*Research Article*

# Membrane Transport of Nonelectrolyte Solutions in Concentration Polarization Conditions: $H^r$ Form of the Kedem–Katchalsky–Peusner Equations

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In this paper, the Kedem–Katchalsky equations in matrix form for nonhomogeneous ternary nonelectrolyte solutions were applied for interpretation of transport through the membrane mounted in horizontal plane. Coefficients $H^r_{ij}$, $H^r_{ij}$, and $H^r_{det} = \det[H^r]$ (for nonhomogeneous solutions), $H_{ij}$ and $H_{det} = \det[H]$ (for homogeneous solutions) ($i$, $j \in \{1, 2, 3\}$, $r = A$, B), $\psi_{ij} = (H^A_{ij} - H^B_{ij})/H_{ij}$, and $\psi_{det} = (H^A_{det} - H^B_{det})/H_{det}$ were calculated on the basis of experimentally determined coefficients ($L_p$, $\sigma_1$, $\sigma_2$ $\omega_{11}$, $\omega_{22}$, $\omega_{21}$, $\omega_{12}$, $\zeta^r_1$, and $\zeta^r_2$) for glucose in aqueous ethanol solutions and two configurations of the membrane system. From the calculations, it results that the values of coefficients $H^r_{12}$, $H^r_{13}$, $H^r_{22}$, $H^r_{23}$, $H^r_{32}$, $H^r_{33}$, and $H^r_{det}$ depend nonlinearly on solution concentration as well as on a configuration of membrane system. Besides, the values of coefficients $H^r_{21}$, $H_{12}$, $H_{21}$, $H_{22}$, $H^r_{33}$, and $H_{det}$ depend linearly on solution concentration. The value of coefficients $H_{13}$, $H_{23}$, and $H_{33}$ do not depend on solution concentration. The coefficients $\psi_{12}$, $\psi_{13}$, $\psi_{22} = \psi_{23}$, $\psi_{32} = \psi_{33}$, and $\psi_{det}$ depend nonlinearly on solution concentration and for $\overline{C}_1 \approx 9.24 \, \mathrm{mol \, m^{-3}}$ are equal to zero. For $\overline{C}_1 < 9.24 \, \mathrm{mol \, m^{-3}}$, the values of coefficients $\psi_{12}$ and $\psi_{13}$ are negative and for $\overline{C}_1 > 9.23 \, \mathrm{mol \, m^{-3}}$, positive. In contrast, the values of coefficients $\psi_{22} = \psi_{23}$, $\psi_{32} = \psi_{33}$, and $\psi_{det}$ for $\overline{C}_1 < 9.24 \, \mathrm{mol \, m^{-3}}$ are positive and for $\overline{C}_1 > 9.24 \, \mathrm{mol \, m^{-3}}$, negative. For $\psi = 0$, we can observe nonconvective state, in which concentration Rayleigh number reaches the critical value $R_C = 1691.09$, for $\psi < 0$ is convective state with convection directed straight down and for $\psi > 0$ is convective state with convection directed straight up.

## 1. Introduction

The membrane transport study is still a scientific challenge fulfilling the cognitive and application criteria in many processes occurring in living organisms (in membranes and biological cells, kidneys, etc.) observed in laboratory conditions and used in industry (hemodialysis, desalination of water, concentration of juices, etc.) [1, 2]. Moreover, in recent years, experimental techniques using microfluidic methods and nanoscale interface engineering to study interalia colloidal flows have been developed [3, 4]. The exchange of fluids and substances dissolved in it by biological membranes facilitates the transport of substances needed by living organisms to maintain their metabolic activity and regulation of

pressure equilibrium by membranes in order to maintain the structural integrity of biological systems [5]. Passive and active transport processes control permeation of these substances. Passive transport mechanisms allow the flows of water and/or solutes to reduce their concentration gradients without energy using. In turn, active transport mechanisms allow penetration of dissolved substances against their concentration gradients, at the expense of energy supplied from metabolic reactions. The interaction between these mechanisms determines the hydrostatic fluid pressure and osmotic pressure differences through biological membranes, which are important features of biological systems [6, 7].

To describe transport phenomena in biological and technical systems, models developed in the framework of

nonequilibrium thermodynamics [8–12], diffusion models [13, 14], and friction [15–17]; models developed in the framework of statistical physics [18–20]; and models developed as part of the network thermodynamics [21–26] are used. In order to characterize the relationship between generalized streams of liquid and solutes and their generalized driving forces, which are derived from the electrochemical and/or chemical affinity gradient, the thermodynamics of Onsager are used [9, 11, 27, 28]. This model is based on the existence of the dissipation function, which describes the total change in the entropy of the system. In near-equilibrium systems, i.e., those in which the dissipation rate of free energy is small, linear dependencies between the flow and the driving forces (thermodynamic stimuli) can be assumed [10, 11]. The classic Kedem–Katchalsky model equations were developed in accordance with these principles [9]. This model takes into account the interaction between solvents and nonionic solutes. This gives a set of phenomenological membrane coefficients that can be easily determined experimentally in a series of independent experiments. Moreover, this formalism provides the theoretical basis for the analysis of the volume and solute fluxes in various membrane systems [10, 11]. Therefore, the KK equations are one of the basic research tools for membrane transport in both biological and artificial systems. Many versions of these equations are used: classical [7], Kargol's [29], Chang and Pinsky [6], and network thermodynamics [22–26]. These versions of KK equations for nonelectrolytes show the relationship between volume flow ($J_v$) and dissolved matter ($J_s$) and thermodynamic forces: osmotic ($\Delta\pi$) and/or hydrostatic ($\Delta P$). The network form of KK equations is obtained by symmetrical and/or hybrid transformation of classical KK equations using Peusner network thermodynamics (Peusner's Network Thermodynamics, Peusner's NT) [22]. For homogeneous and nonhomogeneous binary solutions of nonelectrolytes, two symmetrical and two hybrid forms of KK equations are known. The symmetrical form of these equations contain Peusner coefficients: $L_{ij}$ and $R_{ij}$ (for homogeneous solutions) and hybrid forms- $L_{ij}^r$ and $R_{ij}^r$ (for inhomogeneous solutions) ($i, j \in \{1, 2\}$, $r =$ A, B) [22, 24].

