The PSK Channel Capacity Estimation under Dynamic Plasma Sheath Channel

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Received 13 November 2019; Revised 4 March 2020; Accepted 14 March 2020; Published 9 April 2020

During the reentry process, the plasma sheath covering the surface of the hypersonic aircraft will cause the amplitude attenuation and phase jitter of the communication electromagnetic waves. Channel parameters such as the electron density and collision frequency of the plasma sheath reflect the changing trend of the plasma sheath, and these parameters can be measured by physical means. However, these parameters cannot directly reflect the change of the channel communication ability and cannot directly serve the design of communication methods in the plasma sheath. Due to the particularity of the plasma sheath, the traditional channel estimation method for Additive White Gaussian Noise channels will no longer be applicable. This paper presents a channel capacity estimation method for dynamic plasma sheath. First, the plasma sheath is equivalent to a discrete input continuous output memoryless channel, and then the channel capacity expression is derived according to Shannon formula. Finally, the channel capacity of the dynamic plasma sheath is estimated by calculating the transition probability density function. The simulation results show that the channel capacity of the dynamic plasma sheath is affected by both the signal-to-noise ratio (SNR) and the dynamic parameters of the plasma sheath. When the electron density is small, the channel capacity is mainly affected by the SNR. As the electron density increases, the dynamic parameters of the plasma sheath gradually become the main factor affecting the channel capacity. This method is a theoretical analysis of the channel capacity when the channel parameters of the plasma channel are known, and it is meaningful for conducting the work of communication methods design.

1. Introduction

When a hypersonic vehicle is flying in the atmosphere at hypersonic speed, owing to the tremendous heat generated by air compression and ablation surrounding the vehicle, a plasma sheath covering the vehicle is generated [1]. The plasma sheath attenuates the energy of communication signal and influences the quality of the signal which will lead to lost or at least severely degraded communication, telemetry, navigation signals, etc. [2–4].

The communication blackout caused by plasma sheath is one of the main factors that restrict the development of hypersonic vehicle. At present, a large number of theories and experimental studies have been proposed to solve the radio blackout [5–11]. However, the problem has not yet been completely resolved. In addition, the attenuation of the electromagnetic wave by the plasma sheath varies dynamically with the plasma sheath’s electron density. He et al. [12, 13] assumed a finite-state Markov channel (FSMC) model, to represent the dynamical effects on electromagnetic wave propagation through the plasma sheath. Shi et al. [14] proposed a new multistate Markov channel modeling method to adaptively estimate the number of plasma sheath channel states and the channel parameters [14]. Yang et al. [15] simulated the propagation of QPSK signals even though the dynamic plasma and constellation of the receiving signals circunvolve. The EM signals propagation through the dynamic plasma was analyzed, and the parasitic modulation of...
amplitude and phase in the experiment of wave propagation in the dynamic plasma was observed [16]. In conclusion, even if most of the wave energy could penetrate through the plasma sheath, the parasitic modulation of amplitude and phase due to the dynamic plasma sheath would cause communication failure. However, further appropriate quantitative analysis of this problem needs to proceed.

To design the communication system suitable for the plasma sheath channel, the communication performance limitations caused by plasma sheath channel should be clearly understood. Literature [17] assessed the channel capacity of the reentry channel with plasma sheath based on the communication link budget method and the classical Shannon information theory, which is essentially the capacity of the continuous fading Additive White Gaussian Noise (AWGN) channels. However, only the electromagnetic waves attenuation with different frequencies caused by plasma sheath is considered, and the effect of dynamic plasma is ignored. The parasitic modulation of amplitude and phase caused by dynamic plasma sheath is a special kind of multiplicative interference [18], which heavily interferes with the demodulation of the angle modulated signals. In addition, considering the multidimensional modulation scheme, the Shannon capacity of the continuous AWGN channels is not suitable, and the capacity of the discrete input and discrete output channels should be calculated.

