Research Article

Numerical Simulation of Galvanic Corrosion between Carbon Steel and Low Alloy Steel in a Bolted Joint

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The galvanic corrosion of a bolt joint combining carbon steel end plate and low alloy steel bolt was investigated electrochemically in a 1 M HCl solution. The corrosion parameters of the joint components were used for numerical simulation using Comsol Multiphysics software to analyze the galvanic corrosion behavior at the contact zone between the head bolt and the end plate. In this research work we evaluate the variation of the corrosion rate in the steel end plate considered as the anode, in order to determine the lifetime of the bolted assembly used in steel structures. Three materials (20MnCr5, 42CrMo4, and 32CrMoV13) and three bolts (M12, M16, and M20) were tested in two thicknesses of electrolyte 1 M HCl ($e = 1$ mm, $e = 20$ mm). It is found that the corrosion rate of the anode part (end plate) is higher for 32CrMoV13 materials and it increases if both diameter of the bolt and thickness of the electrolyte increase ($\text{Cr}(M20) > \text{Cr}(M16) > \text{Cr}(M12)$ and $\text{Cr}(e = 20 \text{ mm}) > \text{Cr}(e = 1 \text{ mm})$). This corrosion rate is higher in the contact area between the bolt head and the end plate, and it decreases if we move away from this contact area.

1. Introduction

Galvanic corrosion can simply be defined as the corrosion that occurs as a result of one metal being in contact with another in a conducting, corrosive environment. The corrosion is stimulated by the potential difference that exists between the two metals: the more noble material acting as a cathode where some oxidizing species is reduced and the more active metal, which corrodes, acting as the anode. The anodic reaction is, by definition, some form of metal dissolution; the cathodic reaction is, in the majority of practical situations, either oxygen reduction or hydrogen evolution, or a combination of both. Many factors affecting galvanic corrosion are already discussed to determining whether or not galvanic corrosion will occur in a particular instance and if so at what rate; it is important when considering the theory of galvanic corrosion to be aware of these factors including electrode potential, reaction kinetics, alloy composition, protection film characteristics, bulk solution environment, total geometry, and type of joint [1].

There is a high incidence of past scientists taking an interest in corrosion to understand what causes it and what limits or accelerates the process. Numerous studies have

been conducted; some take a more global outlook [2], whereas some take a more focused approach [3]. The study conducted in [2] looked at many different galvanic couples commonly used in seawater applications. The study focused on developing reasonable models for systems experiencing varying periods of exposure to the corrosive environment.

Simulation of galvanic corrosion between magnesium and aluminum has been performed by Lacroix et al. [4], Deshpande [5–7], Jia et al. [8], and Trinh et al. [9] who have studied the corrosion of magnesium alloys in contact to mild steel under static conditions. The publications of Murer et al. [10–12] and Shi and Kelly [13] in this context also gave an extended insight into the topic, especially in the very important choice of boundary conditions. New studies of Sun et al. [14], who applied the mathematical approach of Yan et al. [15] to the modeling of deposit formation under seawater conditions, clearly introduce a possible way of a useful model built up for the mentioned purpose. The following studies and results are based on the progress achieved by them. Basic galvanic current density computations were modified by layer growth aspects leading to time dependent variations in the electrochemical response of the electrodes.

In the work of Höchez [18], simulation of galvanic corrosion between Mg and Al on a geometry like a self-pierce punch rivet joint has been performed. Starting from the initial model setup and related to the scientific challenge, the mathematical model requires fundamental assumptions. As always in modeling action they define the quality and accuracy of simulation results. The analysis via computer simulations showed the ability to study parameters influencing corrosion performance like the affected surface fraction by a reaction product.

In the works of Johnson and Abbott [19] and Xu et al. [20], the impact of the nature of the materials on the corrosion rate of mild steel in a galvanic coupling was studied. These articles show that the corrosion rate of mild steel changes if the material is changed in the galvanic corrosion. The addition of Cr in steel alloys improves the corrosion resistance of the steel. It is found that as the Cr content increases, the corrosion rate of the mild steel decreases.

Finite element analysis (FEA) of potential and current distributions in galvanic systems has long been studied in the literature [21–27]. Such studies are often carried out to investigate fundamental effects of electrolyte geometry [26, 28, 29], electrode kinetics [25], and unique part geometries [21]. Early modeling attempts have provided an analytical solution [26, 30] for the galvanic current and potential distribution around a galvanic junction involving two dissimilar metals kept side by side and covered by a corroding solution. These theoretical predictions were also later validated with experimental measurement of the potential and current density across the junction. Recently, semianalytical and numerical methods [29, 31] were developed to solve for current density distribution on anode and cathode in a galvanic couple and similar electrochemical phenomena such as electrodeposition. Galvanic corrosion is found to be better modeled with a continuum approach than a lattice model since the curvature of the interface is an important parameter in determining the electrochemical dissolution rate.

