdot e^{j[kx_1 \sin \theta - \pi m (2t_1 + \tau_1)]}, \dots, \frac{\sin(\pi m \tau_n)}{\pi m \tau_n} \right. \\ &\quad \left. \cdot e^{j[kx_n \sin \theta - \pi m (2t_n + \tau_n)]} \right)^T, \end{aligned} \quad (9)$$

where t_n ($0 \leq t_n < 1$) is the switch-on time instant.

Comparing formulas (8) with (9), we can see that the complex excitation of VAS modulation and PS modulation is the same as the steering vector at the center frequency. And, the pattern of the antenna array at the center frequency is only related to τ_n . PS modulation adds switch-on time instant t_n to VAS modulation. By optimizing variables t_n and τ_n reasonably, the amplitude excitation value of array elements can be optimized, and the desired pattern can be obtained.

3.2. PSO-CVX Algorithm. According to the definition of the convex optimization algorithm, formula (5) is a convex optimization model under the premise that the position of array elements is fixed, and the objective function of side lobe suppression can be obtained as follows:

$$\begin{aligned} \min_{W, X} \quad & \max_{\theta \in \Omega_s} |W^H B_{\theta_0}|, \\ \text{s.t.} \quad & |W^H B_{\theta_M 0}| = 1, \\ & 0 < \tau_n \leq 1, \end{aligned} \quad (10)$$

where $W = [I_1 \cdot e^{j\alpha_1 \tau_1}, I_2 \cdot e^{j\alpha_2 \tau_2}, \dots, I_N \cdot e^{j\alpha_N \tau_N}]^T$ is the excitation weight, $B_{\theta_M 0} = (e^{jkx_1 \sin \theta_M}, \dots, e^{jkx_N \sin \theta_M})^T$ is the steering vector of main radiation direction θ_M at the central frequency band f_0 , and $B_{\theta_0} = (1, e^{jkx_1 \sin \theta}, \dots, e^{jkx_N \sin \theta})^T$ is the steering vector of side lobe region at the central frequency band f_0 . $\text{fitness} = \max_{\theta \in \Omega_s} |W^H B_{\theta_0}|$ is the fitness function.

In order to achieve sideband suppression in the time modulation sparse array, the variables needed to be optimized are the normalized switch-on time instant t_n , switch-on duration time τ_n , and position variables x_n of element. Particle swarm optimization (PSO) is used to optimize the position and turn-on time of the array elements. The optimization model of formula (7) is implemented by the convex optimization algorithm. The objective function of sideband suppression can be obtained as follows:

$$\begin{aligned} \min \quad & \max_{\theta, M} |W^H B_{\theta_M}|, \\ \text{s.t.} \quad & |W^H B_{\theta_M 0}| = 1, \\ & \max_{\theta \in \Omega_s} |W^H B_{\theta_0}| \leq \alpha, \end{aligned} \quad (11)$$

where W , $B_{\theta_M 0}$, and B_{θ_0} are the same as those of formula (10), α is the constraint value of PSLL, which depends on different the requirements of different projects, and B_{θ_M} which is the same as that in formula (9), is a complex steering vector in the sideband region at the first harmonic frequency modulated by PS. When using VAS modulation, B_{θ_m} is the same as that in formula (8); $\text{fitness} = \max_{\theta, M} |W^H B_{\theta_M}|$ is the fitness function.

Scholars Clerc and Kennedy proposed a compression factor method as a new speed update formula for the PSO algorithm [24]. The formula is as follows:

$$\begin{aligned} v_{id}^{t+1} &= \text{CFa} \times [v_{id}^t + \varphi_1 \cdot r_1 \cdot (p_{id}^t - x_{id}^t) \\ &\quad + \varphi_2 \cdot r_2 \cdot (p_{gd}^t - x_{id}^t)], \end{aligned} \quad (12)$$

where P_{id} and P_{gd} are iteration optimum values of particles in the single optimization process and global optimum values of all particles, respectively, x_{id}^t and v_{id}^t denote the d th position component and velocity component of the i particle in the t th iteration, and r_1 and r_2 represent independent random numbers in the range $[0, 1]$. The definition of the constriction factor (CFa) is

$$CFa = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|}, \quad (13)$$

$$\varphi = \varphi_1 + \varphi_2, \quad \varphi > 4.$$

Usually, φ_1 and φ_2 change in the range of [1.5, 2.05], and the corresponding optimal values are different when the optimization objective is different. A lot of research results show that ideal optimization results can be obtained, when $\varphi = \varphi_1 = \varphi_2 = 2.05$.