In this paper, the Multiphase Shift Key (MPSK) channel capacity under dynamic plasma sheath channel is quantitatively analyzed. The rest of this paper is organized as follows. Section 2 shows the MPSK channel capacity estimation method with the parasitic modulation interference for the dynamic plasma sheath channel. Section 3 introduces and discusses the results of channel capacity under different simulation conditions. Finally, Section 4 summarizes the paper.

2. MPSK Capacity Estimation Method under Dynamic Plasma Sheath Channel

The plasma sheath will attenuate the transmitted electromagnetic waves. In addition, the dynamic characteristics of the plasma sheath will cause double parasitic modulation of the amplitude and phase of the electromagnetic wave [15], and this is a multiplication interference that seriously interferes with the demodulation of the angle modulation signals. Therefore, the Shannon capacity of the continuous AWGN channel is not suitable for calculating the channel capacity of the plasma sheath. It is necessary to analyze and calculate the influence of the dynamic plasma sheath on the transition probability of the MPSK signal from the modulation system and then to calculate the MPSK capacity under dynamic plasma sheath channel.

For a quadrature modulation communication system, such as BPSK, QPSK, or DQPSK, the modulation channel has discrete inputs and discrete outputs, which could be described as Discrete Memoryless Channel (DMC). Therefore, the channel capacity can be calculated as

\[
C = \log_2(q) - \frac{1}{d} \sum_{k=0}^{q-1} \log_2 \sum_{y=0}^{q-1} p(y|x_k) \cdot \log_2 \frac{\sum_{i=0}^{q-1} p(y|x_i)}{\sum_{i=0}^{q-1} p(y|x_i)}
\]

where \( q \) is the dimension of MPSK modulation, \( E \) is the desired operator, \( x_i \) is the discrete input, \( y \) is the continuous output, and \( p(y|x_i) \) is the transition probability density function, which indicates the conditional probability that a given Markov chain is in one state at a certain moment and then reaches another state after a certain time.

Assuming the channel is a symmetric channel and \( x_i \) is equal probability distribution, then

\[
\sum_{i=0}^{q-1} p(y|x_i) = 1,
\]

\[
p(y_j|x_i) = p(y_i|x_j).
\]

The channel capacity of DMC can be further expressed as

\[
C = \log_2(q) - \frac{1}{d} \sum_{k=0}^{q-1} p(y=0|x_k) \cdot \log_2 \frac{1}{p(y=0|x_k)}
\]

The channel capacity of DMC in BPSK and QPSK modulation modes can be expressed as follows:

BPSK:

\[
C = \log_2(2) - \frac{1}{2} \sum_{k=0}^{1} p(y=0|x_k) \cdot \log_2 \frac{1}{p(y=0|x_k)}
\]

QPSK:

\[
C = 2 - \sum_{k=0}^{1} p(y=0|x_k) \cdot \log_2 \frac{1}{p(y=0|x_k)}
\]
electron density increases, and in severe cases the con-
stellation points will overlap. The constellation points are dif-
ticult to distinguish in the overlap region, which will seri-
ously interfere with the demodulation decision.

Therefore, the transition probability in the dynamic sheath channel cannot be calculated by the optimal receiver theory under the additive noise channel.

To solve the above transfer probability density function, we need to analyze the multiplicative noise caused by the dynamic plasma sheath channel.

The plasma sheath channel is different from the tradi-
tional AWGN channel, and its output is related to many parameters of the plasma sheath. The most important plasma parameter that affects electromagnetic wave propagation is the electron density of the plasma sheath. The main factors that affect plasma dynamics are the flight status and the internal flow disturbances that change the physical parameters at different rates. If the electron density distribution \( n_e(t) \) of the plasma sheath is known, then the plasma frequency \( \omega_p(t) \) can be expressed as \[ \omega_p(t) = \sqrt{\frac{n_e(t) e^2}{\varepsilon_0 m_e}} \] (6)

where \( n_e(t) \) is the electron density, \( e \) is the electron charge, \( m_e \) is the mass of an electron, and \( \varepsilon_0 \) is the dielectric constant in free space.