In our work the finite element method was used to perform a parametric study of the galvanic corrosion of a column-beam bolted assembly used in steel structures. This study takes into consideration the material of the cathode “bolt,” the thickness of the electrolyte “ e ,” the dimensions of the bolt head, and the distance from the contact area between the head bolt and the end plate in order to determine the parameters which can cause the maximum corrosion rate of the anode “end plate.”

2. Materials and Methods

2.1. Bolted Joint and Materials. Three types of materials used for the bolt were tested to predict the corrosion rate of the end plate. Thus three types of galvanic connections were studied in a bolted joint. The material composition of the elements of the assembly is stated in Table 1. The bolt was made by three types of low alloy steel which have the higher corrosion potential, and the metal of the end plate is the carbon steel (S235JR), Table 2. Galvanic corrosion therefore took place in the contact zone between end plate and head bolt in each bolted joint. In other words, the end plate acted as anode, and the bolt becomes cathode.

TABLE 1: Bolt and end plate materials.

Bolted joint	Bolt	End plate
1	20MnCr5	S235JR
2	42CrMo4	S235JR
3	32CrMoV13	S235JR

FIGURE 1: Column-beam bolted joint with end plate.

e : electrolyte thickness
 w : end plate width
 K : head height
 S : head width

FIGURE 2: Geometric characteristics of the modeling area.

2.2. Geometry and Modeling. The bolted assembly is column-beam type and it is used in steel structures, Figure 1. The developed model is Cartesian 2D type and it is used for a representation of the plane modeling of the elements entering into the galvanic corrosion, Figure 2. The modeled part is the area of contact between the bolt head, the end plate, and the electrolyte. The width of the anode (end plate) is $w = 26.5$ mm and the dimensions of the bolt head depend on the size of the bolt (Table 3).

2.3. Study Parameters. The analysis of galvanic corrosion is parametric, taking into account the effect of the bolt material, the bolt size, the electrolyte thickness, and the immersion time on the corrosion rate of the end plate, Table 4 and Figure 3.

Three types of low alloys were used for the bolt (20MnCr5, 42CrMo4, and 32CrMoV13) and will represent the cathode. For the bolt size three diameters were taken in this study (M12, M16, and M20); the dimensions that will be taken into consideration during the galvanic analysis are the dimensions of the bolt head as it is the part that will be in contact

TABLE 2: Chemical composition of materials.

	C	Cr	Mo	V	Mn	Si	P	S	Cu
S235JR	0.17	-	-	-	1.4	-	0.035	0.035	0.4
42CrMo4	0.38–0.45	0.9–1.2	0.15–0.3	-	0.6–0.9	0.4	0.025	0.035	-
32CrMoV13	0.32	3	1	0.2	0.35	0.25	<0.05	<0.05	-
20MnCr5	0.17–0.22	1–1.3	-	-	1.1–1.4	0.4	0.025	0.035	-

TABLE 3: Bolt size.

Bolt size	K (mm)	S (mm)
M12	7,5	18
M16	10	24
M20	12,5	30

TABLE 4: Study parameters for the galvanic connection.

Bolt material	Bolt size	Electrolyte thickness	Immersion time
20MnCr5	M12	1 mm	1 month
42CrMo4	M16	20 mm	6 months
32CrMoV13	M20		12 months

with the end plate. Galvanic corrosion will be studied in two thicknesses of the 1 M HCl electrolyte ($e = 1$ mm, $e = 20$ mm) and for the following period range: 1 month, 6 months, and 12 months.

3. Method and Boundaries Conditions

3.1. Analysis of the Electrode Potential Distribution. The present analysis is based on a model galvanic corrosion couple consisting of two elements shown in Figure 2. The couple consists of a cathodic element (bolt) of a width “S” in the +X direction and a height “K” in +Z direction; the width of the anode (plate) is “w” in the -X direction. The couple is covered by an electrolyte of depth “e” in the +Z direction. The electrolyte is bounded by perfect insulators at $z = e$, $x = -w$, and $x = +S$.

