Research Article

Spectrum Sharing with Vehicular Communication in Cognitive Small-Cell Networks

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An increasing number of vehicles make spectrum resources face serious challenges in vehicular cognitive small-cell networks. The means of spectrum sharing can greatly alleviate this pressure. In this paper, we introduce a supermodular game theoretic approach to analyze the problem of spectrum sharing. The small-cell BS (primary service provider, PSP) and the vehicle (secondary service provider, SSP) can share the spectrum, where the PSP can sell idle spectrum resources to the SSP. This is taken as a spectrum trading market, and a Bertrand competition model is considered to depict this phenomenon. Different PSPs compete with each other to maximize their individual profits. The Bertrand competition model can be proved as a supermodular game, and the corresponding Nash equilibrium (NE) solution is provided as the optimal price solution. Hence, an improved genetic simulated annealing algorithm is designed to achieve NE. Simulation results demonstrate that the NE point for the price of the primary service provider exists. The change of the exogenous variable is also analyzed on the equilibrium point.

1. Introduction

With the improvement of people’s living standards and the continuous development of vehicular networks technologies, more and more people choose to buy a car as a means of transportation [1]. The explosive growth of the number of vehicles directly changes the traditional car network communication. Increased vehicle data business and bandwidth-hungry applications in Long Term Evolution Vehicle-to-Everything (LTE-V2X) networks are a challengeable problem, and inappropriate handling way will cause quality of service (QoS) deterioration [2]. Hence, reasonable use of base station (BS) resource and cognitive radio can effectively improve QoS of vehicular communication for small-cell networks. In this context, vehicle as unlicensed user can adopt idle frequency bands which are not occupied by licensed users through trading to enhance the utilization of spectrum and capacity of small-cell networks. For spectrum trading, the price of spectrum resource is the key consideration based on licensed and unlicensed users.

Game theory is an effective tool to help analyze strategy selection problem. In fact, many works adopt it to study and analyze spectrum sharing problem. The spectrum sharing problem can be translated into the spectrum trading problem with different types, such as market equilibrium and cooperative and competitive market, and the price with analysis of the proposed spectrum market equilibrium has been studied in [3]. In [4] and [5], the authors studied the spectrum sharing problem for two PSPs and multiple PSPs on solving competitive market, respectively. Most of these works are carried out on cognitive networks, and fewer applications are in small-cell networks, especially for vehicular communication.
The explosive growth in the mobile devices puts a heavy burden on the vehicular networks in terms of increased spectrum resources and operational costs. Scarcity of spectrum resources has always been an important challenge for vehicular communication. Inspired by [6], we consider spectrum trading between PSPs and SSPs to improve spectrum efficiency and utilization in this paper. We utilize the smooth supermodular game theory to model the spectrum sharing issue from the perspective of spectrum price. Also, we derive the solution of spectrum pricing model. To the best of our knowledge, these problems have not been studied in existing literature.

The main contributions of this paper are listed as follows:

1. We propose the Bertrand competition model on spectrum sharing problem between small-cell BSs and vehicles in vehicular cognitive small-cell networks. Furthermore, the proposed Bertrand competition game model is verified as a supermodular game in detail.

2. We present the solution of the Bertrand competition model. Specifically, we can adopt genetic simulated annealing algorithm to decide the NE of Bertrand competition model quickly and maintain the optimal spectrum efficiency and utilization.

3. Based on numerical results, we verify that the proposed Bertrand competition model can solve the spectrum sharing problem of the vehicular cognitive small-cell networks. We also find out that the spectrum replacement coefficient brings significant influences on the spectrum price in the process of Bertrand competition game.

The rest of this paper is organized as follows. Section 2 describes the related works that address the spectrum sharing problem in cognitive small-cell networks. In order to promote our research, the basic theoretical knowledge is given in Section 3 to facilitate researchers’ reading. In Section 4, we provide the scenario description and system model of Bertrand competition between the PSP and the SSP. Based on Section 4, Section 5 discusses the solution of the proposed spectrum pricing model. In Section 6, simulation results display the numerical performance analysis. Finally, Section 7 summarizes and concludes this paper.