The complex relative dielectric constant of dynamic plasma sheath is

\[ \varepsilon_r(t) = 1 - \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2} - j \frac{\nu_{ce}}{\omega} \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2}, \] (7)

where \( \omega \) is the frequency of the electromagnetic wave and \( \nu_{ce} \) is the collision frequency. The propagation vector can be expressed as follows:

\[ k(t) = \beta(t) - j\alpha(t) = \omega \sqrt{\mu_0 \varepsilon_r \varepsilon_r(t)}. \] (8)

The real part and imaginary part of the propagation vector are

\[ \alpha(t) = \frac{\omega}{\sqrt{2c}} \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2} - 1 + \left( 1 - \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2} \right)^2 \left( \frac{\nu_{ce}}{\omega} \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2} \right)^2. \]

\[ \beta(t) = \frac{\omega}{\sqrt{2c}} \left( 1 - \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2} \right) + \left( 1 - \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2} \right)^2 \left( \frac{\nu_{ce}}{\omega} \frac{\omega_p^2(t)}{\omega^2 + \nu_{ce}^2} \right)^2. \] (9)

The real part \( \beta(t) \) and the imaginary part \( \alpha(t) \) represent the phase coefficient and the attenuation coefficient, respectively. The electromagnetic wave in the plasma can be written as

\[ E = E_0 e^{j(\omega t - \beta t)z} e^{-j\alpha(t)z} = E_0 e^{j(\omega t - \beta t)z} \] (10)

where \( z \) is the thickness of the plasma sheath.

Therefore, the electromagnetic waves attenuation and phase shift vary dynamically with the fluctuation of the electron density. Viewed as communication signals, the parasitic modulation of both amplitude and phase could be described as multiplicative noises, which can be specifically expressed as

\[ h(t, \omega) = e^{-j\beta(t)z} = e^{-\alpha(t)z} \] (11)

The transmission signal through the plasma sheath channel could be mathematically modeled as

\[ y(t) = x(t) + n(t) = x(t) \cdot h(t, \omega) + n(t), \] (12)

where \( x(t) \) is input signals, \( y(t) \) is output signals, and \( n(t) \) is additive noise.

According to the above analysis, we can get the transition probability of the dynamic plasma sheath channel shown in (13), which is a two-dimensional function related to amplitude and phase. From (3) and (13), we can get the channel capacity of the PSK channel:

![Figure 1: Constellation diagram of QPSK signal after dynamic plasma. (a) \( \omega_p/\omega = 0.7 \); (b) \( \omega_p/\omega = 0.8 \); (c) \( \omega_p/\omega = 0.9 \).](image)
The electron density of the dynamic plasma mentioned in the text is the greater the change in the electron density between each layer, and the greater the jitter variance of the electron density. DO he greater the jitter variance of the electron density, the greater the change in the electron density between each layer.

\[
p(y|x; \theta) = \int_{-\infty}^{\infty} p_s(s|x; \theta) p_n(y-s|x) ds
\]

\[
= \int_{-\infty}^{\infty} p_s(s|x; \theta) \cdot \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(s-y)^2}{2\sigma^2}\right) ds
\]

\[
= \int_{-\infty}^{\infty} p\left(x e^{-\alpha(t) z} e^{-j\beta(t) z}\right) | x; \theta \cdot \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(n(t))^2}{2\sigma^2}\right) d(x e^{-\alpha(t) z} e^{-j\beta(t) z}).
\]

(13)

3. Numerical Simulation

Employing the theory described above, this section evaluates the channel capacity of the dynamic plasma sheath under different modulation schemes according to (3)–(5).

3.1. Channel Simulation Methods and Conditions. During reentry, the parameters of the plasma sheath are non-uniformly distributed due to the shape and attitude of the aircraft. Electron density is one of the key parameters. According to the literature [19], the nonuniformly distributed plasma model can be simplified into multiple uniform plasma layered overlays. The plasma of each layer is regarded as uniform distribution, and the electron density between layers is different. In this way, the propagation of electromagnetic wave in dynamic inhomogeneous plasma can be equivalent to that in multilayer homogeneous medium.