The various current versus electrode potential relationships for the anodic and cathodic reactions are assumed to be subject to activation control with logarithmic (Tafel) polarization behavior. Thus the net cathodic current density per unit length of corrosion couple $i_c(x)$ on the bolt at an electrode potential E_x is

$$i_c(x) = i_{0(\text{bolt})} \left\{ \exp\left(\frac{E_{\text{bolt}} - E_x}{\beta_{\text{bolt}}}\right) - \exp\left(\frac{E_{\text{bolt}} - E_x}{-\alpha_{\text{bolt}}}\right) \right\}, \quad (1)$$

where E_{bolt} is the free corrosion potential of element A, $i_{0(\text{bolt})}$ is the free corrosion current density, and α_{bolt} and β_{bolt} are the Tafel parameters for the anodic and cathodic reactions,

FIGURE 3: Study parameters for the galvanic connection. (a) $e = 1$ mm, (b) $e = 20$ mm.

respectively. Similarly for the end plate, the net anodic current density per unit length of corrosion couple $i_a(x)$ on end plate at an electrode potential E_x is

$$i_a(x) = i_{0(\text{end plate})} \left\{ \exp\left(\frac{E_{\text{end plate}} - E_x}{-\alpha_{\text{end plate}}}\right) - \exp\left(\frac{E_{\text{end plate}} - E_x}{\beta_{\text{end plate}}}\right) \right\}, \quad (2)$$

where $E_{\text{end plate}}$ is the free corrosion potential of end plate, $i_{0(\text{end plate})}$ is the free corrosion current density, and $\alpha_{\text{end plate}}$ and $\beta_{\text{end plate}}$ are the Tafel parameters for the anodic and cathodic reactions, respectively.

The equation governing the potential distribution in the electrolyte becomes Laplace's equation (for constant conductivity):

$$\nabla^2 \phi = 0. \quad (3)$$

The governing equation for the potential must be solved using software based on finite element method (FEM); this equation must be subjected to appropriate boundary conditions. At any electrode-electrolyte interface, both anodic and cathodic processes take place simultaneously and the current density through the interface is the result of the

FIGURE 4: Boundary conditions. (a) $e = 1$ mm, (b) $e = 20$ mm.

electronic exchanges in both processes. In a galvanic couple, the cathodic process dominates in the more noble metal (the bolt) while the anodic process is dominant in the less noble metal (the end plate).

3.2. Boundary Conditions. The solution of (3) is based on the boundary conditions shown in Figure 4.

For $z = 0$ the normal derivative of electrode potential E is given by

$$\begin{aligned} \frac{dE}{dz} &= \frac{i_{c(x)}}{\sigma} \quad \text{For } -w < x < 0, \\ \frac{dE}{dz} &= \frac{i_{a(x)}}{\sigma} \quad \text{For } 0 < x < S, \end{aligned} \quad (4)$$

where σ is the conductivity of the corrosion electrolyte.

Since there is no current flow normal to the insulating boundaries,

$$\begin{aligned} \frac{dE}{dx} &= 0 \quad \text{at } x = -w, \quad x = +S, \\ \frac{dE}{dz} &= 0 \quad \text{at } z = e. \end{aligned} \quad (5)$$

The numerical solution of the Laplace equation requires the definition of the electrochemical parameters of the anode, the cathode, and the electrolyte in a 1M HCl electrolyte whose characteristics are cited in the Table 5 [16]. These parameters are determined from experimental tests already carried out in the literature [17] and they are cited in Table 6.

4. Results and Discussion

4.1. Electrolyte Potential. Once the electrochemical parameters were defined, the geometry was applied, the boundary

FIGURE 5: Electrolyte potential for M12 bolt and electrolyte thickness, $e = 1$ mm at 12 months with deformed geometry. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

conditions and governing equation were applied, and the appropriate mesh was found for the geometries, the model for galvanic corrosion was solved. The resolution of Laplace's equation enables us to present electrolyte potential on the surfaces of electrodes bolt/end plate. Figures 5, 6, and 7 show us the surface plot of electrolyte potential with deformed

(a)

(b)

(c)

FIGURE 6: Electrolyte potential for M16 bolt and electrolyte thickness, $e = 1$ mm at 12 months with deformed geometry. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

TABLE 5: Electrolyte characteristics [16].

Conductivity (S/m)	33.2 (S/m)
Low thickness (mm)	1 (mm)
High thickness (mm)	20 (mm)

(a)

(b)

(c)

FIGURE 7: Electrolyte potential for M20 bolt and electrolyte thickness, $e = 1$ mm at 12 months with deformed geometry. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

TABLE 6: Electrochemical parameters in 1 M HCl [17].