Therefore, the concepts of $L$, $R$, $H$, and $P$ have been introduced in the form of Kedem–Katchalsky network equations for homogeneity conditions and $L^r$, $R^r$, $H^r$, and $P^r$ in the form of Kedem–Katchalsky network equations for polarization concentration conditions of solutions separated by the membrane [25]. For the homogeneity conditions of nonelectrolytic ternary solutions, there are two symmetrical and six hybrid forms of network KK equations. The symmetric forms of these equations, similarly to homogeneous ternary solutions, contain $L_{ij}$ and $R_{ij}$ coefficients and are derived directly from Onsager's thermodynamics and hybrid forms- $P_{ij}$, $H_{ij}$, $K_{ij}$, $N_{ij}$, $S_{ij}$, and $W_{ij}$ ($i, j \in \{1, 2, 3\}$), which are a consequence of the application of network thermodynamics techniques [30]. For concentration polarization conditions, these coefficients should be written in the form $L_{ij}^r$, $R_{ij}^r$, $H_{ij}^r$, $K_{ij}^r$, $P_{ij}^r$, $N_{ij}^r$, $S_{ij}^r$, and $W_{ij}^r$ ($i, j \in \{1, 2, 3\}$, $r =$ A, B). Therefore, the concept of the form $L$, $R$, $H$, $K$, $P$, $N$, $S$, and $W$ can be introduced in the network equations KK for the conditions of homogeneity and the form $L^r$, $R^r$, $H^r$, $K^r$, $P^r$, $N^r$, $S^r$, and $W^r$

of the KK equations for the concentration polarization conditions of ternary solutions separated by the membrane.

The aim of the next series of papers is to present the form $L^r$, $R^r$, $H^r$, $K^r$, $P^r$, $N^r$, $S^r$, and $W^r$ of the KK equations for concentration polarization conditions of ternary solutions. The aim of this work is to develop the form of $H^r$ of the KK equations, containing the Peusner coefficients $H_{ij}^r$ ($i, j \in \{1, 2, 3\}$, $r =$ A, B). Besides, we compare $H_{ij}^r$ and $H_{ij}$ coefficients and matrix coefficients $H_{det}^r = \det [H^r]$ and $H_{det} = \det [H]$. We will present the results of calculations of coefficients $H_{ij}^r$ and $H_{ij}$ matrix coefficients $H_{det}^r = \det [H^r]$ and $H_{det} = \det [H]$ and the quotients $\psi_{ij} = (H_{ij}^A - H_{ij}^B)/H_{ij}$ and $\psi_{det} = (H_{det}^A - H_{det}^B)/H_{det}$.

## 2. Materials and Methods

*2.1. Membrane System.* Similarly as in previous papers [31, 32], we will consider transport of nonhomogeneous ternary nonelectrolyte solutions with concentrations at the initial moment ($t = 0$) $C_{kh}$ and $C_{kl}$ ($C_{kh} > C_{kl}$, $k = 1, 2$) through the membrane (M) in the single-membrane system (Figure 1). This membrane separates compartments $l$ and $h$ and is isotropic, symmetric, electroneutral, and selective for solvent and nonionized dissolved substances. In the case of membrane located in horizontal plane that is perpendicularly to the gravity vector, there are configurations A or B of arrangement of solutions in relation to the membrane ($r =$ A or B). In configuration A, the solution with concentration $C_{kl}$ is in the compartment over the membrane while the solution with concentration $C_{kh}$ is in compartment under the membrane. In configuration B of the membrane system, location of solutions is reversed. We will consider only isothermal and stationary processes of membrane transport, for which the measure is the volume fluxes ($J_v^r$) and solutes fluxes ($J_k^r$) ($k = 1, 2$ and $r =$ A, B). These fluxes can be described by the KK equations for nonhomogeneous ternary nonelectrolyte solutions [33].

Under such conditions, solutes, which diffuse through the membrane, create the concentration boundary layers (CBLs) on both sides of the membrane signed by $l_h^r$ and $l_l^r$ [32, 33]. The CBL $l_h^r$ has a thickness marked as $\delta_h^r$ and CBL $l_l^r$ has a thickness marked as $\delta_l^r$.

The mean concentrations of solutes "1" and "2" in membrane ($\overline{C}_1$, $\overline{C}_2$) can be calculated using expressions $\overline{C}_k = (C_{kh} - C_{kl})$ $[\ln (C_{kh} C_{kl}^{-1})]^{-1}$ ($k = 1, 2$). Appearance of CBLs causes those concentrations at the interfaces of the membrane and solutions, respectively, to decrease from $C_{kh}$ to $C_{ki}^r$ and increase from $C_{kl}$ to $C_{ke}^r$. For steady state, the following relation is fulfilled: $C_{ki}^r > C_{ke}^r$, $C_{ke}^r > C_{kl}$, $C_{kh} > C_{ki}^r$ ($k = 1, 2$) [31, 34]. In addition, $\rho_e^{~r}$ and $\rho_i^{~r}$ denote the densities at interfaces $l_l^r$/M and M/$l_h^r$, respectively, while $\rho_l$ and $\rho_h$ ($\rho_l < \rho_h$ or $\rho_l > \rho_h$) denote the densities of solutions outside the layers. Moreover, $\rho_e^r > \rho_l$ or $\rho_e^r < \rho_i^r$, $\rho_e^r > \rho_i^r$ or $\rho_e^r < \rho_i^r$, and $\rho_i^{~r} > \rho_h$ or $\rho_i^{~r} < \rho_h$ [34]. If the solution with lower density is under the membrane the system, $l_h^r$/M/$l_l^r$ loses its hydrodynamic stability, and convective instabilities in the near membrane area are observed [33, 34].

The measure of the concentration polarization is the concentration polarization factor ($\zeta_k^r$). Its value depends on both the concentration of solutions separated by the membrane ($\overline{C}_k$) and the configuration of the membrane system
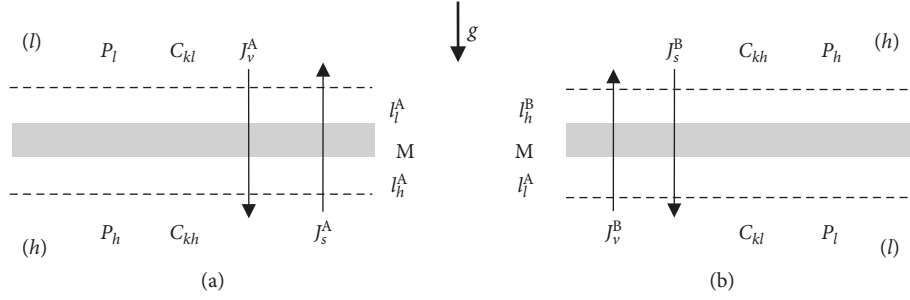
FIGURE 1: The model of single-membrane system: M, membrane; $l_l^A$ and $l_h^A$, the concentration boundary layers in configuration A; $l_l^B$ and $l_h^B$, the concentration boundary layers in configuration B; $P_h$ and $P_l$, mechanical pressures; $C_{kh}$ and $C_{kl}$, concentrations of solutions; $J_k^A$ and $J_v^A$, solute and volume fluxes in configuration A; and $J_k^B$ and $J_v^B$, solute and volume fluxes in configuration B.