3.3. The Procedure of PSO-CVX Algorithm. This paper combines the advantages of PSO and convex optimization algorithms. Because of its high efficiency and simplicity, the PSO algorithm is used as a global optimization algorithm. The optimal element position and turn-on time are obtained by particle iterative evolution. Considering that the convex optimization algorithm has the characteristic that the local optimal solution is the global optimal solution, the CVX algorithm is used as the local optimization algorithm to obtain the optimal weight variable W_{best} . All these features guarantee that W_{best} is the global optimal with the best local position and turn-on time. Figure 2 is the corresponding flow chart of detailed descriptions of the PSO-CVX algorithm.

- (1) Set the number of array elements, the lobe width of zero power point, and the sampling interval of angle for array model
- (2) Initialize the particle population, position, iterations, and speed of each particle
- (3) Calculate the value of the steering vector
- (4) WHILE the maximum number of cycles has not been reached, DO
 - (a) Obtain weight coefficients of each array element by the CVX algorithm
 - (b) Compute fitness values
 - (c) Update global optimal solution and historical optimal solution
 - (d) Update positions and velocities of the particles
 - (e) Increase the loop counter
- (5) End cycle and display the best results of pattern synthesis

4. Simulation Results

When the number of array elements is constrained in the limited space platforms such as aircraft and ships, in order to improve the performance of the antenna array and restrain the grating lobe caused by the spacing of array elements larger than $\lambda/2$, it is necessary to adopt the sparse array structure of array elements. Therefore, this paper focuses on the research and simulation of small scale antenna array with limited layout space. Sparse ratio is a measurement of the sparsity of a sparse array, which is defined as follows:

FIGURE 2: Flow chart of PSO-CVX.

$$\gamma = \left(1 - \frac{N}{M}\right) \times 100\%, \quad (14)$$

where N is the number of elements in the antenna array and M is the equivalent number of elements which are arranged at a half-wavelength spacing in the antenna array.

Standard deviation is a measure of the discreteness of the test results, which can reflect the stability of the optimization algorithm. Its formula is as follows:

$$\sigma = \sqrt{\frac{\sum_{k=1}^K (X_k - \bar{X})^2}{K}}, \quad (15)$$

where σ is the standard deviation, K is the number of independent experiment, X_k is the result of the k th experiment, and $\bar{X}$ is the average of the results of all experiments.

In order to prove the superiority of the algorithm mentioned above, this section gives several simulation and comparison results of the time modulation sparse linear array optimized by the PSO-CVX algorithm. Firstly, the TMSLA is compared with the time-modulated linear uniform array, traditional linear uniform array, and Chebyshev array to verify the effect of time modulation on side lobe suppression. Then, the sparse array optimized by the PSO-CVX algorithm is compared with the references from different aspects. Secondly, this section analyzes the influence of phase change on the TMSLA. Finally, by optimizing the position, the switch-on duration time, and the switch-on time instant of array elements, the sideband level suppression of the TMSLA is simulated and compared with the algorithms in some references.

4.1. Simulations of the Side Lobe Suppression. The simulation in this section adopts the time modulation mode of VAS. As

mentioned in Section 3.1, it has the same steering vector as PS modulation at the central frequency, so the side lobe suppression effect is the same.

The antenna array consists of 17 elements, and the central frequency is 5×10^9 Hz. Because the sparse array element spacing is usually larger than $\lambda/2$, and it will produce a higher grating lobe. Considering the grating lobe problem of the array, the position parameters of the array are added, and the position of the antenna is optimized by the PSO algorithm. The array arrangement is centrosymmetric, so the position variable of the element to be optimized is 8. The particle number is 100, the maximum number of iterations is 100, and the time modulation period is 10 ms. In order to reduce the randomness of array arrangement and avoid the occurrence of too narrow or too wide spacing between adjacent elements, the spacing between adjacent elements is limited between $[\lambda/2, \lambda]$. At the same time, considering the switching time of the RF switch, the switch-on duration time of the array element is limited between $[0.01T_p, T_p]$.