	β_a (mV/dec)	β_c (mV/dec)	i_0 (A/m ²)	E_{eq} (mV)
S235JR	65	122	3.55	-553
42CrMo4	209.9	249.9	7.1	-495
32CrMoV13	263.7	173.1	10	-500
20MnCr5	106.2	124.3	1.6	-488

FIGURE 8: Electrolyte potential for M12 bolt and electrolyte thickness, $e = 20$ mm at 12 months with deformed geometry. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

geometry at different bolt sizes and different materials of bolt. It can be seen that the potential of the electrolyte increases if the bolt size increases: $E(M20) > E(M16) > E(M12)$. These potential values also depend on the bolt material: $E(32CrMoV13) > E(42CrMo4) > E(20MnCr5)$.

FIGURE 9: Electrolyte potential for M16 bolt and electrolyte thickness, $e = 20$ mm at 12 months with deformed geometry. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

The potential of the electrolyte also depends on the thickness of the electrolyte; it increases if the thickness of the electrolyte increases, and this applies to the two parameters: bolt size and bolt material, $E(e = 20 \text{ mm}) > E(e = 1 \text{ mm})$, Figures 8, 9, and 10.

FIGURE 10: Electrolyte potential for M20 bolt and electrolyte thickness, $e = 20$ mm at 12 months with deformed geometry. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

4.2. Corrosion Rate C_R . The corrosion rate was calculated using the following relation [32]:

$$V_c = \frac{I_{\text{corr}} \times M}{z \times F \times \rho}, \quad (6)$$

where V_c is the corrosion rate (cm/yr), I_{corr} is the corrosion current density (A/cm^2), M is the molar mass

FIGURE 11: Corrosion rate for electrolyte thickness, $e = 1$ mm. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

$M(\text{Fe}) = 55.85$ g/mol, z is the valence of iron $z = 2$, F is the faraday constant $F = 96500$ A·s/mol, and ρ is the density of steel $\rho = 7.87$ g/cm³.

Table 7 and Figures 11 and 12 show that the corrosion rate depends on the bolt size, the bolt material, and the electrolyte thickness. It can be seen that the corrosion rate

FIGURE 12: Corrosion rate for electrolyte thickness, $e = 20$ mm. (a) 20MnCr5, (b) 42CrMo4, (c) 32CrMoV13.

TABLE 7: Corrosion rate values as a function of bolt size, material, and electrolyte thickness.

Bolt size	Bolt material	Corrosion rate C_R of end plate (mm/yr)			
		Electrolyte thickness $e = 1$ mm		Electrolyte thickness $e = 20$ mm	
		$x = 0$	$x = -26.5$ mm	$x = 0$	$x = -26.5$ mm
M12	20MnCr5	4.55	3.65	4.70	4.40
	42CrMo4	11.80	9.40	12.30	10.90
	32CrMoV13	14.10	11.30	15.25	13.30
M16	20MnCr5	5.10	4.10	5.55	5.20
	42CrMo4	13.25	10.70	14.80	13.20
	32CrMoV13	15.30	12.50	17.70	15.60
M20	20MnCr5	5.50	4.40	6.30	5.90
	42CrMo4	14.40	11.70	17.00	15.25
	32CrMoV13	16.10	13.40	19.60	17.55

of the end plate increases if the bolt size increases: $C_R(\text{M20}) > C_R(\text{M16}) > C_R(\text{M12})$. These corrosion rate values depend also on the bolt material: $C_R(32\text{CrMoV13}) > C_R(42\text{CrMo4}) > C_R(20\text{MnCr5})$.

The corrosion rate of the end plate depends also on the thickness of the electrolyte; it increases if the thickness of the electrolyte increases, and this applies to the two parameters: bolt size and bolt material, $C_R(e = 20 \text{ mm}) > C_R(e = 1 \text{ mm})$.

5. Conclusion

The choice of bolts in galvanic bolted joints of steel structures must take into account the effect of the bolt size and bolt material. It is shown in this study that as the size of the bolt increases, the corrosion rate increases. Also if the steel alloys of the bolt change, the corrosion rate changes. The increasing of the corrosion rate in the process of galvanic corrosion of the end plate leads to a reduction in lifetime of the bolted assembly. Hence a great interest must be shown by engineers/designers in the choice of the bolts when designing the steel structures to ensure long lifetime.

Conflicts of Interest

The authors declare no conflicts of interest.

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