2. Related Works

Cognitive radio techniques can be introduced to be applied to spectrum access networks [7]. Spectrum management technology satisfies requirements of users in cognitive radio networks by adjusting and controlling the spectrum access network. Generally, researchers designed an optimization problem to find the optimal solution which could improve the utility for the users [8]. Reference [9] analyzed secondary user using a continuous-time Markov chain model to improve the performance of cognitive radio networks. Subsequently, game theory as an effective tool for spectrum management was used in cognitive radio networks [10, 11], such as rate control [12] and power control [13]. A game theory adaptive spectrum allocation scheme was presented for cognitive radio networks [14]. The cooperative competition process of the players was displayed in the game. However, these works did not consider the price problem of the players for spectrum trading in the cognitive radio networks.

For the spectrum trading problem, the spectrum resource allocation and the price of spectrum are related to each other. The service provider wants to get the maximizing benefit and the user wants to get the most benefits on QoS and price. In [15], a spectrum allocation algorithm based on static game model was proposed in cognitive radio networks. In particular, the utility function was given by the dynamic pricing function and cost function of the primary user. Reference [16] considered transmission rate and reliability in designed utility maximization scheme with price. In [17], price-based resource allocation schemes were investigated for femtocell networks. A Stackelberg game approach was proposed to maximize the joint utility with the macrocell and femtocell networks on interference power constraint. Reference [18] studied resource allocation in wireless ad hoc networks and proposed price-based resource allocation algorithm to achieve an optimal resource allocation and fairness among competing users. However, these works only considered the spectrum price problems in static spectrum access scenarios. Then, some researchers studied the problem of price in dynamic spectrum scenario [19–21]. Reference [19] described resource allocation based on space, time, and frequency for cognitive radio wireless networks. It formulated the new cognitive radio system for the primary user transmission and gave the corresponding dynamic resource allocation solution. In heterogeneous applications networks, [20] designed resource allocation algorithm by making online control decisions for a virtualized data center. The proposed optimal resource allocation algorithm could improve joint utility of the heterogeneous applications throughput of the data center. Cloud computing technology could assist the users achieve their requirements for resource usage. Based on cloud computing [21], authors proposed a dynamic data allocation method according to application demands and further designed the concept of green computing to optimize the number of users. However, these works did not discuss the effect of equilibrium and stability of the competition with the users on pricing.

Most works in cognitive radio networks considered competition and taken spectrum sharing into noncooperative game account [22–25]. In [22], the authors considered interaction between the spectrum holder, the service provider, and the user to propose a three-layer spectrum allocation model for dynamic spectrum access networks. Yang et al. [23] designed a market-based model for primary users and secondary users and proposed a pricing-based spectrum allocation mechanism to enhance spectrum utilization and revenue in cognitive radio networks. This method has higher complexity and lower spectrum utilization due to the pricing contention. To address this issue, one novel pricing-based spectrum allocation scheme was designed in [24] to reduce complexity and to enhance spectrum utilization. The primary user’s utility and its revenue were also studied in [24].
when the spectrum of primary user was controllable. These previous works considered the primary user’s revenue ignoring the case of secondary users in trading market. Hence, Li et al. [25] studied the choice of the secondary users facing different spectrum qualities. It focused on the optimal pricing solution in secondary buyers by designing a novel spectrum pricing mechanism for secondary users according to their selection preferences. These works considered pricing and spectrum allocation by formulating an optimization problem. However, other applications in spectrum allocation would be useful to help reader to further solve the spectrum sharing problem.

More recently, the interest in cellular-based vehicular networks research had been dramatically investigated [26]. As far as we know, the spectrum resource applications in vehicular communication had been widely discussed by the authors in [27, 28]. In cellular networks, device-to-device (D2D) technology can provide efficient and reliable vehicular communications. The high-speed movement characteristic of the vehicle brings great challenges for spectrum sharing and power allocation due to large-scale fading channel information. Liang et al. [27] proposed an optimal resource allocation algorithm which was robust to channel information in vehicular networks. Meanwhile, this paper considered sum capacity of all vehicle-to-infrastructure (V2I) links and taken it as an optimization objective to enhance V2I link throughput. Traffic safety and efficiency are important considerations in vehicular networks. Vehicle platooning could improve these two aspects by sharing and exchanging control information among vehicles. In [28], a resource allocation algorithm based on platoon and users’ transmission rate was proposed to improve spectrum resource utilization and the platoon stability. Zhang et al. [29] focused on V2I communication scenario and proposed a joint power and subcarrier allocation mechanism to improve QoS of vehicular networks. Cellular V2X communication has been studied in [30], which proposed two modes on LTE vehicular communications. The native features for time-division LTE were also given for the centralized architecture, and the radio resource allocation scheme was optimized for better supporting V2I communication. Note that these previous works considered resource allocation in cellular-based vehicular networks. However, most proposed schemes were complicated and few works analyzed the spectrum sharing issues for vehicular communication in cognitive small-cell networks.