($r = $ A, B). More specifically for this case, the thicknesses of CBLs $\delta_h^r$ and $\delta_l^r$ exceed values $(\delta_h^r)_{\text{crit}}$ and $(\delta_l^r)_{\text{crit}}$, and coefficient of concentration polarization $(\zeta_k^r)$ exceeds its critical value $(\zeta_k^r)_{\text{crit}}$ suitably [34–36]. For diluted ternary non-electrolyte solutions, the concentration polarization factor $(\zeta_{ks}^r)$ and the thickness of concentration the boundary layers $(\delta_h^r$ and $\delta_l^r)$ can be described by the expression $\zeta_{ks}^r (\overline{C}_k, \overline{C}_s) = \{1 + RT\omega_{ks}[\delta_l^r (\overline{C}_k, \overline{C}_s) + \delta_h^r (\overline{C}_k, \overline{C}_s)] D_{ks}^{-1}\}^{-1}$ [31].

### 2.2. Matrix Form of the Kedem–Katchalsky Equations.

According to the Kedem–Katchalsky formalism [7, 10, 33], transport properties of the membrane are determined for solutions containing a solvent and dissolved two substances (ternary solution) by practical coefficients: hydraulic permeability $(L_p)$, reflection $(\sigma_k, k = 1, 2)$, and permeability of solute $(\omega_{kf}, k, f \in \{1, 2\})$. In turn, the transport properties of the complex $l_l^r/\text{M}/l_h^r$ are characterized by hydraulic permeability $(L_p^r)$, reflection $(\sigma_k^r, \sigma_a^r)$, and permeability coefficients of solute $(\omega_{kf}^r)$. The coefficients of hydraulic, osmotic, advective, and diffusive concentration polarization are defined by expressions $\zeta_p^r = L_p^r/L_p$, $\zeta_{vk}^r = \sigma_k^r/\sigma_k$, $\zeta_a^r = \sigma_a^r/\sigma_k$, and $\zeta_s^r = \omega_{kf}^r/\omega_{kf}$ [23]. The decrease of the value of volume and solute fluxes from $J_v$ and $J_k$ (in conditions of homogeneous solutions) to $J_v^r$ and $J_k^r$ (in condition of concentration polarization) is caused by formation of the concentration boundary layers $l_l^r$ and $l_h^r$, respectively [33].

The classical form of Kedem–Katchalsky equations for concentration polarization conditions can be written as

$$J_v^r = \zeta_p^r L_p (\Delta P - \zeta_{v1}^r \sigma_1 \Delta \pi_1 - \zeta_{v2}^r \sigma_2 \Delta \pi_2), \tag{1}$$

$$J_1^r = \zeta_{s11}^r \omega_{11} \Delta \pi_1 + \zeta_{s12}^r \omega_{12} \Delta \pi_2 + \overline{C}_1 (1 - \zeta_{a1}^r \sigma_1) J_v^r, \tag{2}$$

$$J_2^r = \zeta_{s21}^r \omega_{21} \Delta \pi_1 + \zeta_{s22}^r \omega_{22} \Delta \pi_2 + \overline{C}_1 (1 - \zeta_{a2}^r \sigma_2) J_v^r, \tag{3}$$

where $J_v^r$, $J_1^r$, and $J_2^r$ are volume and solutes "1" and "2" fluxes, respectively; $L_p$ is the hydraulic permeability coefficient; $\sigma_1$ and $\sigma_2$ are reflection coefficients suitably for solutes "1" and "2"; $\omega_{11}$ and $\omega_{22}$ are the solute permeability coefficients for solutes "1" and "2" generated by forces with indexes "1" and "2" and $\omega_{12}$ and $\omega_{21}$ are the cross coefficients of permeability for substances "1" and "2" generated by forces with indexes "2" and "1" respectively. $\Delta P = P_h - P_l$ is the hydrostatic pressure difference ($P_h$, $P_l$ are higher and lower

values of hydrostatic pressure suitably). $\Delta \pi_k = RT(C_{kh} - C_{kl})$ is the difference of osmotic pressure ($RT$ is the product of gas constant and thermodynamic temperature whereas $C_{kh}$ and $C_{kl}$ are solutes concentrations, $k = 1, 2$). $\overline{C}_k$ is the mean solute concentration in membrane and is expressed by $\overline{C}_k = (C_{kh} - C_{kl}) [\ln (C_{kh} C_{kl}^{-1})]^{-1}$ $(k = 1, 2)$.

Relatively simple algebraic transformations allow transforming equations (1)–(3) into the form

$$\Delta P - \Delta \pi_1 - \Delta \pi_2 = \frac{J_v^r}{\zeta_p^r} - \overline{C}_1 (1 - \zeta_{v1}^r \sigma_1) \frac{\Delta \pi_1}{\overline{C}_1}$$
$$- \overline{C}_2 (1 - \zeta_{v2}^r \sigma_2) \frac{\Delta \pi_2}{\overline{C}_2}, \tag{4}$$

$$J_1^r = \overline{C}_1 (1 - \zeta_{a1}^r \sigma_1) J_v^r + \overline{C}_1 \zeta_{s11}^r \omega_{11} \frac{\Delta \pi_1}{\overline{C}_1} + \overline{C}_2 \zeta_{s12}^r \omega_{12} \frac{\Delta \pi_2}{\overline{C}_2}, \tag{5}$$

$$J_2^r = \overline{C}_2 (1 - \zeta_{a2}^r \sigma_2) J_v^r + \overline{C}_1 \zeta_{s21}^r \omega_{21} \frac{\Delta \pi_1}{\overline{C}_1} + \overline{C}_2 \zeta_{s22}^r \omega_{22} \frac{\Delta \pi_2}{\overline{C}_2}, \tag{6}$$

The above equations are one of the forms of Kedem–Katchalsky equations obtained by the hybrid transformation of Peusner's thermodynamic networks.

Equations (4) and (5) can be transformed by simple algebraic transformations to the matrix form:

$$\begin{bmatrix} \Delta P - \Delta \pi_1 - \Delta \pi_2 \\ J_1^r \\ J_2^r \end{bmatrix} = \begin{bmatrix} H_{11}^r & H_{12}^r & H_{13}^r \\ H_{21}^r & H_{22}^r & H_{23}^r \\ H_{31}^r & H_{32}^r & H_{33}^r \end{bmatrix} \begin{bmatrix} J_v^r \\ \dfrac{\Delta \pi_1}{\overline{C}_1} \\ \dfrac{\Delta \pi_2}{\overline{C}_2} \end{bmatrix} = [H^r] \begin{bmatrix} J_v^r \\ \dfrac{\Delta \pi_1}{\overline{C}_1} \\ \dfrac{\Delta \pi_2}{\overline{C}_2} \end{bmatrix}, \tag{7}$$

where $H_{11}^r = (\zeta_p^r L_p)^{-1}$, $H_{12}^r = -\overline{C}_1 (1 - \zeta_{v1}^r \sigma_1)$, $H_{13}^r = -\overline{C}_2 (1 - \zeta_{v2}^r \sigma_2)$, $H_{21}^r = \overline{C}_1 (1 - \zeta_{a1}^r \sigma_1)$, $H_{22}^r = \overline{C}_1 \zeta_{s11}^r \omega_{11}$, $H_{23}^r = \overline{C}_2 \zeta_{s12}^r \omega_{12}$, $H_{31}^r = \overline{C}_2 (1 - \zeta_{a2}^r \sigma_2)$, $H_{32}^r = \overline{C}_1 \zeta_{s21}^r \omega_{21}$, $H_{33}^r = \overline{C}_2 \zeta_{s22}^r \omega_{22}$, and $[H^r]$ is the matrix of Peusner's coefficients $H_{ij}^r$ $(i, j \in \{1, 2, 3\})$ for ternary nonelectrolyte solutions in conditions of concentration polarization.