Figure 3 shows the comparison among the simulation results of the TMLA, TMSLA, Chebyshev optimization and not the optimized normalized antenna pattern. Except for the TMSLA, the element spacing of other algorithms is $\lambda/2$. In this figure, the PSLL of the traditional linear array is -13.17 dB, and zero power bandwidth of the main lobe is 13.6° . The PSLL of the TMLA is -20.77 dB, which coincides with the array pattern optimized by Chebyshev. With the introduction of time modulation, the PSLL decreases by 7.6 dB while the main lobe width remains unchanged. The optimum PSLL of the TMSLA is -36.54 dB, which is 15.77 dB lower than that of the uniform linear array and Chebyshev array. Therefore, the method of optimizing the antenna array by changing the excitation amplitude can be replaced by adjusting the switch-on duration time of time modulation technology while keeping the pattern consistent. Also, better optimization results can be obtained by adding location variables. The above analysis shows that the introduction of the time modulation technology effectively improves the side lobe suppression ability of the antenna system and avoids the problem that high dynamic amplitude ratio is difficult to achieve in a hardware circuit.

Figure 4 shows the optimal switch-on duration time of the TMSLA. For the uniform linear uniform array with symmetrical distribution, the optimal timing obtained by the CVX optimization algorithm shows a trend that the closer to the center of the array, the longer the switch-on duration time of the array elements, and the switch-on duration time series is centrosymmetric.

The PSO-CVX algorithm proposed in this paper has been tested 20 times independently for the TMSLA. Figure 5 shows the fitness curve statistical result of the PSO-CVX hybrid algorithm. It can be seen that the PSO-CVX algorithm proposed in this paper achieves stability after about 70 iterations, which illustrates the efficiency of the PSO-CVX algorithm. The optimum peak side lobe level is -36.54 dB, the worst of which is -35.50 dB, and the average of which is -35.78 dB. The difference between the results of optimization is small, which reflects the stability of the optimization effect of the algorithm.

FIGURE 3: TMLA, TMSLA, Chebyshev, and traditional patterns.

FIGURE 4: Histogram of normalized switch-on duration time distribution of 17-element TMSLA.

FIGURE 5: Fitness curve of the PSO-CVX algorithm.

Using the same parameters to simulate on the same computer, the PSL of antenna array is -31.26 dB by optimizing the excitation amplitude and position with the PSO algorithm, which is 5.28 dB higher than that with the PSO-CVX algorithm. Meanwhile, the beam width of 3 dB is 5.4 degree, which is 0.8 degree larger than that optimized by the PSO-CVX algorithm, as shown in Figure 6. In one optimization process, the time cost of the PSO-CVX algorithm and the PSO algorithm is 1026.2 seconds and 16.5 seconds, respectively. Subsequently, the number of iterations of the PSO algorithm was changed from 100 to 7500 and another simulation was made. The time cost of the PSO algorithm was 1192.3 seconds, but the side lobe level was not lower than -31.26 dB. Compared with the PSO algorithm, the PSO-CVX algorithm requires more time cost, but helps realizing the antenna radiation pattern with a lower side lobe level, and further demonstrates the advantage of the time-modulated sparse array in realizing the low/ultralow side lobe level.

Figure 7 is the corresponding element positions of the TMSLA optimized by the PSO-CVX algorithm in VAS strategy. The aperture width is 13.7926λ . The position distribution of the array elements is generally uniform, and there is no case of too narrow or too wide. Calculated by formula (14), the sparse ratio is 41.4% .

In order to further verify the superiority of time modulation technology, some simulations are made to compare the time modulation sparse array with the 16-element local sparse array by using the exploratory harmony search (EHS) optimization algorithm mentioned in [25]. Their array arrangement is centrosymmetric, and the spacing of 10 elements near the center is $\lambda/2$. The remaining six positions are optimized and the remaining parameters of the array are the same. The optimized result is as shown in Table 1. The value of standard deviation (SD) is calculated by formula (15).