3. Preliminary Knowledge

3.1. Supermodular Game. The supermodular game provided a general method based on the lattice planning for analyzing games with complementary strategies [31]. Hence, it did not require the convexity and the differentiability hypothesis in the traditional optimization theory, but only a certain order structure of the strategy space and certain weak continuity and monotonicity of the objective function were needed. It had a pure strategy Nash equilibrium, and the Nash equilibrium set also had a certain order structure.

The following describes some properties of the supermodular game: In set \( \Phi \), there are any two elements, \( x \) and \( y \), and their upper and lower bounds are all in set \( \Phi \). The upper bound is marked as \( x \vee y \) and the lower bound is denoted as \( x \wedge y \). Hence, set \( \Phi \) is called lattice. \( f \) is the function which is from the lattice \( S \) to real number \( R \), denoted by \( f: S \rightarrow R \). If \( \forall x, y \in \Phi \),

\[
f(x) + f(y) \leq f(x \wedge y) + f(x \vee y),
\]

then \( f(x) \) is a supermodular function in set \( \Phi \). Furthermore, if \( \log f(x) \) is a supermodular function, \( f(x) \) is a log-supermodular function. Supermodular functions are widely used in various disciplines, such as economics and engineering science. For economic price market, supermodularity represents a complementary investment in the economy. Here, the supermodularity is taken as cardinal property as the value range of the function is the set of real numbers \( R \). In addition, the quasi-supermodularity is the supermodular ordinal numbers corresponding to the cardinal supermodular numbers [32]. If \( \Phi_1 \) is a lattice and \( \Phi_2 \) is a poset, there is a function mapping relationship, \( f: \Phi_1 \rightarrow \Phi_2 \). For any two elements \( x' \) and \( x'' \), if \( f(x' \wedge x'') < f(x') \),

\[
f(x') < f(x' \vee x''),
\]

then function \( f(x) \) is a quasi-supermodular function in set \( \Phi_1 \). Apparently, if one function is a supermodular function, it must also be a quasi-supermodular function.

In supermodular game, increasing difference is another important concept. Supposing \( \Phi_1 \) and \( \Phi_2 \) are two lattices, there is a function mapping relationship, \( f: \Phi_1 \times \Phi_2 \rightarrow R \). For any two elements \( x \) and \( x' \), \( x \geq x' \), in set \( \Phi_1 \), \( f(x, y) - f(x', y) \) has incremental relationship for any element \( y \) in set \( \Phi_2 \). Therefore, function \( f(x, y) \) has increasing difference on \( x \).

Assume that the game strategy space is continuous; the real number \( R \) has the usual order relationship. In the \( n \) dimension Euclid space, \( x \geq y \Rightarrow x_i \geq y_i, i \in \{1, 2, \ldots, n\} \) for vectors \( x = (x_1, \ldots, x_n) \) and \( y = (y_1, \ldots, y_n) \). Generally, the object function is smooth supermodular. The following lemmas describe the relationship between supermodular and increasing difference on object smoothness function.

Lemma 1 (see [33]). If and only if the function \( f \) has increasing difference in \( R^n \) (increasing difference in multiple variables means having an increasing difference in any two pairs of variables), \( f \) is supermodular function.

Lemma 2 (see [33]). For a interval \( W = [x, X] \) in Euclid space \( R^n \), if the function \( f: R^n \rightarrow R \) is twice continuously differentiable in an open interval including \( W \), then the necessary and sufficient condition for \( f \) to be a supermodular function on \( W \) is

\[
\frac{\partial^2 f(x, y)}{\partial x_i \partial x_j} \geq 0, \quad \forall i \neq j.
\]
that the cross partial derivative is nonnegative, the second-order partial derivative is not required, and the concave of the function is not needed. Section 3.2 will further elaborate the characteristics of the smooth supermodular game.