Figure 2 shows the transmission model of the signal in the plasma sheath. The channel in the model mainly includes an N-layer plasma channel with different electron density and a Gaussian white noise channel.

In the dynamic plasma channel model, the electron density \(Ne(z)\) distribution model of the plasma sheath uses a double Gaussian distribution [19–21]:

\[
Ne(z) = \begin{cases} 
N_{e(\text{max})}e^{-\alpha_1(z-z_0)^2}, & 0 \leq z \leq z_0, \\
N_{e(\text{max})}e^{-\alpha_2(z-z_0)^2}, & z_0 \leq z \leq z_2.
\end{cases}
\]

(14)

Here, \(z\) is the vertical distance from the surface of the aircraft, \(N_{e(\text{max})}\) is the peak electron density at the location of \(z_0\), and \(\alpha_1\) and \(\alpha_2\) represent the curve’s shape. According to the literature [19], the model parameters of equation (14) are set as \(\alpha_1 = 2000\), \(\alpha_2 = 1700\), \(z_0 = 0.035\ m\), and \(z_2 = 0.2\ m\).

Figure 3 shows the probability distribution curve of normalized electron density under different jitter variance \(\sigma\). The distribution of electron density obeys Gaussian distribution. The greater the jitter variance of the electron density, the greater the change in the electron density between each layer, and the greater the dynamics of the plasma. The electron density of the dynamic plasma mentioned in the following is the average electron density.

The jitter model of the dynamic plasma sheath mainly includes sinusoidal jitter model and Gaussian jitter model.

The sinusoidal jitter model can be expressed as (15) whose electron density changes in a sinusoidal shape over time:

\[
N_e(t) = N_{e0} \times (1 + A \cdot \sin(2\pi F_r t)),
\]

(15)

Here, \(N_e(t)\) is the electron density of plasma sheath, \(N_{e0}\) is the offset of the electron density, \(A\) is the jitter value of the electron density, and \(F_r\) is the frequency of the plasma sheath change.

The Gaussian jitter model can be expressed as (16), whose electron density changes in Gaussian shape over time:

\[
N_e(t) = \overline{N_e} - \Delta(t).
\]

(16)

Here, \(\overline{N_e}\) is the average electron density and \(\Delta(t)\) is the coefficient of variation. The change law of the electron density satisfies the pink noise distribution; that is, the spectrum energy is inversely proportional to the frequency.

In the simulation of this paper, the basic model of the electron density distribution of the plasma sheath adopts the double Gaussian nonuniform distribution model. The dynamic model uses Gaussian jitter model with random jitter in the form of pink noise. The information transmission rate is 9 GHz.

The plasma channel is modeled by the wave impedance method. It is assumed that the source information is equally probabilistic and the channel is a symmetric channel. The modulated signal passes through a dynamic plasma sheath, and then the transition probability matrix between the received signal and the transmitted signal is calculated. Finally, the channel capacity under different modulation modes is obtained according to (13).

3.2. Simulation Results. The communication performance assessment of the typical telemetry modulations is essential to the research on the blackout problem. Channel parameters such as the electron density and collision frequency of the plasma sheath reflect the changing trend of the plasma sheath. Although these parameters can be measured by physical means, they cannot directly reflect the change of the channel communication ability. The channel capacity is an important parameter to evaluate the performance of a communication system. In this section, we take BPSK and QPSK signals as examples to evaluate and compare the channel capacity performance of MPSK under the dynamic plasma sheath at different signal-to-noise ratio (SNR) and dynamic parameters. This conclusion has important implications for researching blackout problem under the plasma sheath.

