Research Article

An Efficient Laguerre-Based FDTD Iterative Algorithm in 3D Cylindrical Coordinate System

Da-Wei Zhu,1,2 Hai-Lin Chen,2 Zi-Yu Pan,1 Bo-Ao Xu,3 and Bin Chen2

1School of Information and Communication Engineering, Nanjing Institute of Technology, Nanjing, China
2National Key Laboratory on Electromagnetic Environmental Effects and Electro-Optical Engineering, Army Engineering University of PLA, Nanjing, China
3Rocket Force Academy, Beijing, China

Correspondence should be addressed to Hai-Lin Chen; hylinchen@126.com

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Here an efficient Laguerre-based finite-difference time-domain iterative algorithm is proposed. Different from the previously developed iterative procedure used in the efficient FDTD algorithm, a new perturbation term combined with the Gauss–Seidel iterative procedure is introduced to form the new Laguerre-based FDTD algorithm in the 3D cylindrical coordinate system. Such a treatment scheme can reduce the splitting error to a low level and obtain a good convergence; in other words, it can improve the efficiency and accuracy than other algorithms. To verify the performance of the proposed algorithm, two scattering numerical examples are given. The computation results show that the proposed algorithm can be better than the ADI-FDTD algorithm in terms of efficiency and accuracy. Meanwhile, the proposed algorithm is extremely useful for the problems with fine structures in the 3D cylindrical coordinate system.

1. Introduction

In recent years, the unconditionally stable Laguerre-based finite-difference time-domain (FDTD) algorithm has been applied to simulate transient electromagnetic problems in the Cartesian coordinate. By transforming the time-domain problem to the Laguerre domain using the temporal Galerkin’s testing procedure, the transient solution is independent of time discretization. Thus, Laguerre-based FDTD formulation has the advantage of less numerical error when a larger time step is used.

The main drawback of the conventional Laguerre-based FDTD algorithm is that it requires solving a very large sparse matrix. To overcome this problem, a factorization-splitting scheme [1] was used to resolve the huge sparse matrix into six tridiagonal matrices, and then the chasing method was used to solve them [2]. This provided excellent computational accuracy and can be efficiently parallel-processed on a computing cluster. To further reduce the splitting error, the modified perturbation term was introduced in [3, 4]; meanwhile, the Gauss–Seidel iterative procedure was applied, which made the new Laguerre-based FDTD scheme more efficient and accurate. Obviously, the good feasibility of the efficient Laguerre scheme had been confirmed in the rectangular coordinate system.

However, in many applications, we have to deal with 3D cylindrical structures such as in optical fiber communication, integrated optics, and defense industry. Moreover, the geometry of interest may consist of fine structures. If we adopt the conventional FDTD method to discretize the cylindrical structure with the Cartesian grid, a significant staircasing error appears. In fact, due to the existence of the $1/\rho$ term in the 3D cylindrical coordinate system, the decomposition of the fields along the $\phi$ direction will lead the splitting error. How to reduce this kind of error in the iterative FDTD calculation has become a difficult problem. In
addition, how to conduct ϕ direction transformation of the
fields on the z-axis is also a research difficult in the 3D
cylindrical coordinate system. Perhaps this is why, since
2000, the FDTD method has done little applications in the
3D cylindrical coordinate system. There is not even an article
on the Laguerre technology.

Therefore, in order to expand the research field of the
FDTD method, we propose an efficient Laguerre-based fi-
nite-difference time-domain iterative algorithm in the 3D
cylindrical coordinate system. Firstly, according to the
ideology of the weighted Laguerre polynomial (WLP) FDTD
scheme, the WLP-FDTD equations of the proposed algo-
rum in the 3D cylindrical coordinate system are deduced
by introducing the new perturbation term and the non-
physical intermediate variables. Secondly, the field com-
ponents on the z-axis are amended with special treatment.
Finally, to verify the proposed algorithm, two scattering
numerical examples are given. Numerical results show that
the proposed algorithm can be better than the ADI-FDTD

2. Iterative Theory and Formulations

2.1. WLP-FDTD Equations in 3D Cylindrical Coordinate
System. Introducing the WLP technology [6] into Maxwell’s
equations of the 3D cylindrical coordinate system [7], one can
obtain the equations of the conventional WLP-FDTD in the
3D cylindrical coordinate system, directly:

\[ e^q_{\rho} + 2 \sum_{k=0}^{q-1} e^k_{\rho} = \left( \frac{2}{\mu_0} \right) \left( \frac{1}{\rho} \partial \phi \right) h^q_z - \partial z h^q_{\phi}, \]