It results from equation (7) for the nondiagonal coefficients $H_{12}^r \neq H_{21}^r$, $H_{13}^r \neq H_{31}^r$, and $H_{23}^r \neq H_{32}^r$. On the basis of equation (7), we get $H_{12}^r = -H_{21}^r (1 - \zeta_{v1}^r \sigma_1)(1 - \zeta_{a1}^r \sigma_1)^{-1}$,

$H^r_{31} = -H^r_{31}(1 - \zeta^r_{v2}\sigma_2)(1 - \zeta^r_{a2}\sigma_2)^{-1}$, and $H^r_{23} = H^r_{32}\overline{C}_2\omega_{12}$ $\zeta^r_{s12}(\overline{C}_1\omega_{21}\zeta^r_{s21})^{-1}$. Moreover, the determinant of the matrix $[H^r]$ is equal to

$$\det[H^r] = H^r_{\det} = \overline{C}_1\overline{C}_2\zeta^r_p - 1L_p^{-1}\gamma_1 + \overline{C}_1\overline{C}_2^2(1 - \zeta^r_{a2}\sigma_2)\gamma_2$$
$$+ \overline{C}_1^2\overline{C}_2(1 - \zeta^r_{a1}\sigma_1)\gamma_3, \tag{8}$$

where $\gamma_1 = \omega_{11}\zeta^r_{s11}\omega_{22}\zeta^r_{s22} - \omega_{12}\zeta^r_{s12}\omega_{21}\zeta^r_{s21}$, $\gamma_2 = \omega_{11}\zeta^r_{s11}$ $(1 - \zeta^r_{v2}\sigma_2) - \omega_{12}\zeta^r_{s12}(1 - \zeta^r_{v1}\sigma_1)$, $\gamma_3 = \omega_{22}\zeta^r_{s22}(1 - \zeta^r_{v1}\sigma_1) - \omega_{21}\zeta^r_{s21}(1 - \zeta^r_{v2}\sigma_2)$.

Index "$r$" in equations (3)–(10) indicates that the fluxes $J^r_v$, $J^r_1$, and $J^r_2$, coefficients $H^r_{ij}$ ($i, j \in \{1, 2, 3\}$ and $r$ = A, B) and matrix $[H^r]$ of these coefficients, depend on configuration of the membrane system. For homogeneous conditions ($\zeta^r_p = \zeta^r_{v1} = \zeta^r_{v2} = \zeta^r_{a1} = \zeta^r_{a2} = \zeta^r_{s11} = \zeta^r_{s12} = \zeta^r_{s22} = \zeta^r_{s21} = 1$), we get

$$\begin{bmatrix} \Delta P - \Delta\pi_1 - \Delta\pi_2 \\ J_1 \\ J_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} J_v \\ \dfrac{\Delta\pi_1}{\overline{C}_1} \\ \dfrac{\Delta\pi_2}{\overline{C}_2} \end{bmatrix} = [H^r] \begin{bmatrix} J_v \\ \dfrac{\Delta\pi_1}{\overline{C}_1} \\ \dfrac{\Delta\pi_2}{\overline{C}_2} \end{bmatrix}, \tag{9}$$

where $H_{11} = L_p^{-1}$, $H_{12} = -\overline{C}_1(1 - \sigma_1) = -H_{21}$, $H_{13} = -\overline{C}_2$ $(1 - \sigma_2) = -H_{31}$, $H_{22} = \overline{C}_1\zeta^r_{s11}\omega_{11}$, $H_{23} = \overline{C}_2\omega_{12}$, $H_{32} = \overline{C}_1\omega_{21}$, $H_{33} = \overline{C}_2\omega_{22}$, and $[H]$ is the matrix of Peusner's coefficients $H_{ij}$ ($i, j \in \{1, 2, 3\}$) for ternary nonelectrolyte homogeneity solutions.

On the basis of equation (9), we get $H_{23} = H_{32}\overline{C}_2\omega_{12}\overline{C}_1^{-1}\omega_{21}^{-1}$. The determinant of the matrix $[H]$ is equal to

$$\det[H] = H_{\det} = \overline{C}_1\overline{C}_2L_p^{-1}\gamma_1 + \overline{C}_1\overline{C}_2^2(1 - \sigma_2)\gamma_2$$
$$+ \overline{C}_1^2\overline{C}_2(1 - \sigma_1)\gamma_3, \tag{10}$$

where $\gamma_1 = \omega_{11}\omega_{22} - \omega_{12}\omega_{21}$, $\gamma_2 = \omega_{11}(1 - \sigma_2) - \omega_{12}(1 - \sigma_1)$, and $\gamma_3 = \omega_{22}(1 - \sigma_1) - \omega_{21}(1 - \sigma_2)$.

In order to show the relationship between coefficients $H^r_{ij}$ and $H_{ij}$ and between $H^r_{\det}$ and $H_{\det}$ for A and B configurations of the membrane system ($r$ = A, B), we can calculate using equations (6), (10), (11), and (12), the coefficients $\psi_{ij} = (H^A_{ij} - H^B_{ij})/H_{ij}$ and $\psi_{\det} = (H^A_{\det} - H^B_{\det})/H_{\det}$. The expressions for the coefficients $\psi_{ij}$ and $\psi_{\det}$ are given as follows:

$$\psi_{11} = \frac{H^A_{11} - H^B_{11}}{H_{11}} = \frac{\zeta^B_p - \zeta^A_p}{\zeta^A_p\zeta^B_p}, \tag{11}$$

$$\psi_{12} = \frac{H^A_{12} - H^B_{12}}{H_{12}} = \frac{\sigma_1\left(\zeta^B_{v1} - \zeta^A_{v1}\right)}{1 - \sigma_1}, \tag{12}$$

$$\psi_{13} = \frac{H^A_{13} - H^B_{13}}{H_{13}} = \frac{\sigma_2\left(\zeta^B_{v2} - \zeta^A_{v2}\right)}{1 - \sigma_2}, \tag{13}$$