3.2.1. Influence of SNR on Channel Capacity. In order to prevent the influence of other parameters of the plasma sheath, we fixed the jitter variance, collision frequency, and thickness of the plasma sheath to 0.2, 1 GHz, and 30 mm, respectively.

Figure 4 shows the relationship between channel capacity and signal-to-noise ratio at different electron densities after BPSK and QPSK signals pass through the plasma sheath, respectively. Taking the BPSK signal as an example,
as shown by the blue curve ($N_e = 4 \times 10^{17} \text{m}^{-3}$), when the SNR is increased from $-5 \text{ dB}$ to $5 \text{ dB}$, the channel capacity is increased by about 60%. This is mainly because the proportion of useful information in the channel increases with the increase of SNR, which leads to an increase in the system transition probability, so the overall channel capacity shows an upward trend. In addition, the plasma sheath will affect the transmission of electromagnetic waves. The electromagnetic waves will be reflected and transmitted through the sheath, and the transmittable energy determines the channel capacity. When SNR is increased to $8 \text{ dB}$, the channel capacity is close to the maximum energy that the sheath can transmit when the electron density of the plasma sheath is $4 \times 10^{17} \text{m}^{-3}$. Therefore, increasing SNR has little effect on the channel capacity. At this time, the channel capacity is mainly influenced by the electron density of the plasma sheath. As shown in Figure 4(a), under the same SNR, the channel capacity decreases as the electron density increases.

As shown in Figure 4(b), compared with the BPSK signal, the QPSK signal channel capacity has the same trend with the SNR, but the latter changes faster.

The simulation results show that when the electron density of the plasma sheath is small, the channel capacity is mainly affected by the SNR. As the electron density increases, the influence of the plasma sheath’s own parameters on the channel capacity is gradually intensified. When the channel capacity is close to the maximum transmittable energy of the plasma sheath, the method of increasing the channel capacity by increasing SNR is no longer applicable.

3.2.2. Influence of Dynamic Characteristics of Plasma Sheath on Channel Capacity. The influence of the dynamic plasma sheath on electromagnetic waves has an uncertain characteristic that depends on many factors including the electron density ($N_e$), collision frequency ($v_{cn}$), jitter variance ($\sigma$), and thickness ($d$).

(1) Electron Density ($N_e$). Figure 5 shows the relationship between the channel capacity and the electron density of BPSK and QPSK. Compared with the BPSK signal, the QPSK signal has faster decay of the channel capacity when the electron density increases from $2 \times 10^{17} \text{m}^{-3}$ to $6 \times 10^{17} \text{m}^{-3}$. The main reason is that the constellation data of the phase-modulated signal is subject to severe rotational distortion due to the dynamic characteristics of the plasma. For MPSK signals, the phase range over which BPSK can correctly distinguish symbols is $\pm \pi/2$; for QPSK, this is $\pm \pi/4$. Because the phase decision range of the QPSK signal is half that of the BPSK signal, the transmission performance of the QPSK signal in dynamic plasma is worse.

(2) Collision Frequency ($v_{cn}$). The relationship between the channel capacity and the electron density of BPSK with $v_{cn}$ at 0.1 GHz, 1 GHz, and 2 GHz is displayed in Figure 6(a). It is observed that when the electron density is less than $4 \times 10^{17} \text{m}^{-3}$, the collision frequency change has little effect on the channel capacity. As the electron density increases, the impact of the collision frequency on the channel capacity increases. For example, when the electron density is $8 \times 10^{17} \text{m}^{-3}$, the channel capacity at the collision frequency of 2 GHz is 40% larger than that at the collision frequency of 1 GHz. This is mainly due to the rotation of the constellation of the BPSK signal caused by the dynamic plasma sheath, which can cause phase blurring in severe cases. The degree of rotation of the constellation is related to the dynamic parameters of the plasma sheath. Increasing the collision frequency can weaken the rotation of the constellation and reduce the decision error of the binary code. In this simulation, when the electron density is less than $4 \times 10^{17} \text{m}^{-3}$, the rotation of the
constellation is not obvious, so the change in collision frequency has little effect on the channel capacity. Figure 6(b) shows the relationship between the channel capacity and the electron density of QPSK with $v_{\text{en}}$ at 1 GHz, 2 GHz, and 3 GHz. Compared with the BPSK signal, the QPSK signal channel capacity has the same trend with the collision frequency. Meanwhile, we also find that the communication performance follows the order of $QPSK > BPSK$ at the same collision frequency.