[1]

\[ e^q_{\phi} + 2 \sum_{k=0}^{q-1} e^k_{\phi} = \left( \frac{2}{\mu_0} \right) \left( \frac{1}{\rho} \partial \phi \right) h^q_z. \]

[2]

\[ e^q_z + 2 \sum_{k=0}^{q-1} e^k_z = \left( \frac{2}{\epsilon_0} \right) \left( \frac{1}{\rho} \partial \phi \right) h^q_{\phi} - \partial \rho \phi h^q_z, \]

[3]

\[ h^q_{\rho} + 2 \sum_{k=0}^{q-1} h^k_{\rho} = \left( \frac{2}{\mu_0} \right) \left( \partial z e^q_{\phi} - \left( \frac{1}{\rho} \right) \partial \phi e^q_z \right), \]

[4]

\[ h^q_{\phi} + 2 \sum_{k=0}^{q-1} h^k_{\phi} = \left( \frac{2}{\mu_0} \right) \left( \partial \rho e^q_z - \partial z e^q_{\rho} \right), \]

[5]

\[ h^q_z + 2 \sum_{k=0}^{q-1} h^k_z = \left( \frac{2}{\mu_0} \right) \left( \frac{1}{\rho} \partial \phi e^q_{\rho} \right) - \left( \frac{1}{\rho} \right) \left( \partial \left( \rho e^q_{\phi} \right) \right). \]

[6]

where \( s > 0 \) is a time-scale factor, \( q \) is the order of weighted
Laguerre polynomials, and \( \epsilon \) and \( \mu \) are the electric and
magnetic permeability, respectively.

2.2. Iterative Equations for the Off-Axis Fields. Equations
(1)–(6) can be written in the following matrix form:

\[ W^q - V^{q-1} = (A + B)W^q, \]

[7]

where \( W^q \) and \( V^{q-1} \) are the general form of \( q \)-order WLP
fields and the summation of \( q - 1 \) order WLP fields, respec-
respectively, and \( A \) and \( B \) are the coefficient matrices.

Obviously, solving equation (7) must involve large
sparse matrix problems, and it is impossible to solve it.
Therefore, the efficient algorithm must be adopted to avoid
the direct solution of the large sparse matrix.

Referring to the brief form of the 3-D FDTD equation in
[4], when the excitation sources \( JM \) are added, the equations
of the efficient Laguerre-based FDTD in the 3D cylindrical
coordinate can be rewritten as

\[ (I - A)(I - B)W^q = -ABW^{q-1} + V^{q-1} + JM, \]

[8]

\[ (I - A)(I - B)W^q_p = ABW^{q-1} + V^{q-1} + JM, \]

[9]

where \( p \) is the \( p \)th iteration and equations (8) and (9) are
the initial and iterative computation, respectively. Clearly, (9)
have the same form with (8). So, one can combine them as fol-
follows:

\[ W^q = -W^{q-1}, \]

[10a]

\[ (I - A)(I - B)W^q_p = ABW^{q-1} + V^{q-1} + JM, \]

[10b]

where

\[ W^q = [ E^q_0 H^q_0 ]^T, \]

\[ V^{q-1} = [ E^{q-1}_0 H^{q-1}_0 ]^T, \]

\[ E^q_0 = [ e^q_{\rho} e^q_{\phi} e^q_z ]^T, \]

\[ H^q_0 = [ h^q_{\rho} h^q_{\phi} h^q_z ]^T, \]

\[ E^{q-1}_0 = [ -2 \sum_{k=0}^{q-1} e^k_{\rho} -2 \sum_{k=0}^{q-1} e^k_{\phi} -2 \sum_{k=0}^{q-1} e^k_z ]^T, \]

\[ H^{q-1}_0 = [ -2 \sum_{k=0}^{q-1} h^k_{\rho} -2 \sum_{k=0}^{q-1} h^k_{\phi} -2 \sum_{k=0}^{q-1} h^k_z ]^T, \]

\[ JM^q = [ JM^q_{\rho} JM^q_{\phi} JM^q_z ]^T, \]

\[ JM^{q-1}_0 = [ -a f^q_{\rho} -a f^q_{\phi} -a f^q_z ]^T, \]

\[ JM^{q-1}_0 = [ b M^{q-1}_{\rho} b M^{q-1}_{\phi} b M^{q-1}_z ]^T. \]