The statistical results in Table 1 show that under the same parameters, the best and worst PSL of the sparse array based on PSO-CVX is 3.9 dB and 3.63 dB lower than that in [25] at the cost of the standard deviation of 0.11 . Compared with those in [25], the optimized array aperture is 9.392 wavelength, which is 1.108 wavelength less, and the -3 dB bandwidth is 5.4 degrees, which is 5 degrees less in this paper. After only 40 iterations, the above results are achieved, which demonstrates the superiority of time modulation technology in side lobe suppression of antenna array.

The modified Bayesian optimization algorithm (M-BOA) proposed in [6] can get a lower peak side lobe level than the differential evolution algorithm and the modified genetic algorithm. In this paper, we use the same parameters as in [6]. The number of elements is 37 . The spacing of the array element is greater than $\lambda/2$, and the aperture of arrays is not greater than 24λ . The algorithm contains 100 independent particles. Twenty independent experiments are carried out with 1000 iterations in each experiment. The comparison between the sparse linear array based on PSO and modified Bayesian optimization arrays is shown in Figures 8 and 9. In Figure 9(a), the optimal, worst, and average fitness curves of the TMSLA are basically the same, and they reach a stable state after 10 iterations, while the fitness curve optimized by

FIGURE 6: Contrast between the PSO-CVX and PSO algorithms.

FIGURE 7: Diagram of optimized array layout of PSO-CVX.

M-BOA algorithm reaches a stable state after 800 iterations, as shown in Figure 9(b). The PSL optimized by PSO-CVX is 2.2 dB lower than that of M-BOA. Detailed comparisons are shown in Table 2.

In [16], the standard PSO algorithm is applied to sparse linear arrays. The position and excitation amplitude of arrays with a different number of elements are optimized, respectively. The peak side lobe level by optimized spacing is -27.6 dB, and that by optimized excitation amplitude is -35.3 dB. In this paper, we adopt the same parameters as the array in [16]. The simulation results show that the peak side lobe level is 39.12 dB, and the array aperture is 18.08λ . The comparison of parameters between two algorithms is shown in Table 3. The sparse rate of the array obtained by the algorithm in this paper is consistent with that in [16], which is 31.4% . At the same time, it is 3.82 dB lower than the lowest side lobe level -35.3 dB in [16], as shown in Figure 10. The fitness curve in [16] reaches a stable state after 200 iterations, as shown in Table 3, and that in this paper reaches a stable state after 100 iterations.

4.2. Simulation of the Effect of Phase Change on Array Performance. In order to verify the influence of phase change on the peak side lobe level, the peak sideband level, the switch-on duration time, the and sparse ratio of array, some simulations are made in the PS mode for in-phase and out-phase sparse arrays, respectively. Considering a 17-element symmetric sparse array with a peak side lobe level constraint of -30 dB, the optimization objective is to minimize the peak sideband level of the first and second

TABLE 1: PSLL comparison between the array mentioned in [25] and time-modulated sparse array.

Comparison object	Optimal PSLL (dB)	Worst PSLL (dB)	SD of PSLL	-3 dB bandwidth (°)	Optimized location (right side)		
[25]	-13.59	-13.47	0.07	10.4	3.66	4.36	5.25
PSO-CVX	-17.49	-17.1	0.18	5.4	2.99	3.83	4.696

FIGURE 8: Pattern comparison of PSO-VAS and M-BOA.

sideband. The normalized turn-on time t_n is $[0, 1]$, and the other element parameters are set exactly the same as those in Section 4.1.

4.2.1. Simulations of the TMSLA with In-Phase Excitation. Selected from 20 independent experiments, the best normalized antenna pattern is shown in Figure 11, which shows the sideband level of a 17-element sparse array with equal amplitude and in-phase excitation, using the PSO-CVX hybrid algorithm in the PS mode. The corresponding PSLL is -31.05 dB, which satisfies the PSLL constraint of -30 dB.

4.2.2. Simulations of the TMSLA with Out-Phase Excitation. The simulation parameters in this section are consistent with those in the previous section, except that the phase excitation of the array element becomes an adjustable optimization variable which participates in the PSO-CVX algorithm. The best normalized antenna pattern is shown in Figure 12. The corresponding peak side lobe level is -30.13 dB, which also achieves a peak side lobe level of -30 dB.