$$\psi_{21} = \frac{H^A_{21} - H^B_{21}}{H_{21}} = \frac{\sigma_1\left(\zeta^B_{a1} - \zeta^A_{a1}\right)}{1 - \sigma_1}, \tag{14}$$

$$\psi_{22} = \frac{H^A_{22} - H^B_{22}}{H_{22}} = \zeta^A_{s11} - \zeta^B_{s11}, \tag{15}$$

$$\psi_{23} = \frac{H^A_{23} - H^B_{23}}{H_{23}} = \zeta^A_{s12} - \zeta^B_{s12}, \tag{16}$$

$$\psi_{31} = \frac{H^A_{31} - H^B_{31}}{H_{31}} = \frac{\sigma_2\left(\zeta^B_{a2} - \zeta^A_{a2}\right)}{1 - \sigma_2}, \tag{17}$$

$$\psi_{32} = \frac{H^A_{32} - H^B_{32}}{H_{32}} = \zeta^A_{s21} - \zeta^B_{s21}, \tag{18}$$

$$\psi_{33} = \frac{H_{33}^A - H_{33}^B}{H_{33}} = \zeta_{s22}^A - \zeta_{s22}^B, \tag{19}$$

$$\psi_{\det} = \frac{H_{\det}^A - H_{\det}^B}{H_{\det}} = \frac{\overline{C}_1\overline{C}_2\left(\gamma_1^A\zeta_p^B - \gamma_1^B\zeta_p^A\right) + L_p\zeta_p^A\zeta_p^A\left[\overline{C}_1\overline{C}_2^{\,2}\left(\gamma_2^A - \gamma_2^B\right) + \overline{C}_1^{\,2}\overline{C}_2\left(\gamma_3^A - \gamma_3^B\right)\right]}{\zeta_p^A\zeta_p^B\overline{C}_1\overline{C}_2\left[\gamma_1 + L_p\left(\gamma_2 + \gamma_3\right)\right]}, \tag{20}$$

where $\gamma_1^A = \omega_{11}\zeta_{s11}^A\omega_{22}\zeta_{s22}^A - \omega_{12}\zeta_{s12}^A\omega_{21}\zeta_{s21}^A$, $\gamma_1^B = \omega_{11}\zeta_{s11}^B\omega_{22}$ $\zeta_{s22}^B - \omega_{12}\zeta_{s12}^B\omega_{21}\zeta_{s21}^B$, $\gamma_2^A = (1 - \zeta_{a2}^A\sigma_2)[\omega_{11}\zeta_{s11}^A(1 - \zeta_{v2}^A\sigma_2) - \omega_{12}\zeta_{s12}^A(1 - \zeta_{v1}^A\sigma_1)]$, $\gamma_2^B = (1 - \zeta_{a2}^B\sigma_2)[\omega_{11}\zeta_{s11}^B(1 - \zeta_{v2}^B\sigma_2) - \omega_{12}\zeta_{s12}^B(1 - \zeta_{v1}^B\sigma_1)]$, $\gamma_3^A = (1 - \zeta_{a1}^A\sigma_1)[\omega_{22}\zeta_{s22}^A(1 - \zeta_{v1}^A\sigma_1) - \omega_{21}\zeta_{s21}^A(1 - \zeta_{v2}^A\sigma_2)]$, $\gamma_3^B = (1 - \zeta_{a1}^B\sigma_1)[\omega_{22}\zeta_{s22}^B(1 - \zeta_{v1}^B\sigma_1) - \omega_{21}\zeta_{s21}^B(1 - \zeta_{v2}^B\sigma_2)]$, $\gamma_1 = \omega_{11}\omega_{22} - \omega_{12}\omega_{21}$, $\gamma_2 = \overline{C}_2(1 - \sigma_2)$ $[\omega_{11}(1 - \sigma_2) - \omega_{12}(1 - \sigma_1)]$, and $\gamma_3 = \overline{C}_1(1 - \sigma_1)[\omega_{22}\,(1 - \sigma_1) - \omega_{21}(1 - \sigma_2)]$.

The values of coefficients $\psi_{ij}$ and $\psi_{\det}$ show the influence of concentration polarization and natural convection on the membrane transport.

## 3. Results and Discussion

The values of coefficients $H_{ij}$, $H_{ij}^r$, $H_{\det}^r$, $\psi_{ij} = (H_{ij}^A - H_{ij}^B)/H_{ij}$, and $\psi_{\det} = (H_{\det}^A - H_{\det}^B)/H_{\det}$, $(i, j \in \{1, 2, 3\}, r = A, B)$, which describe equations (7)–(20), are calculated for polymer membrane *Nephrophan* and glucose solutions in aqueous solution of ethanol. The glucose concentration was marked by index "1" and the ethanol concentration by index "2". The concentration of substance "1" in the compartment $(h)$ take values from $C_{1h} = 1\,\text{mol}\cdot\text{m}^{-3}$ to $C_{1h} = 101\,\text{mol}\cdot\text{m}^{-3}$. In turn, concentration of a substance "2" in the compartment $(h)$ was constant and amounted to $C_{2h} = 201\,\text{mol}\cdot\text{m}^{-3}$. The concentrations of both components in the compartment $(l)$ were established and amounted to $C_{1l} = C_{2l} = 1\,\text{mol}\cdot\text{m}^{-3}$. In expressions under equations (7)–(20), there are coefficients that describe transport properties of membrane $(L_p$, $\sigma_1$, $\sigma_2$ $\omega_{11}$, $\omega_{22}$, $\omega_{21}$ and $\omega_{12})$, average concentrations of solutions "1" and "2" in the membrane $(\overline{C}_1, \overline{C}_2)$, and coefficients of concentration polarization $(\zeta_p^r$, $\zeta_{a1}^r$, $\zeta_{a2}^r$, $\zeta_{v1}^r$, $\zeta_{s11}^r$, $\zeta_{s12}^r$, $\zeta_{v2}^r$, $\zeta_{s22}^r$, and $\zeta_{s21}^r)$. The values of these coefficients are calculated, using conditions $\zeta_p^r = \zeta_{a1}^r = \zeta_{a2}^r = 1$, $\zeta_{v1}^r = \zeta_{s11}^r = \zeta_{s12}^r = \zeta_1^r$, and $\zeta_{v2}^r = \zeta_{s22}^r = \zeta_{s21}^r = \zeta_2^r$ [33, 37].