In addition, the \( A \) and \( B \) in (10b) should be
\[
A = \begin{bmatrix}
0 & A_{12} \\
A_{21} & 0
\end{bmatrix},
\]
\[
B = \begin{bmatrix}
0 & B_{12} \\
B_{21} & 0
\end{bmatrix},
\]
\[
A_{12} = \begin{bmatrix}
0 & -a\partial z & 0 \\
0 & 0 & -a\partial \rho \\
-a\left(\frac{\partial \phi}{\rho}\right) & 0 & 0
\end{bmatrix},
\]
\[
B_{12} = \begin{bmatrix}
0 & 0 & -b\left(\frac{\partial \phi}{\rho}\right) \\
0 & a\left(\frac{\partial \phi}{\rho}\right) & 0 \\
0 & 0 & b\partial \rho
\end{bmatrix},
\]
\[
B_{21} = \begin{bmatrix}
0 & b\partial z & 0 \\
0 & 0 & b\partial \phi \\
b\left(\frac{\partial \phi}{\rho}\right) & 0 & 0
\end{bmatrix}
\]

where \(a = 2/\varepsilon e\) and \(b = 2/\varepsilon u\). \(\partial \rho\), \(\partial \phi\), and \(\partial z\) are the first-order central difference operators along \(\rho\), \(\phi\), and \(z\) axes, respectively.

In fact, equation (10a) can be implemented by simple programming, and we only need to implement the design of the iterative calculation. Therefore, we actually only need to introduce the perturbation term \(AB(W_{p+1}^{g} - W_{p}^{g})\) in [4] into equation (7) to obtain (10b), and the advantage of this is that it greatly simplifies the programming process and improves the computational efficiency.

In accordance with the design theory of the proposed algorithm in this paper, we introduce the nonphysical intermediate variables \(W^{*g} = [E_{0}^{g} H_{0}^{g}]^{T} = [e_{x}^{*g} e_{x}^{*g} e_{z}^{*g} h_{p}^{*g} h_{p}^{*g} h_{z}^{*g}]^{T}\), and it can divide (10b) into

\[
(I - A)W_{p+1}^{g} = BW_{p}^{g} + V_{p}^{g-1} + JM_{p}^{g}, \quad (13a)
\]
\[
(I - B)W_{p+1}^{g} = W_{p}^{g} - BW_{p}^{g}. \quad (13b)
\]

Expanding (13a) and (13b), one can obtain

\[
E_{0,p+1}^{g} - A_{12} H_{0,p+1}^{g} = B_{12} E_{0,p}^{g} + E_{0}^{g-1} + JM_{1}^{g}, \quad (14a)
\]
\[
H_{0,p+1}^{g} - A_{21} E_{0,p+1}^{g} = B_{21} H_{0,p}^{g} + H_{0}^{g-1} + JM_{2}^{g}, \quad (14b)
\]
\[
E_{0,p+1}^{g} - B_{12} H_{0,p+1}^{g} = E_{0}^{g-1} - B_{12} H_{0,p}^{g}, \quad (15a)
\]
\[
H_{0,p+1}^{g} - B_{21} E_{0,p+1}^{g} = H_{0}^{g-1} - B_{21} E_{0,p}^{g}. \quad (15b)
\]

Substituting (14b) into (14a), one can obtain

\[
(I - A_{12} A_{21}) E_{0,p+1}^{g} = A_{12} H_{0,p}^{g} + B_{12} H_{0,p}^{g}, \quad (16)
\]

Expanding (16), one can obtain three implicit equations \(e_{x,p+1}^{q}, e_{z,p+1}^{q}\), and \(e_{z,p+1}^{q}\). Here, the Gauss–Seidel iterative procedure should be used, and the equations after the Gauss–Seidel procedure are