As shown in Table 4, the in-phase excitation array reached a stable state at about 85 iterations, while the out-phase excitation sparse array reached a stable state only after nearly 90 iterations. Obviously, the phase variable slows down the optimization speed. The average PSBL is basically the same. The stability of the sparse array with in-phase excitation is slightly worse. The above results are due to the increase of optimization variables and more complex calculations. The data in Figures 11 and 12 are shown in Table 5. From the statistical results in Table 5, it can be seen that the PSLL optimized by the PSO-CVX algorithm with the PS

mode is -30.13 dB, which realizes -30 dB peak side lobe suppression.

The experimental results of the first 30 sideband levels of the time-modulated sparse array with out-phase excitation and in-phase excitation are compared as shown in Figure 13. The optimized sidebands of the two algorithms are lower than 22.7 dB and show a downward trend. The sideband level of in-phase excitation is lower than that of out-phase excitation, which further confirms that the increase of variable parameters reduces the “robustness” of the algorithm.

In Figure 14, the red “o” indicates the element position of the optimized out-phase excitation antenna array and the blue “x” represents the element position of the in-phase excitation antenna array. The results show that the sparse ratio of both arrays is 32%.

From the above comparison, in terms of element timing and element position arrangement, the difference is little between the simulation results of the time-modulated sparse array with out-phase excitation and in-phase excitation. In addition, due to the introduction of phase excitation variables, the optimization variables and computational complexity of the algorithm increase, resulting in a lower optimization speed and an increase in the number of iterations of the algorithmic convergence.

4.3. Simulation of Sideband Level Suppression. Taking formula (11) as the objective function, the array is simulated with VAS and PS modulation, respectively. The parameters of the array element are exactly the same as those of Section 4.1. The sideband level comparison between the two modulation methods is shown in Figure 15. As seen in Figure 15 and Table 6, the PSO-CVX algorithm in the PS mode adds the expected PSLL as a constraint, and the peak sideband level in the first and second sidebands are 11.5 dB and 11.7 dB lower than those of the PSO-CVX optimization algorithm in the VAS mode, respectively. It illustrates that the PSO-CVX hybrid optimization algorithm in the PS mode can effectively suppress the sideband level and improve the radiation efficiency and gain of the antenna array.

Figure 16 shows the optimal position comparison of the PSO-CVX algorithm under the two time modulation modes. The red “o” indicates the element position of the optimized antenna array in the VAS mode, and the blue “x” represents the element position of the antenna array in the PS mode. The sparse ratio of the sparse array with the PS mode is 32%. Compared with the 41.4% sparse ratio of the VAS modulated array, the sparse ratio of the sparse array in the PS mode reduces by 9.4%. Therefore, in the layout of the array, the PSO-CVX algorithm optimized in the PS mode has narrower spacing, smaller sparse ratio, and more intensive

FIGURE 9: Fitness curve of PSO-VAS and M-BOA. (a) Fitness of PSO-VAS. (b) Fitness of M-BOA.

TABLE 2: Comparison between the time modulation array and modified Bayesian array.

Comparison object	Optimal PSLL (dB)	Worst PSLL (dB)	Average PSLL
M-BOA in [6]	-23.9	-23.15	-23.38
TMSLA	-26.0989	-26.0989	-26.0989

TABLE 3: Performance comparison of the TMSLA and SLA in [16].

Comparison object	Sparse ratio	PSLL	Iterations	Array aperture (λ)	Element spacing (λ)
Array in [16]	31.4%	-35.3 dB	200	18.3	0.5~1
PSO-CVX	31.4%	-39.12 dB	100	18.08	0.5~1

FIGURE 10: PSLL comparison between PSO-CVX and the graph of the optimized excitation amplitude in [16].

FIGURE 11: In-phase excitation pattern of using the PSO-CVX algorithm in the PS mode.

FIGURE 12: Pattern with equal amplitude and out-phase excitation in the PS mode.

TABLE 4: Comparison of robustness between in-phase and out-phase TMSLA.