3.2. Smooth Supermodular Game. Supposing that a non-cooperative game \( \Omega = [N, \{\Phi_i\}_{i \in N}, \{f_i\}_{i \in N}] \), \( \Phi_i \in \mathbb{R}^k \), \( N \) denotes the number of players in a game, the subscripts are denoted by \( i \) and \( j \), and there are \( k_i \) strategy elements for the player \( i \), subscripted by \( n \) and \( m \). \( \Phi_i \) is the strategy set for each player \( i \), and \( f_i \) is a supermodular function which represents a relationship between \( \Phi_i \) and real number field \( R, f : \Phi_i \rightarrow R \). \( x \) and \( y \) represent any two elements in set \( \Phi_i \). If game \( \Omega \) satisfies the following conditions,

1. \( \Phi_i \) is the closed interval in \( \mathbb{R}^k \);
2. \( f_i \) is twice continuously differentiable in \( \Phi_i \);
3. \( f_i \) is supermodular on \( \Phi_i \), that is,

\[
\frac{\partial^2 f_i(x,y)}{\partial x_i \partial x_j} \geq 0, \quad (4)
\]

where \( n \neq m, 1 \leq n, m \leq i \);
4. \( f_i \) has increasing differences in \( \Phi_i \), that is,

\[
\frac{\partial^2 f_i(x,y)}{\partial x_i \partial x_j} \geq 0, \quad (5)
\]

where \( j \neq i, 1 \leq n \leq k_i, 1 \leq m \leq k_j \);
then \( \Omega \) is a smooth supermodular game.

4. Scenario Description and System Model

4.1. Scenario Description. To improve spectrum resources scarcity and efficiency, it is a beneficial method to share spectrum among multiple users in wireless networks. The users compete for spectrum resources in order to maximize their own benefits. Against this background, one typical spectrum sharing model with vehicular communication for cognitive small-cell networks is depicted in Figure 1. Small-cell BSs are randomly deployed in small-cell networks, and every small-cell network has only one small-cell BS, which has the same transmission power, to provide services for attaching a few vehicles. The moving vehicle enters one small-cell network and builds communication link to small-cell BSs. We assume the vehicle has a large amount of burst data that needs to be transmitted. Then, the vehicle can charge small-cell BS for idle frequency spectrum resource usage to enhance data transmission efficiency. For the convenience of the following description, the small-cell BS is referred to as PSP and the vehicle is referred to as secondary service provider (SSP). The SSP requests leased spectrum and the PSPs can sell their idle spectrum resources to the SSP. So far, a spectrum trading market is formed. In this way, the SSP can lease spectrum of multiple PSPs. We assume that different PSPs know about each other’s existence and compete with each other to maximize their individual profits.

In spectrum sharing model of cognitive small-cell networks, there are \( N \) PSPs, and the PSP \( i, i \in \{1, 2, \ldots, N\} \), has its own \( B_i^{\text{max}} \) spectrum bandwidth to serve \( M_i \) primary users (PU). The price of unit spectrum with \( B_i^{\text{max}} \) is denoted by \( p_i \). When SSP agrees to the \( p_i \) of PSP \( i \), it will adopt the idle spectrum resource of the corresponding PSP. In Additive White Gaussian Noise (AWGN) channel, an SSP adopts adaptive modulation for data transmission in communication link.

4.2. Wireless Communication. Based on transmission channel quality, the transmission rate of the SSP, by adjusting the corresponding channel parameter settings, will be changed. Hence, the spectral efficiency (bits/sec/Hz) can be expressed as [34]

\[
k_i = \log_2 (1 + \gamma_k), \quad (6)
\]

where \( \gamma_k \) is signal-to-noise ratio (SNR) between the SSP and the \( k \)th PSP, \( K = 1.5/\ln (0.2/\text{BER}_{\text{tar}}) \), and \( \text{BER}_{\text{tar}} \) is the target bit error rate (BER) of the \( k \)th PSP.

4.3. Price Problem Model. As described in Section 4.1, the spectrum sharing problem in this paper can be designed as the competitive pricing model. As the PSP is aware of the existence of other PSPs, all of the PSPs compete with each other to maximize their individual profit in trading market. Each PSP has a selfish character, only considers its own profits, and performs spectrum resource management according to its own information and policies. Obviously, all PSPs compete for price. Therefore, one PSP could set a price...
on its own spectrum to maximize its individual profit when the spectrum price in networks is given by the other PSPs. This is a strategy choice problem.