\[
(1 - ab\partial z)^{2} e_{x,p+1}^{*g} = -ab\partial z \partial e_{z,p}^{q} + a\left(\frac{\partial \phi}{\rho}\right) h_{z,p}^{q} + V_{1}^{g-1} + JM_{1}^{g}, \quad (17a)
\]
\[
(1 - ab\partial \rho) \left(\frac{\partial \phi}{\rho}\right) e_{z,p+1}^{*g} = -ab\partial \rho \left(\frac{\partial \phi}{\rho}\right) e_{z,p+1}^{*g} + a\partial z H_{z,p+1}^{q} + V_{2}^{g-1} + JM_{2}^{g}, \quad (17b)
\]
\[
(1 - ab\left(\frac{\partial \phi}{\rho}\right)^{2}) e_{z,p+1}^{*g} = -ab\left(\frac{\partial \phi}{\rho}\right) e_{z,p+1}^{*g} + a\left(\frac{\partial \rho}{\rho}\right) h_{z,p+1}^{q} + V_{3}^{g-1} + JM_{3}^{g}, \quad (17c)
\]

\[
V_{1}^{g-1} = 2a\partial z \sum_{k=0}^{q-1} h_{z}^{k} - 2 \sum_{k=0}^{q-1} e_{z}^{k}, \quad (17a-1)
\]
\[
V_{2}^{g-1} = 2a\partial \rho \sum_{k=0}^{q-1} h_{z}^{k} - 2 \sum_{k=0}^{q-1} e_{z}^{k}, \quad (17b-1)
\]
\[
V_{3}^{g-1} = 2a\left(\frac{\partial \phi}{\rho}\right) \sum_{k=0}^{q-1} h_{z}^{k} - 2 \sum_{k=0}^{q-1} e_{z}^{k}, \quad (17c-1)
\]
\[
JM_{1}^{g} = -ab \partial z M_{z}^{q} - aJ_{z}^{q}, \quad (17a-2)
\]
\[
JM_{2}^{g} = -ab \partial \rho M_{z}^{q} - aJ_{z}^{q}, \quad (17b-2)
\]
\[ \mathbf{J}^q_{m} = \rho a \left( \frac{\partial \phi}{\rho} \right) M^q_{\rho \phi} - a \mathbf{J}_{\phi}^q. \]  
\text{(17c-2)}

Obviously, the necessary nonphysical variables \( e_{\phi, p+1}^q \), \( e_{\phi, p}^q \), and \( e_{z, p+1}^q \) for solving (15a) have been obtained through solving (17a)–(17c).

Next is how to solve the fields of the nonphysical variables \( H_{\phi, p+1}^q \) for solving (15b). Here, it is not necessary because substituting (14b) into (15b), equation (15b) can be rewritten as

\[ \mathbf{H}_{\phi, p+1}^q = \mathbf{B}_{21} \mathbf{E}_{\phi, p+1}^q + \mathbf{A}_{21} \mathbf{E}_{\phi, p+1}^q + \mathbf{H}_{0}^{q-1} + \mathbf{J}^q_{\rho}. \]  
\text{(18)}

Clearly, the nonphysical variables \( H_{\phi, p+1}^q \) are eliminated already. Meaning that, the nonphysical variables \( H_{\phi, p+1}^q \) do not have to be solved in the whole iteration.

Substituting (18) into (15a), one can obtain a new equation which is similar with (16):

\[ (I - \mathbf{B}_{12} \mathbf{B}_{21}) \mathbf{E}_{\phi, p+1}^q = \mathbf{E}_{\phi, p+1}^q + \mathbf{B}_{12} \mathbf{A}_{21} \mathbf{E}_{\phi, p+1}^q + \mathbf{B}_{12} \mathbf{H}_{\rho}^q + \mathbf{B}_{12} \mathbf{H}_{\phi}^{q-1} + \mathbf{B}_{12} \mathbf{J}^q_{\rho}. \]  
\text{(19)}

Expanding (18) and (19), one can obtain the computational equations of the electric and magnetic fields of the proposed algorithm:

\[ \left(1 - a b \left( \frac{\partial \phi}{\rho} \right)^2 \right) e_{\phi, p+1}^q = e_{\phi, p+1}^q \]

\[ - ab \left( \frac{\partial \phi}{\rho} \right) \left( \frac{1}{\rho} \right) \left( \frac{\partial}{\partial \rho} \right) h_{\phi, p+1}^q \]

\[ - a \left( \frac{\partial}{\partial \rho} \right) h_{\phi, p+1}^q + V_4^{q-1} + \mathbf{J}_{\rho}^q. \]  
\text{(20)}

\[ \left(1 - a b \left( \frac{\partial z}{\rho} \right)^2 \right) e_{\phi, p+1}^q = e_{\phi, p+1}^q \]