Excitation mode	Stable iterations	Average PSBL (dB)	Best PSBL (dB)	Worst PSBL (dB)
In-phase	85	-21.32	-22.7	-20.08
Out-phase	90	-21.05	-23.07	-19.38

TABLE 5: Comparison of PSBL between in-phase and out-phase TMSLA.

Excitation mode	Expected PSLL (dB)	Optimized PSLL (dB)	First PSBL (dB)	Second PSBL (dB)
In-phase	-30	-31.05	-22.7	-27.1
Out-phase	-30	-30.13	-23.07	-23.10

FIGURE 13: Sideband level of in-phase and out-phase excitation.

spacing than the PSO-CVX algorithm optimized in the VAS mode. The experimental results are in agreement with the theoretical analysis.

FIGURE 14: Position arrangement of the PS-modulated sparse array with in-phase and out-phase excitation.

In order to further verify the superiority of the optimization algorithm in this paper, it is compared with [21]. In [21], the array elements are arranged with an equal spacing of $\lambda/2$. This paper discusses a sparse array with a sparse spacing of $\lambda/2 \sim \lambda$. The other parameters are the same as those in [21]. The pattern of sparse array simulation in this paper is shown in Figure 17, and the comparison of the first 30 sideband levels between them is shown in Figure 18. The performance parameters of the two patterns are compared as shown in Table 7. It can be seen that by using the same optimization algorithm and maintaining the same side lobe level constraint of -30 dB, the sideband level of sparse arrays is 2.25 dB lower and the beam width of -10 dB is 3.6° narrower than those of uniformly spaced arrays.

FIGURE 15: Sideband level comparison of the time-modulated sparse array with equal amplitude and phase excitation. (a) Comparison of first side band level. (b) Comparison of second side band level.

TABLE 6: Sideband level comparison of two time modulation modes.

Algorithm	Expected PSLL (dB)	Optimized PSLL (dB)	First PSBL (dB)	Second PSBL (dB)
PSO-CVX with VAS	NULL	-36.54	-11.2	-15.4
PSO-CVX with PS	-30	-31.05	-22.7	-27.1

FIGURE 16: Position arrangement comparison of the sparse array with PS modulation and VAS modulation.

In [20], the position of array elements is changed from $\lambda/2$ to a sparse array with unfixed spacing, and the time modulation mode of VAS is adopted. Using the same array parameters as those in [20], the simulation results of the central frequency, the first sideband, and the second sideband are shown in Figure 19. Their parameters are shown in Table 8, and the side lobe constraint is -25 dB. Compared with the -20.08 dB sideband level in [20], the sideband level is -21.32 dB in this paper, which is 1.24 dB lower, and the beam width is 1.3° narrower. Also, the aperture width in this paper is 7.836λ , and the aperture width in [20] is close to 7 wavelengths. The sparse ratio of this paper is 26.3% and that of [20] is 15.4%.

As shown in from Figure 20, the first 20 sidebands of the two time modulation algorithms show a downward trend, ten of which in the PS mode have a lower sideband level than that in the VAS mode. The simulation results show that the two time modulation algorithms have similar robustness.

FIGURE 17: 16-element equal-amplitude in-phase excitation in the PS mode.

5. Conclusions

This paper focuses on the application of the PSO-CVX algorithm in side lobe suppression of the TMSLA. Compared with the traditional uniform array, the introduction of time

FIGURE 18: Sideband level comparison of the sparse array and array with a spacing of $\lambda/2$.

TABLE 7: Parametric comparison of 16-element between TMSLAs in the PS mode and the results in [21].

Comparison object	Modulation method	PSLL (DB)	PSBL (DB)	-10 dB bandwidth (DB)
[21]	PS	-30	-19.5	13
This paper	PS	-30	-21.75	9.4

FIGURE 19: Pattern of 11-element equal-amplitude in-phase excitation in the PS mode.

TABLE 8: Parametric comparison of 16-element between TMSLAs in the PS mode and the results in [20].