In order to calculate $H_{ij}$, $H_{ij}^r$, $H_{\det}^r$, $\psi_{ij} = (H_{ij}^A - H_{ij}^B)/H_{ij}$, and $\psi_{\det} = (H_{\det}^A - H_{\det}^B)/H_{\det}$, $(i, j \in \{1, 2, 3\}, r = A, B)$ on the basis of equations (7)–(20), we used the characteristics $\zeta_1^r = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ and $\zeta_2^r = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ presented in Figure 2 and following data: $L_p = 4.9 \times 10^{-12}\,\text{m}^3\cdot\text{N}^{-1}\cdot\text{s}^{-1}$, $\sigma_1 = 0.068$, $\sigma_2 = 0.025$, $\omega_{11} = 0.8 \times 10^{-9}\,\text{mol}\cdot\text{N}^{-1}\cdot\text{s}^{-1}$, $\omega_{12} = 0.81 \times 10^{-13}\,\text{mol}\cdot\text{N}^{-1}\cdot\text{s}^{-1}$, $\omega_{22} = 1.43 \times 10^{-9}\,\text{mol}\cdot\text{N}^{-1}\cdot\text{s}^{-1}$, $\omega_{21} = 1.63 \times 10^{-12}\,\text{mol}\cdot\text{N}^{-1}\cdot\text{s}^{-1}$, $\overline{C}_1 = 2.79 \div 21.67\,\text{mol}\cdot\text{m}^{-3}$ and $\overline{C}_2 = 37.71\,\text{mol}\cdot\text{m}^{-3}$.

The calculations based on equations (7) and (9) show that $H_{11}^A = H_{11}^B = H_{11} = 0.204 \times 10^{12}\,\text{N}\cdot\text{s}\cdot\text{m}^{-3}$, and $H_{31}^A = H_{31}^B = H_{31} = 36.77\,\text{mol}\cdot\text{m}^{-3}$ are independent of both concentration of solution and configuration of the membrane system.

Dependencies of coefficients $H_{12}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{12}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{12} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{21}^A = H_{21}^B = H_{21} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ are shown in Figure 3. Graphs 1A, 1B, and 1 show that values of coefficients $H_{12}^A$, $H_{12}^B$, and $H_{12}$ are decrease almost linearly with the increase in $\overline{C}_1$ for $\overline{C}_2 = 37.71\,\text{mol}\cdot\text{m}^{-3}$. These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{12}^A = H_{12}^B = -8.98\,\text{mol}\cdot\text{m}^{-3}$, for $\overline{C}_1 < 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{12} > H_{12}^A > H_{12}^B < 0$, and for $\overline{C}_1 > 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{12} > H_{12}^B > H_{12}^A < 0$.

Graph 2 shows that values of coefficients $H_{21}^A$, $H_{21}^B$, and $H_{21}$ increase linearly with the increase in $\overline{C}_1$ for $\overline{C}_2 = 37.71\,\text{mol}\cdot\text{m}^{-3}$. These coefficients fulfill the condition $H_{21}^A = H_{21}^B = H_{21} > 0$. Dependencies of coefficients $H_{13}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{13}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{13} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ are presented in Figure 4. Graphs 1A and 1B show that values of coefficients $H_{13}^A$ and $H_{13}^B$ decrease and increase nonlinearly with the increase in $\overline{C}_1$ for $\overline{C}_2 = 37.71\,\text{mol}\cdot\text{m}^{-3}$, respectively. The value of coefficient is equal to $H_{13} = -36.77\,\text{mol}\cdot\text{m}^{-3}$ and is independent of both concentration of solution and configuration of the membrane system. These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24\,\text{mol m}^{-3}$, $H_{13}^A = H_{13}^B = -37.49\,\text{mol}\cdot\text{m}^{-3}$, for $\overline{C}_1 < 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{13} < H_{13}^A < H_{13}^B < 0$, and for $\overline{C}_1 > 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{13} < H_{13}^B < H_{13}^A < 0$.

The graphs 1A, 1B, and 1 shown in Figure 5 illustrate the dependencies $H_{22}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{22}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{22} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$. Graph 1A shows that the values of coefficient $H_{22}^A$ first increase in a nonlinear way and then decrease in a nonlinear manner with the increase of $\overline{C}_1$ value and graph 1B shows that the $H_{22}^B$ values increase nonlinearly with the increase of the of $\overline{C}_1$ value (for $\overline{C}_2 = 37.71\,\text{mol}\cdot\text{m}^{-3}$). In turn, the course of graph 1 shows that $H_{22}$ is a linear function of $\overline{C}_1$ (for $\overline{C}_2 = 37.71\,\text{mol}\cdot\text{m}^{-3}$). These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{22}^A = H_{22}^B = 1.81 \times 10^{-9}\,\text{mol}^2\text{N}^{-1}\cdot\text{s}^{-1}\cdot\text{m}^{-3}$, for $\overline{C}_1 < 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{22} > H_{22}^A > H_{22}^B > 0$, and $\overline{C}_1 > 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{22} > H_{22}^B > H_{22}^A > 0$.

Dependencies of coefficients $H_{23}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{23}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{23} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ are presented in Figure 6. Graphs 1A and 1B shows that values of coefficients $H_{23}^A$ and $H_{23}^B$ decrease and increase almost nonlinearly with the increase in $\overline{C}_1$ for $\overline{C}_2 = 37.71\,\text{mol}\cdot\text{m}^{-3}$, respectively. The value of coefficient is equal to $H_{23} = 30.54\,\text{mol}\cdot\text{m}^{-3}$ and is independent of both concentration of solution and configuration of the membrane system. These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{23}^A = H_{23}^B = 6.98 \times 10^{-13}\,\text{mol}^2\cdot\text{N}^{-1}\cdot\text{s}^{-1}\cdot\text{m}^{-3}$, for $\overline{C}_1 < 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{23} > H_{23}^A > H_{23}^B > 0$, and $\overline{C}_1 > 9.24\,\text{mol}\cdot\text{m}^{-3}$, $H_{23} > H_{23}^B > H_{23}^A > 0$.
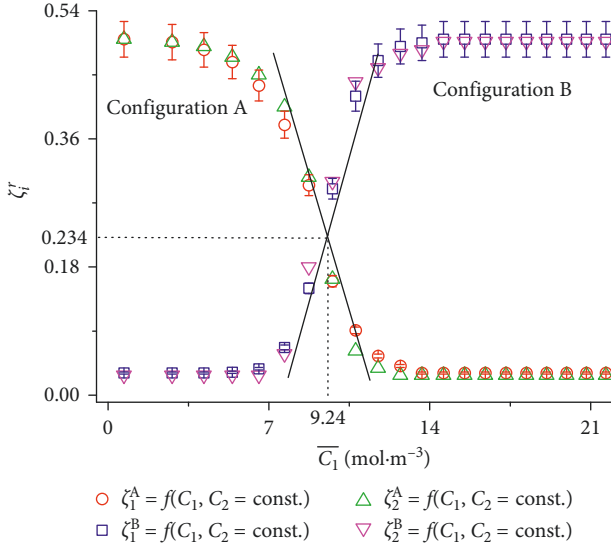
FIGURE 2: Dependencies of concentration polarization coefficient ($\zeta_k^r$, $k = 1, 2$) on glucose concentration in 201 mol·m$^{-3}$ aqueous ethanol solution for configuration A ($r = $ A and B ($r = $ B)) of the single-membrane system [31, 32].
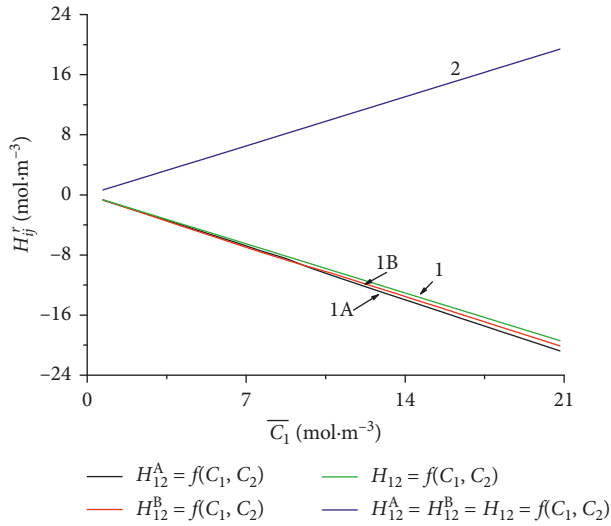