\[ - a b \left( \frac{1}{\rho} \right) \left( \frac{\partial}{\partial \rho} \right) \left( \frac{\partial \phi}{\rho} \right) h_{\phi, p+1}^q \]

\[ - a \left( \frac{\partial}{\partial \rho} \right) h_{\phi, p+1}^q + V_4^{q-1} + \mathbf{J}_{\rho}^q. \]  
\text{(21)}

\[ V_4^{q-1} = -2 a \left( \frac{\partial \phi}{\rho} \right) \sum_{k=0}^{q-1} h_{\rho}^k, \]  
\text{(20-1)}

\[ \mathbf{J}^q_{\rho} = ab \left( \frac{\partial \phi}{\rho} \right) M^q_{\rho \phi}, \]  
\text{(20-2)}

\[ \mathbf{V}_5^{q-1} = -2 a \rho \sum_{k=0}^{q-1} h_{\rho}^k, \]  
\text{(21-1)}

\[ \mathbf{J}^q_{\rho} = ab \left( \frac{\partial \phi}{\rho} \right) M^q_{\rho \phi}, \]  
\text{(21-2)}

\[ \mathbf{V}_6^{q-1} = -2 a \left( \frac{1}{\rho} \right) \left( \frac{\partial \phi}{\rho} \right) \sum_{k=0}^{q-1} h_{\rho}^k, \]  
\text{(22-1)}

\[ \mathbf{J}^q_{\rho} = ab \left( \frac{1}{\rho} \right) \left( \frac{\partial \phi}{\rho} \right) \rho M^q_{\rho \phi}, \]  
\text{(22-2)}

\[ h_{\phi, p+1}^q = b \left( \frac{\partial \phi}{\rho} \right) e_{\phi, p+1}^q - b \left( \frac{\partial \phi}{\rho} \right) e_{\phi, p+1}^q \]

\[ - 2 \sum_{k=0}^{q-1} h_{\rho}^k + b M_{\rho \phi}^q, \]  
\text{(23)}

\[ h_{\phi, p+1}^q = b \left( \frac{\partial \phi}{\rho} \right) e_{\phi, p+1}^q - b \left( \frac{\partial \phi}{\rho} \right) e_{\phi, p+1}^q \]

\[ - 2 \sum_{k=0}^{q-1} h_{\rho}^k + b M_{\rho \phi}^q, \]  
\text{(24)}

\[ h_{\phi, p+1}^q = b \left( \frac{\partial \phi}{\rho} \right) e_{\phi, p+1}^q - b \left( \frac{1}{\rho} \right) \left( \frac{\partial \phi}{\rho} \right) e_{\phi, p+1}^q \]

\[ - 2 \sum_{k=0}^{q-1} h_{\rho}^k + b M_{\rho \phi}^q, \]  
\text{(25)}

At this point, the implementation scheme of the proposed algorithm has been elaborated, and the overall execution process is described in Figure 1.

### 3. Special Treatment for the On-Axis Fields

#### 3.1. Theory and Analysis of the Special Treatment for the On-Axis Field Components

Because of the strangeness of the cylindrical coordinate system, drastic change of the field nearby the axis will lead the splitting error to be great [8]. Therefore, on-axis fields need to be specially treated in the 3D cylindrical coordinate system.

The positions of the electric/magnetic fields of the 3D cylindrical coordinate are shown in Figure 2, deducting the fields nearby the axis, and the \( \rho \phi \) section is a partial circular ring. Seeing the black shadow sector in Figure 3, clearly, the \( \rho \phi \) section can be added by many sectors. That is to say, all the electric/magnetic fields on the axis exhibit mathematical strangeness. As shown in Figure 3, when \( \rho = 0 \), there are \( E_z \) and \( H_z \) field components on the axis; therefore, we should treat \( E_z \) \((0, j, k + (1/2)) \) and \( H_z \) \((1/2, j + (1/2), k) \) specially.