Comparison object	Modulation method	PSLL (DB)	PSBL (DB)	-3 dB bandwidth (DB)	Aperture width (λ)
[20]	VAS	-25.0	-20.08	8.5	7.836
This paper	PS	-25.11	-21.32	7.2	6.850

modulation can replace the amplitude excitation with the timing control of array element to effectively suppress the side lobe level of the antenna array due to the introduction of

time modulation. In order to realize the sparse ratio of the array and to avoid the occurrence of gating lobe, the element position variable is introduced, and the optimization model

FIGURE 20: Sideband level comparison in the PS and VAS mode.

becomes a nonlinear high-dimensional complex optimization, which is transformed into a lower-dimensional particle swarm optimization and a locally convex optimization. The PSO algorithm is used to optimize the position variables of the particles, and the CVX algorithm is used to solve the equivalent complex excitation consisting of switch-on duration time and static phase excitation. The pattern optimized by the PSO-CVX hybrid algorithm shows that the introduction of position variables can effectively reduce the side lobe level of the antenna array, and the TMSLA has more advantages in synthesizing low/ultralow side lobe patterns. At the same time, by optimizing the objective function of variables, the degree of freedom of optimization is increased, and the number of optimization variables is effectively controlled. The algorithm reduces the number of iterations needed to converge to the optimal solution, which shows the effectiveness of the algorithm. In addition, under the same side lobe suppression effect, the PSO-CVX algorithm using the PS mode can synthesize a lower sideband level and narrower main beam. The array antenna system has better robustness and overall radiation characteristics.

Data Availability

Previously reported formula (1), (3), (9), and (12) data were used to support this study and are available at DOI: 10.1049/el.2018.0333, DOI: 10.1109/LAWP.2018.2875530, DOI: 10.1049/iet-map.2009.0042, and DOI: 10.1109/LAWP.2014.2299285. These prior studies (and datasets) are cited at relevant places within the text as references [22], [23], [21], and [24].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (Grant nos. NS2016043 and kfjj20180406), the National Natural Science Foundation of China (Grant no. 61671239), and the Aeronautics Science Foundation of China (Grant nos. 2017ZC52036 and 20172752019).

References

- [1] T. Wang, K.-W. Xia, and N. Lu, "Pattern synthesis for sparse arrays by compressed sensing and low-rank matrix recovery methods," *International Journal of Antennas and Propagation*, vol. 2018, Article ID 6403269, 11 pages, 2018.
- [2] X. Wang, M. Amin, X. Wang, and X. Cao, "Sparse array quiescent beamformer design combining adaptive and deterministic constraints," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 11, pp. 5808–5818, 2017.
- [3] K. Liu, X. Tu, X. Li, and Y. Cheng, "Design method of high-efficiency sparse array for ultra-wideband radar," *Electronics Letters*, vol. 52, no. 3, pp. 225–226, 2016.
- [4] R. Bhattacharya, T. K. Bhattacharyya, and R. Garg, "Position mutated hierarchical particle swarm optimization and its application in synthesis of unequally spaced antenna arrays," *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 7, pp. 3174–3181, 2012.
- [5] Y. Liu, Q. H. Liu, and Z. Nie, "Reducing the number of elements in multiple-pattern linear arrays by the extended matrix pencil methods," *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 2, pp. 652–660, 2013.
- [6] B. Van Ha, R. E. Zich, M. Mussetta, and P. Pirinoli, "Linear sparse array synthesis using modified bayesian optimization algorithm," in *Proceedings of the 2013 IEEE Antennas and Propagation Society International Symposium (APSURSI)*, pp. 594–595, Orlando, FL, USA, July 2013.
- [7] F. Viani, G. Oliveri, and A. Massa, "Compressive sensing pattern matching techniques for synthesizing planar sparse arrays," *IEEE Transactions on Antennas and Propagation*, vol. 61, no. 9, pp. 4577–4587, 2013.
- [8] Y. Li and Y. Li, "Investigation on SIW slot antenna array with beam scanning ability," *International Journal of Antennas and Propagation*, vol. 2019, Article ID 8293624, 7 pages, 2019.
- [9] L. Cen, Z. L. Yu, W. Ser, and W. Cen, "Linear aperiodic array synthesis using an improved genetic algorithm," *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 2, pp. 895–902, 2012.
- [10] Z.-K. Chen, F.-G. Yan, X.-L. Qiao, and Y.-N. Zhao, "Sparse antenna array design for MIMO radar using multiobjective differential evolution," *International Journal of Antennas and Propagation*, vol. 2016, Article ID 1747843, 12 pages, 2016.
- [11] X.-K. Wang and G.-B. Wang, "A hybrid method based on the iterative fourier transform and the differential evolution for pattern synthesis of sparse linear arrays," *International Journal of Antennas and Propagation*, vol. 2018, Article ID 6309192, 7 pages, 2018.
- [12] J. Euziere, R. Guinvarc'h, B. Uguen, and R. Gillard, "Optimization of sparse time-modulated array by genetic algorithm for radar applications," *IEEE Antennas and Wireless Propagation Letters*, vol. 13, pp. 161–164, 2014.