5. Solution of Spectrum Pricing Models

5.1. Demand Function of the SSP. To calculate spectrum demand, we consider the quadratic utility function [35] and design the demand function; that is,

\[ D_i(\bar{p}, \epsilon) = w_i k_i^r - p_i + \epsilon \sum_{j \neq i} p_j, \]  

where the price vector \( \bar{p} \) has element \( p_i \) composition, \( w_i \) is weight parameter, the product of \( w_i \) and \( k_i^r \) represents the market capacity of the spectrum \( B_i^{\text{own}} \), \( k_i^r \) denotes the transmission spectral efficiency of the SSP, and \( \epsilon \) denotes the spectrum replacement factor. That is, the value of \( \epsilon \) reflects spectral similarity and shows whether SSP can switch on its own spectrum to maximize its individual profit when the spectrum price in networks is given by the other PSPs. This is a strategy choice problem.

\[ \frac{\partial D_i(\bar{p}, \epsilon)}{\partial p_i} = D_i(\bar{p}, \epsilon) + (p_i - c_i) \frac{\partial D_i(\bar{p}, \epsilon)}{\partial p_i} - 2 \frac{d_i k_i^r}{M_i} \left( B_i^{\text{req}} - k_i^r D_i(\bar{p}, \epsilon) \right), \]

5.2. Profit Function of the PSP. For Bertrand game, there is no cooperation between PSPs, and strategy variable is the price vector \( \bar{p} \). Hence, the profit function of the PSP can be calculated as

\[ u_i(\bar{p}, \epsilon) = (p_i - c_i)D_i(\bar{p}, \epsilon) - d_i \left( B_i^{\text{req}} - k_i^r B_i^{\text{own}} - D_i(\bar{p}, \epsilon) \right)^2, \]

where \( c_i \) is the unit spectrum of the \( D_i(\bar{p}, d) \), \( d_i \) is the difference factor, and \( B_i^{\text{req}} \) is the PSP connection required bandwidth. We know from the system model that the PSP \( i, i \in \{1, 2, \ldots, N\} \) has a size of \( B_i^{\text{own}} \) spectrum and it serves \( M_i \) PU.

Hence, the item \( p_i D_i(\bar{p}, \epsilon) \) indicates the profit from sharing the spectrum with SSP from the PSP’s rental spectrum, and the item \( c_i D_i(\bar{p}, \epsilon) \) denotes the cost of the PSP \( i \) for renting out spectrum \( D_i(\bar{p}, \epsilon) \). Obviously, the cost of PSP consumption is due to sharing the spectrum with the SSP, resulting in a QoS performance degradation of PU.

5.3. Supermodular Game Argumentation. Combining the conditions satisfied by the smooth supermodular game, it is easy to judge that equation (8) satisfies the two smooth supermodular game conditions (1) and (2).

First, the first-order derivative of profit function \( u_i(\bar{p}, \epsilon) \) is derived as follows:

\[ \frac{\partial u_i(\bar{p}, \epsilon)}{\partial p_i} = D_i(\bar{p}, \epsilon) + (p_i - c_i) \frac{\partial D_i(\bar{p}, \epsilon)}{\partial p_i} - 2 \frac{d_i k_i^r}{M_i} \left( B_i^{\text{req}} - k_i^r B_i^{\text{own}} - D_i(\bar{p}, \epsilon) \right) \times \frac{\partial D_i(\bar{p}, \epsilon)}{\partial p_i}. \]

By equation (7), the following equality is derived:

\[ \frac{\partial D_i(\bar{p}, \epsilon)}{\partial p_i} = -1, \]

\[ \frac{\partial D_i(\bar{p}, \epsilon)}{\partial p_j} = \epsilon. \]

By substituting equations (10) and (11) into (9), the second derivative of profit function \( u_i(\bar{p}, \epsilon) \) on \( p_i \) is expressed as

\[ \frac{\partial^2 u_i(\bar{p}, \epsilon)}{\partial p_i \partial \epsilon} = \left( 1 + 2d_i \left( \frac{k_i^r}{M_i} \right)^2 \right) \epsilon. \]

Obviously, \( (1 + 2d_i (k_i^r/M_i)^2) \epsilon \geq 0 \), so condition (3) with smooth supermodular game is satisfied.