According Stokes’s theorem, we start from the following integral form of Maxwell’s equations in the time domain:
\[ \epsilon \int_S \left( \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{s} = \oint_I \vec{H} \cdot d\vec{l}, \quad (26) \]
\[ \mu \int_S \left( \frac{\partial \vec{H}}{\partial t} \right) \cdot d\vec{s} = -\oint_I \vec{E} \cdot d\vec{l}, \quad (27) \]

where \( \vec{l} \) is a closed curve surrounding the \( \rho = 0 \) axis and \( \vec{s} \) is the surface bounded by the curve \( \vec{l} \). Equations (26) and (27) are actually the integral form of the Ampere loop theorem.
3.2. Differentia of the Special Treatment for the On-Axis Field Components in 3D Cylindrical Coordinate System. The FDTD difference equation of $E_z(0,j,k + (1/2))$ can be derived from the formula (26), its integral area is the dotted line circle, and the radius is $\Delta p/2$, as shown in Figure 3:

$$n(\Delta p)^2 e\left(\frac{\partial E_z}{\partial t}\right)(0,j,k + (1/2))$$

$$= \sum_{j_1=1}^{j_m} H_\phi\left(\frac{1}{2},j_1,k + \frac{1}{2}\right)(\frac{\Delta \rho}{2}) \Delta \phi,$$

noting $j_1 \neq j$.

Obviously, the form of the right side of equation (28) is different from equations (17a)–(17c-2) in [9]. Supposing $H_\phi(1/2,j_1,k + (1/2))$ in equation (28) is calculated in a body of revolution structure and each $H_\phi(1/2,j_1,k + (1/2))$ is equal for all the different $j_1$ values, equation (28) can be modified as

$$\varepsilon\left(\frac{\partial E_z}{\partial t}\right)(0,j,k + (1/2)) = \left(2\left(\frac{j_m \Delta \phi}{\pi \Delta \rho}\right)\right) H_\phi\left(\frac{1}{2},j_1,k + \frac{1}{2}\right)$$

$$= \left(\frac{4}{\Delta \rho}\right) H_\phi\left(\frac{1}{2},j_1,k + \frac{1}{2}\right),$$

(29)

where $j_m \Delta \phi = 2\pi$.

In this way, the form of the right side of equation (29) is similar with (17a)–(17c-2) in [9]. By comparing equations (28) and (29), we can explain the different characteristics of the special treatment scheme on the axis in the 3D cylindrical coordinate structure, which is clearly distinguished from the scheme in the body of the revolution structure.

3.3. Special Treatment of $E_z^2(0,j,k + (1/2))$ and $H_z^2(1/2,j+k + (1/2),k)$. Introducing the WLP technology [6] into equation (28), one can obtain

$$E_z^2(0,j,k + \frac{1}{2}) + 2 \sum_{k=0}^{q-1} E_z^k(0,j,k + \frac{1}{2})$$

$$= \frac{2\alpha \Delta \phi}{\pi \Delta \rho} \sum_{j_1=1}^{j_m} H_\phi\left(\frac{1}{2},j_1,k + \frac{1}{2}\right).$$

(30)

In order to solve equation (30), we present the first-order FDTD central difference expansion of equation (24) at $\rho = 0$:

$$h^q_{\phi,p+1}\left(\frac{1}{2},\frac{1}{2},j,k + \frac{1}{2}\right) = \left[\frac{\epsilon_{eq}}{\rho_{h+1}}\left(1,j,k + \frac{1}{2}\right)\right]$$

$$- e^{-q}_{\phi,p+1}\left(0,j,k + \frac{1}{2}\right)$$

$$- \left(\frac{b}{\Delta \rho}\right)$$

$$- e^{-q}_{\phi,p+1}\left(0,j,k + \frac{1}{2}\right)$$

$$- e^{-q}_{\phi,p+1}\left(1,j,k + \frac{1}{2}\right)$$

$$- e^{-q}_{\phi,p+1}\left(1,j,k + \frac{1}{2}\right)$$

$$- e^{-q}_{\phi,p+1}\left(1,j,k + \frac{1}{2}\right)$$

(31)

Substituting (31) into (30), one can obtain the special treatment equation of $E_z(0,j,k+(1/2))$ in the 3D cylindrical coordinate system:

$$\left(1 + \left(\frac{2j_{m}ab \Delta \phi}{\pi \Delta \rho^2}\right)\right) e_{z,p+1}(0,j,k + \frac{1}{2})$$

$$= \left(\frac{2ab \alpha \Delta \phi}{\pi \Delta \rho}\right) \sum_{j_1=1}^{j_m} e_{z,p+1}(1,j_1,k + \frac{1}{2})$$

$$= \left(\frac{2ab \alpha \Delta \phi}{\pi \Delta \rho}\right) \sum_{j_1=1}^{j_m} e_{z,p+1}(1,j_1,k + \frac{1}{2})$$

$$= \left(\frac{4a \Delta \phi}{\pi \Delta \rho}\right) \sum_{j_1=1}^{j_m} \left(\sum_{k=0}^{q-1} H_\phi\left(\frac{1}{2},j_1,k + \frac{1}{2}\right)\right) - 2 \sum_{k=0}^{q-1} e_{z}(0,j,k + \frac{1}{2})$$

$$+ \left(\frac{2ab \alpha \Delta \phi}{\pi \Delta \rho}\right) \sum_{j_1=1}^{j_m} M_{eq}\left(\frac{1}{2},j_1,k + \frac{1}{2}\right).$$