- [13] F. Zhang, W. Jia, and M. Yao, "Linear aperiodic array synthesis using differential evolution algorithm," *IEEE Antennas and Wireless Propagation Letters*, vol. 12, pp. 797–800, 2013.
- [14] Y. Jiang, S. Zhang, and Q. Guo, "An effective two-step approach to the synthesis of uniform amplitude linear arrays," *IEEE Antennas and Wireless Propagation Letters*, vol. 16, pp. 437–440, 2017.
- [15] W. Dong, S. Zeng, Y. Wu, D. Guo, L. Qiao, and Z. Liu, "Linear sparse arrays designed by dynamic constrained multi-objective evolutionary algorithm," in *Proceedings of the 2014 IEEE Congress on Evolutionary Computation (CEC)*, pp. 3067–3072, Beijing, China, July 2014.
- [16] N. Ullah, Z. Huiling, T. Rahim, S. ur Rahman, and M. MuhammadKamal, "Reduced side lobe level of sparse linear antenna array by optimized spacing and excitation amplitude using particle swarm optimization," in *Proceedings of the 7th IEEE International Symposium on Microwave, Antenna, Propagation, and EMC Technologies (MAPE)*, pp. 96–99, Xi'an, China, October 2017.
- [17] B. Fuchs, "Synthesis of sparse arrays with focused or shaped beampattern via sequential convex optimizations," *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 7, pp. 3499–3503, 2012.
- [18] P. You, Y. Liu, S.-L. Chen, K. D. Xu, W. Li, and Q. H. Liu, "Synthesis of unequally spaced linear antenna arrays with minimum element spacing constraint by alternating convex optimization," *IEEE Antennas and Wireless Propagation Letters*, vol. 16, pp. 3126–3130, 2017.
- [19] C. Yan, P. Yang, Z. Xing, and S. Huang, "Synthesis of planar sparse arrays with minimum spacing constraint," *IEEE Antennas and Wireless Propagation Letters*, vol. 17, no. 6, pp. 1095–1098, 2018.
- [20] L. Poli, P. Rocca, A. Massa, and M. D'Urso, "Optimized design of sparse time modulated linear arrays," in *Proceedings of the 7th European Conference on Antennas and Propagation (EuCAP)*, pp. 138–141, Gothenburg, Sweden, April 2013.
- [21] L. Poli, P. Rocca, L. Manica, and A. Massa, "Pattern synthesis in time-modulated linear arrays through pulse shifting," *IET Microwaves, Antennas & Propagation*, vol. 4, no. 9, pp. 1157–1164, 2010.
- [22] X. Ma, Y. Liu, K. D. Xu, C. Zhu, and Q. H. Liu, "Synthesising multiple-pattern sparse linear array with accurate sidelobe control by the extended reweighted L1-norm minimisation," *Electronics Letters*, vol. 54, no. 9, pp. 548–550, 2018.
- [23] I. Kanbaz, U. Yesilyurt, and E. Aksoy, "A study on harmonic power calculation for nonuniform period linear time modulated arrays," *IEEE Antennas and Wireless Propagation Letters*, vol. 17, no. 12, pp. 2369–2373, 2018.
- [24] M. Clerc and J. Kennedy, "The particle swarm—explosion, stability, and convergence in a multidimensional complex space," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 1, pp. 58–73, 2002.
- [25] S.-H. Yang and J.-F. Kiang, "Optimization of sparse linear arrays using harmony search algorithms," *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 11, pp. 4732–4738, 2015.