Finally, the second derivative of profit function \( u_i(\bar{p}, \epsilon) \) on \( p_i \) and \( \epsilon \) is denoted as

\[ \frac{\partial^2 u_i(\bar{p}, \epsilon)}{\partial p_i \partial \epsilon} = \left( 1 + 2d_i \left( \frac{k_i^r}{M_i} \right)^2 \right) \sum_{j \neq i} p_j. \]

From equation (13), \( (1 + 2d_i (k_i^r/M_i)^2) \sum_{j \neq i} p_j \geq 0 \), condition (4) is also satisfied for \( \epsilon \). In conclusion, the game is smooth supermodular game. There are a largest and a smallest NE in pure strategies and they are nondecreasing functions of the parameter \( \epsilon \).

According to the basic concept of smooth supermodular game [33], the game with profit function in equation (8) is a supermodular game on Bertrand competition. So, the largest and smallest NE price \( \left( \frac{\partial u_i(\bar{p}, \epsilon)}{\partial p_i} \right) / \epsilon = 0 \) for PSP \( i \) on pure strategies exist and are nondecreasing functions.

5.4. Solution of NE. To solve for \( p_i \) in equation (9), the reaction function method is applied. That is, the best strategy for the PSP \( (p_i) \) is determined by the strategy of the other PSPs. The set of price on the other PSPs is \( \bar{p} \). \( \bar{p} = \bar{p} \cup \{p_i\} \). \( \bar{p} \) indicates the vector of all PSPs \( \{1, 2, \ldots, i, \ldots, N\} \) except for the PSP \( i \). The reaction function is defined as

\[ \pi_i(\bar{p} \cup \{p_i\}) = \arg\max_{p_i} u_i(\bar{p} \cup \{p_i\}) \epsilon. \]

If and only if

\[ p_i^* = \pi_i(\bar{p}^* \cup \{p_i\}), \quad \forall i, \]

then, \( \bar{p}^* = \{p_1^*, p_2^*, \ldots, p_i^*, \ldots, p_N^*\} \) denotes NE of bidding game, where \( p_i^* \) is the best strategy set of the PSP \( j \neq i \). In a nutshell, the PSPs can obtain an optimal benefit when equation (15) holds. The price at this time can be regarded as the optimal pricing.

From the above analysis, the computational complexity of solving equilibrium points is greatly reduced. Different from traditional game theory methodology, the second-order partial derivative and the concaveness of the profit function are not noticed. In this paper, we only consider the nonnegativity condition of the profit function with cross
5.5. Improved Genetic Simulated Annealing Algorithm. The genetic algorithm has underdeveloped solution precision on optimization function. However, it is suitable for nonlinear, nonconvex optimization problem, and the genetic algorithm can effectively solve this problem. The simulated annealing algorithm can be introduced to avoid the search process from falling into the local optimal solution as it has strong local search ability. In this paper, an improved genetic simulated annealing algorithm, which combines the genetic algorithm with the simulated annealing algorithm, is designed to be applied to cognitive small-cell networks. The individual choice in the genetic algorithm is replaced by annealing selection to avoid falling into local optimal solution.

Based on the proposed algorithm, the optimal solutions with equation (14) will be reached. The algorithm flowchart is given in Figure 2. Compared with the traditional genetic algorithm, the proposed improved genetic simulated annealing algorithm searches for the global optimal solution starting from a set of randomly generated initial solutions. In the process of performing genetic operations such as selection, crossover, and mutation, the annealing operation is performed independently on the individual, and the results are taken as individuals in the next generation population. This process is iterated until a certain convergence condition is reached.

In the following, the specific working process of the proposed genetic simulated annealing algorithm is given in Algorithm 1. It is worth noting that this algorithm is a direct application to the analysis of spectrum sharing problems because it is easy to understand. A simple and summarized comparison between traditional genetic algorithms and the proposed genetic simulated annealing algorithm is ignored as a result of the limited scope of this article. The discussions and comparisons of the main contents which are based on this algorithm will be discussed in detail.

6. Simulation Results

This part simulates and discusses the proposed equilibrium game market. The system model with spectrum sharing of cognitive small-cell networks is considered in Figure 1. We only consider the ability difference for the spectrum sharing system, but the utility functions of all the PSPs are the same. There are four PSPs, and each PSP has \( N_i = 10 \) vehicles (as PU). The total available bandwidth of PSPs is 200 MHz, and the service rate required for each PSP connection is 2 Mbps. The target BER is set as \( 10^{-4} \), and the weight parameter for each the PSP is the same, \( w_i = 20 \). The received SNR of the SSP is variable in 10–20 dB. The value of the spectrum replacement coefficient (difference factor) is variable in 0.0–1.0. Some of these parameters may change in a specific simulation environment.