(32)

In a similar way, the FDTD difference equation of $H_z((1/2), j + (1/2), k)$ can be derived from formula (27), its integral area is the dotted line circle, and the radius is $\Delta \rho$, as shown in Figure 3. Its method of obtaining can refer to equations (30) to (32).

In order to solve the special treatment equation (32), one can obtain the following forms of the special treatment of $H_z(1/2), j + (1/2), k)$ at $\rho = 0$.

4. Numerical Results

In order to verify the performance of the proposed algorithm, two scattering numerical examples are given.
First example: an irregular scatter with two grooves is calculated, and the angle of the groove is $\alpha = \pi/4$. The radius of the groove is 19 cm, and the radius and height of the cylinder are 20 cm and 40 cm, respectively. The oblique incident plane wave is added through Huygen’s surface at $\rho = 22.5$ cm and $z = \pm 22.5$ cm, and parallel to the $\phi = 0$ plane, the incident angle is $\theta_i = 45^\circ$, as shown in Figure 4. The incident electric field is

$$E_i(t) = E_0 \cos(2\pi f_0 t) \exp\left(-4\pi \left(\frac{t - t_0}{\tau}\right)^2\right),$$  

(33)

where $E_0 = 10^3$ V/m, $f_0 = 10$ GHz, $\tau = 1.0/f_0$, and $t_0 = 1.25/f_0$. And $s = 7 \times 10^{11}$ and $q = 180$ are chosen for the computation of the field values. The computational domain is meshed using a uniform grid with
Figure 6: Computational results of $E_{\phi}$ at point $P_2$ of the conventional FDTD algorithm, the ADI-FDTD algorithm [5], and the proposed algorithm.

Figure 7: Computational results of $E_{\phi}$ at point $P_3$ of the conventional FDTD algorithm, the ADI-FDTD algorithm [5], and the proposed algorithm.
$\Delta \rho = \Delta z = 1\text{mm}$, leading to the total mesh of $40(\rho) \times 40(\phi) \times 80(z)$, and the first-order Mur absorbing boundary condition is used to truncate the boundary [10].

The observation points of the simulated electric fields are set at $P_1(29, 22, 0)$, $P_2(29, 2, 19)$, and $P_3(25, 25, -15)$, and the iteration of the whole computational domain is denoted $N_t$. By using the conventional FDTD algorithm, the ADI-FDTD algorithm [5], and the proposed algorithm, the time-domain field waveforms are shown in Figures 5–7. The numerical results show an excellent agreement.
between the conventional FDTD and the proposed algorithms.

In addition, to demonstrate the higher accuracy of the proposed algorithm, here two error formulations are provided to describe the accuracy, which are defined as

\[ E_{\phi,\text{error}} = E_{\phi}(t) - E_{\phi,\text{FDTD}}(t), \]  

\[ E_{\phi,\text{error}} = \frac{|E_{\phi}(t) - E_{\phi,\text{FDTD}}(t)|}{E_{\phi}(t)_{\text{max}}} \times 100\%, \]  

where \( E_{\phi}(t) \) are the simulation results of the electric fields of the ADI-FDTD algorithm and the proposed algorithm and \( E_{\phi,\text{FDTD}}(t) \) is the simulation result of the conventional FDTD algorithm.

Figures 8–10 afford the error about \( E \) of the proposed algorithm (with \( N_t = 3 \)). Clearly, the error of the proposed algorithm is much smaller than the ADI-FDTD algorithm (with \( \text{cfln} = 4, \text{cfln} = 5, \text{cfln} = 6, \text{cfln} = \Delta t/\Delta t_{\text{cfl}} \)). The CPU time of the ADI-FDTD algorithm and the proposed algorithm is shown in Table 1.