(3) Jitter Variance ($\sigma$). The relationship between the channel capacity and the electron density of BPSK and QPSK with $\sigma$ at 0.01, 0.05, 0.1, and 0.2 is displayed in Figure 7. It is observed that when the jitter variance of the plasma is less than 0.01, the channel capacity is basically not affected by the electron density. For the BPSK signal, although the overall channel capacity is smaller than the QPSK signal, the fluctuation caused by the jitter variance is smaller than the QPSK signal. As the jitter variance increases, the channel capacity decreases dramatically. Taking the electron density of $8 \times 10^{17} \text{ m}^{-3}$ as an example, when the jitter frequency is 0.05, 0.1, and 0.2, the channel capacity of the BPSK signal is reduced by 20%, 50%, and 80%, respectively. Under the same channel conditions, the channel capacity of the QPSK signal is reduced by 65%, 80%, and 90%, respectively. The increase of the jitter variance intensifies the rotation of the constellation diagram, which increases the bit error rate. Since the decision tolerance of the QPSK signal is smaller than the BPSK signal, the QPSK signal is more significantly affected by the jitter variance.

(4) Thickness ($d$). The relationship between the channel capacity and the electron density of BPSK and QPSK with $d$ at 0.01 m, 0.03 m, 0.06 m, and 0.09 m is displayed in Figure 8. It is observed that the channel capacities of BPSK and QPSK both decrease with the increase of the thickness of the plasma sheath. The thickness of the plasma sheath mainly affects the energy of transmitted electromagnetic waves. The previous theoretical section described the layered model of the plasma sheath. The total attenuation of the electromagnetic wave by the plasma sheath is equal to the sum of the attenuation of each layer. The increase of the plasma thickness means the increase of the number of layers. Therefore, an increase in the thickness of the plasma sheath will reduce the channel capacity.

4. Conclusion

A channel capacity estimation method for dynamic plasma sheath was investigated. The nonlinear dynamic plasma...
channel can be equivalent to a discrete input continuous output memoryless channel. On this basis, the channel capacity calculation method for dynamic plasma sheaths under different modulation systems can be derived from the Shannon formula. DO here variation of the PSK channel capacity under the dynamic plasma sheath channel can be summarized as follows:

1. Parasitic modulation effects can cause severe rotation and distortion of the constellation of the phase-modulated signal. For MPSK signals, the larger the $M$ is, the smaller the phase determination range is. Therefore, the larger the $M$ in the dynamic plasma sheath is, the worse the error resistance of the signal is and the faster the channel capacity will decrease.

2. The channel capacity of the dynamic plasma sheath is affected by both the SNR and the dynamic parameters of the plasma. When the electron density is small, the channel capacity is mainly affected by the SNR. As the electron density increases, the dynamic parameters of the plasma gradually become the main

![Figure 6: Influence of $v_{en}$ on channel capacity. (a) BPSK, (b) QPSK.](image)

![Figure 7: Influence of $\sigma$ on channel capacity. (a) BPSK, (b) QPSK.](image)
factor affecting the channel capacity. Therefore, for dynamic plasma channels, improving the signal-to-noise ratio alone does not necessarily increase the channel capacity. To improve the channel communication quality, we need to change the plasma electron density, collision frequency, and jitter variance by other means.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grants nos. 61627901 and 61601353, the National Basic Research Program of China under Grant no. ICKY2016110C040, the National Key Basic Research Program of China under Grant no. 2014CB340205, and the Aerospace TT&C Innovation Program under Grant no. 201701B.

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