FIGURE 3: The graphic illustration of the dependence $H_{ij}^r = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$, ($i \neq j$, $r = $ A, B) for the glucose in aqueous ethanol solution in conditions of concentration polarization for configurations A and B of the membrane system, respectively: $H_{12}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—curve 1A; $H_{12}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—curve 1B; and $H_{21}^A = H_{21}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—line 2. The lines illustrate the dependence $H_{12} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—line 1 and $H_{21} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ (line 2) in conditions of homogeneity of solutions.

The graphs 1A, 1B, and 1 shown in Figure 7 illustrate the dependencies $H_{32}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{32}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{32} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$. Graph 1A shows that the values of coefficient $H_{32}^A$ first increase in a nonlinear way and then decrease in a nonlinear manner with the increase of $\overline{C}_1$ value, and graph 1B shows that the $H_{32}^B$ values increase nonlinearly with the increase of the of $\overline{C}_1$ value (for
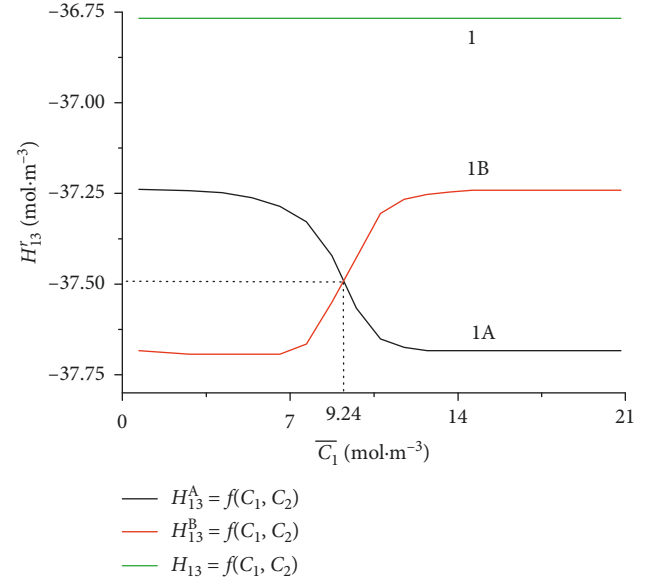


FIGURE 4: The graphic illustration of the dependence $H_{13}^r = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$, ($r = A$, $B$) for the glucose in aqueous ethanol solution in conditions of concentration polarization for configurations A ($H_{13}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, curve 1A) and B ($H_{13}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, curve 1B) of the membrane system. Line 1 illustrates the dependence $H_{13} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ for conditions of homogeneity of solutions.
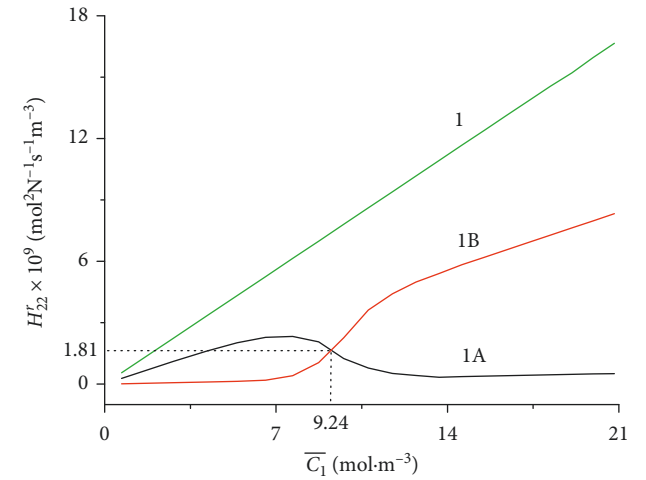


FIGURE 5: The graphic illustration of the dependence $H_{22}^r = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$ ($r = A$, $B$) for the glucose in aqueous ethanol solution in conditions of concentration polarization for configurations A ($H_{22}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, curve 1A) and B ($H_{22}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, curve 1B) of the membrane system. Line 1 illustrates the dependence $H_{22} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ in conditions of homogeneity of solutions.

$\overline{C}_2 = 37.71 \text{ mol·m}^{-3}$). In turn, the course of graph 1 shows that $H_{32}$ is a linear function of $\overline{C}_1$ (for $\overline{C}_2 = 37.71 \text{ mol·m}^{-3}$). These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24 \text{ mol·m}^{-3}$, $H_{32}^A = H_{22}^B = 3.35 \times 10^{-12} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot$ m$^{-3}$, for $\overline{C}_1 < 9.24 \text{ mol·m}^{-3}$, $H_{32} > H_{32}^A > H_{32}^B > 0$, and for $\overline{C}_1 > 9.24 \text{ mol·m}^{-3}$, $H_{32} > H_{32}^B > H_{32}^A > 0$.
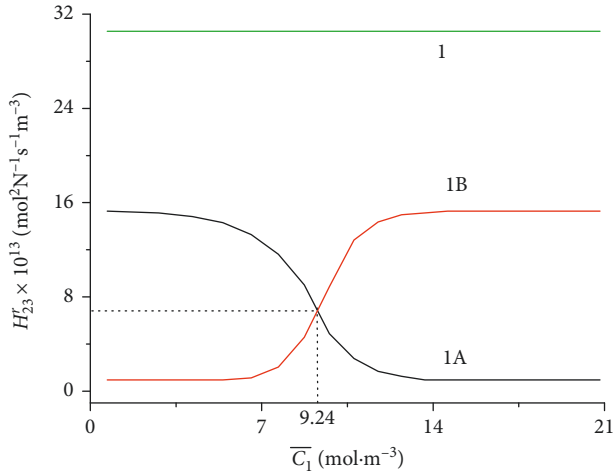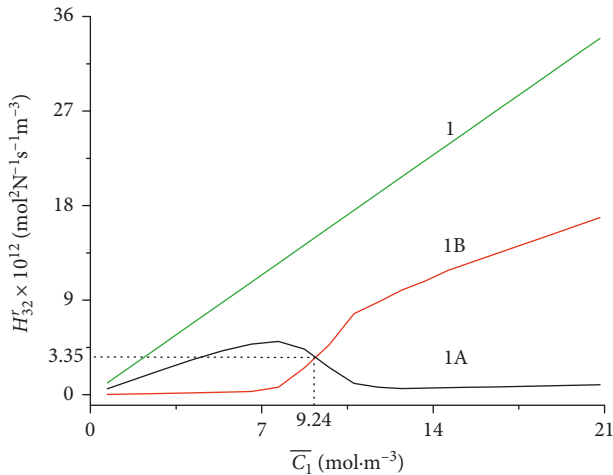
FIGURE 6: The graphic illustration of the dependence $H_{23}^r = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$ ($r = A, B$) for the glucose in aqueous ethanol solution in conditions of concentration polarization for configuration A ($H_{23}^A = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$, curve 1A) and B ($H_{23}^B = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$, curve 1B), of the membrane system. Line 1 illustrates the dependence $H_{23} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ in conditions of homogeneity of solutions.