Observing Figures 8–10 and Table 1, when the proposed algorithm \( N_t = 3 \) and the ADI-FDTD algorithm \( \text{cfln} = 6 \), the CPU time of both algorithms is almost equal; nevertheless, the error of the proposed algorithm can be reduced almost 64% than the ADI-FDTD algorithm at least. Even though, when the ADI-FDTD \( \text{cfln} = 4 \), the comparing results show that the error of the proposed algorithm can be reduced almost 44% at least and the CPU time can be saved almost 35% than the ADI-FDTD algorithm. Obviously, the proposed algorithm can provide better improvement in terms of efficiency and accuracy.

Second example: another irregular scatter with one wedge-shaped bulge is shown in Figure 11: the angle of the

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**Figure 10:** Errors obtained by \( E_{\phi} \) at point \( P_3 \) of the ADI-FDTD algorithm [5] and the proposed algorithm.

**Figure 11:** Illustration of the irregular cylinder with one wedge-shaped bulge under the oblique incident wave.

**Table 1:** Simulation results of the two algorithms in the first example.

<table>
<thead>
<tr>
<th>( \text{cfln} )</th>
<th>ADI-FDTD algorithm [5]</th>
<th>Proposed algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{cfln} = 4 )</td>
<td>CPU time (s) 337.27</td>
<td>( N_t = 3 )</td>
</tr>
<tr>
<td>( \text{cfln} = 5 )</td>
<td>CPU time (s) 270.14</td>
<td></td>
</tr>
<tr>
<td>( \text{cfln} = 6 )</td>
<td>CPU time (s) 224.26</td>
<td></td>
</tr>
<tr>
<td>( \text{cfln} = 6 )</td>
<td>CPU time (s) 218.95</td>
<td></td>
</tr>
</tbody>
</table>

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Figure 12: Computational results of $E_\phi$ at point $P_1$ of the conventional FDTD algorithm, the ADI-FDTD algorithm [5], and the proposed algorithm.

Figure 13: Computational results of $E_\phi$ at point $P_2$ of the conventional FDTD algorithm, the ADI-FDTD algorithm [5], and the proposed algorithm.
bulge is $\pi/4$, the radius of the bulge is 22 cm, and the radius and height of the cylinder are 20 cm and 40 cm, respectively. The other conditions are same with the first example.

The observation points of simulated electric fields are set at $P_1 (29, 10, 15)$, $P_2 (29, 20, 0)$, and $P_3 (15, 38, -29)$, respectively. The waveforms of the proposed algorithm are
shown in Figures 12–14, which also form an excellent agreement with the conventional FDTD algorithm.

To demonstrate the accuracy of the proposed algorithm again, formulations (34-1) and (34-2) are used again here. Figures 15–17 afford the error about $E_\phi$ of the proposed algorithm (with $N_t = 3$) and the ADI-FDTD algorithm (with $\text{cfl n} = 4$, $\text{cfl n} = 5$, and $\text{cfl n} = 6$). The CPU time of the both algorithms is shown in Table 2.

Observing Figures 15–17 and Table 2, when the proposed algorithm $N_t = 3$ and the ADI-FDTD algorithm

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**Figure 16:** Errors obtained by $E_\phi$ at point $P_2$ of the ADI-FDTD algorithm [5] and the proposed algorithm.

**Figure 17:** Errors obtained by $E_\phi$ at point $P_3$ of the ADI-FDTD algorithm [5] and the proposed algorithm.
cfln = 4, cfln = 5 and cfln = 6), the comparison results are similar with the first example, namely, the efficiency and the accuracy of the proposed algorithm are still better than the ADI-FDTD algorithm. Obviously, the second example numerical results verify that the proposed algorithm can provide better improvement than the ADI-FDTD algorithm [5] in terms of efficiency and accuracy again.

## 5. Conclusion

In this paper, an efficient Laguerre-based finite-difference time-domain iterative algorithm in the 3D cylindrical coordinate system is proposed. By adopting a new perturbation term \( AB(W_{q-1}^{p} - W_{q}^{p}) \) and using the Gauss–Seidel iterative procedure method in the iterative algorithm of this paper, this kind of treatment can get better convergence; meanwhile, it also can ensure the enhancing of the efficiency and accuracy. It can be proved by computing the two scattering numerical examples that the proposed algorithm can provide better efficiency and accuracy in high-frequency range than the ADI-FDTD algorithm.

### Data Availability

The simulation data for the 3D cylindrical coordinate structure used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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