The price per spectrum unit versus iteration for four primary services is analyzed in Figure 3. Due to the demand for change and profit competition for the PSPs, price is varied for all iterations and it only changes at a fixed value for each iteration. Obviously, the price increases with the number of iterations until a stable state (NE) is reached when difference factor \( \epsilon = 0.5 \). In this case, any primary service of the PSP cannot improve its own profit, despite increasing its price. Finally, the four primary services at these prices will share their spectrum with the SSP.

Figure 4 depicts the price versus the iteration times on different spectrum replacement factor \( \epsilon \) for one primary service. It is shown that the gain achieved by the proposed genetic simulated annealing algorithm becomes more significant as the number of iterations becomes larger. The price will show a significant difference as the number of iterations increases. The price increases as \( \epsilon \) increases, and the price increase is obvious when \( \epsilon \) takes more than 0.5. This result is also verified in Table 1.

Specifically, the spectrum replacement factor, \( \epsilon \), causes some influence on price in the process of spectrum game, and the results are shown in Table 1. Apparently, the price...
of PSP increases drastically as the value of $\varepsilon$ increases. This is because the increase of $\varepsilon$ makes the SSP easier to switch between different PSP spectra. Increased spectrum demand causes price increases. We can get the interesting result that the price for different PSPs get a sudden increase when $\varepsilon \approx 0.5$. This can provide us with a reference when designing a Bertrand competition in cognitive small-cell networks. It is worth noting that the pricing results are optimal and the PSPs obtain maximum benefits on different $\varepsilon$ when PU = 10. As these results represent an optimal strategy, it is necessary to reveal convergence determined by the real situation.

Figure 5 shows the impact of different $\varepsilon$, where $\varepsilon$ varies from 0 to 1 at intervals of 0.1, on the equilibrium game market to reach a stable state when the price follows iteration changing gradually. Expectedly, the supermodularity of profit function on substitutability coefficient $\varepsilon$ could achieve relative statics. In general, small changes in parameters could cause changes in equilibrium points due to feedback among the different player's strategies. However, it can be seen from the analysis that the supermodularity only tells us the impact of $\varepsilon$ on price strategies $\bar{p}$, not an accurate value. In other words, the supermodularity has weak comparative statics on $\varepsilon$. When $\varepsilon$ gradually increases, it implies that the SSP (vehicle) can switch easily to select suitable frequency spectrum at different PSPs, and the price in equilibrium point also increases correspondingly.
The impact of the number of PSPs on spectrum price is displayed in Figure 6. For convenient observation, we give the statistical results when the number of iterations is 20. Other simulation conditions are unchanged. The spectrum price is gradually decreasing with the increase in the number of PSPs due to higher competition. In spite of increasing the spectrum demand, the PSPs need to reduce their price during competition with each other in order to attract more spectrum demand from the SSP. This can maximize their own interests.

When weight parameter and SNR at the SSP are fixed, the price of the PSP increases with the increase in the spectral substitution coefficient. The significant change in price is at about 0.8 for spectral substitution coefficient. This shows that, as the spectral substitution increases, the degree of spectrum difference decreases. This result is displayed in Figure 7.

7. Conclusions

In this paper, the spectrum trading between the PSP and the SSP is considered as a noncooperative game. During the game, the players are PSPs, the unit spectrum price is the strategy, and the implementation of the corresponding strategy in the game of the PSP is the benefit. A Bertrand model is adopted to analyze the competition process. All PSPs compete with each other to achieve the highest individual profits. The supermodular concept is introduced to simplify analysis, and further the established Bertrand competition model is proved to follow the supermodular model. Meanwhile, the solution of NE point is also developed for the spectrum pricing model, and a genetic simulated annealing algorithm on NE solution is proposed. Finally, we provide the existence of NE with the spectrum unit price for the PSPs and the comparative static analysis is also described on the relationship between the price in NE and the corresponding coefficient.

In future work, the applications of the novel idea related with the genetic algorithm and some improved genetic algorithms in solving spectrum sharing problems will be analyzed and discussed in detail.

Data Availability

The input data used to support the results of this research are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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