FIGURE 7: The graphic illustration of the dependence $H_{32}^r = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$ ($r = A, B$) for the glucose in aqueous ethanol solution in conditions of concentration polarization for configurations A ($H_{32}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, curve 1A) and B ($H_{32}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, curve 1B) of the membrane system. Line 1 illustrates the dependence $H_{32} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ in conditions of homogeneity of solutions.

Dependencies of coefficients $H_{33}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{33}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{33} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ are presented in Figure 8. Graphs 1A and 1B show that values of coefficients $H_{33}^A$ and $H_{33}^B$ decrease and increase almost nonlinearly with the increase in $\overline{C}_1$ for $\overline{C}_2 = 37.71 \text{ mol·m}^{-3}$, respectively. The value of coefficient is equal to $H_{33} = 53.92 \times 10^{-9} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3}$ and is independent of both concentration of solution and configuration of the membrane system. These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24 \text{ mol m}^{-3}$, $H_{33}^A = H_{33}^B = 12.64 \times 10^{-9} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3}$, for $\overline{C}_1 < 9.24 \text{ mol·m}^{-3}$, $H_{33} < H_{33}^A < H_{33}^B > 0$, and for $\overline{C}_1 > 9.24 \text{ mol·m}^{-3}$, $H_{33} < H_{33}^B < H_{33}^A > 0$.
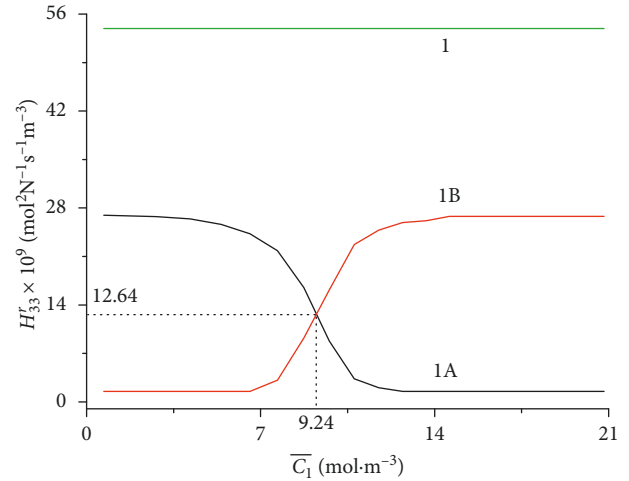


FIGURE 8: The graphic illustration of the dependence $H_{33}^r = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$ ($r = A, B$) for the glucose in aqueous ethanol solution in conditions of concentration polarization for configuration A ($H_{33}^A = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$, curve 1A) and B ($H_{33}^B = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$, curve 1B), of the membrane system. Line 1 illustrates the dependence $H_{23} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ in conditions of homogeneity of solutions.

The graphs 1A, 1B, and 1 shown in Figure 9 illustrate the dependencies $H_{det}^A = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{det}^B = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{det} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$. Graph 1A shows that the values of coefficient $H_{det}^A$ first increase in a nonlinear way and then decrease in a nonlinear manner with the increase of $\overline{C}_1$ value, and graph 1B shows that the $H_{det}^B$ values increase nonlinearly with the increase of the of $\overline{C}_1$ value (for $\overline{C}_2 = 37.71 \text{ mol·m}^{-3}$). In turn, the course of graph 1 shows that det $[H]$ is a linear function of $\overline{C}_1$ (for $\overline{C}_2 = 37.71 \text{ mol·m}^{-3}$). These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24 \text{ mol·m}^{-3}$, $H_{det}^A = H_{det}^B = 6.74 \times 10^{-6} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3}$, for $\overline{C}_1 < 9.24 \text{ mol·m}^{-3}$, $H_{det} > H_{det}^A > H_{det}^B > 0$, and for $\overline{C}_1 > 9.24 \text{ mol·m}^{-3}$, $H_{det} > H_{det}^B > H_{det}^A > 0$.

Figures 3–9 show that there are three groups of characteristics $H_{ij}^r = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{ij} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{det}^r = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, ($i, j \in \{1, 2, 3\}$ and $r = A, B$). The first group includes the characteristics presented in Figure 3, the second Figures 4, 6, and 8, and the third the characteristics shown in Figures 5, 7, and 9. In the case of group 1, which includes concentration characteristics of coefficients $H_{12}^r$, $H_{12}$, $H_{21}^r$, and $H_{21}$ ($r = A, B$), these coefficients are expressed in the same units, and their values are in the range $-21.62 \text{ mol·m}^{-3} \div 20.19 \text{ mol·m}^{-3}$.

In the case of the second group of characteristics, the shape of the concentration characteristics of the coefficients $H_{13}^r$, $H_{13}$, $H_{23}^r$, $H_{23}$, $H_{33}^r$, and $H_{33}$ ($r = A, B$) is very similar. However, the values of these coefficients differ by up to several orders of magnitude: $-37.68 \text{ mol·m}^{-3} \leq H_{13}^r$, $H_{13} \leq -37.23 \text{ mol·m}^{-3}$, $0.95 \times 10^{-13} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3} \leq H_{23}^r$, $H_{23} \leq 30.54 \times 10^{-13} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3}$ and $1.51 \times 10^{-9} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3} \leq H_{23}^r$, $H_{23} \leq 53.92 \times 10^{-9} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3}$.

Moreover, the values of coefficients $H_{13}^r$ and $H_{13}$ are expressed in different units than coefficients $H_{23}^r$, $H_{23}$, $H_{33}^r$, and $H_{33}$.
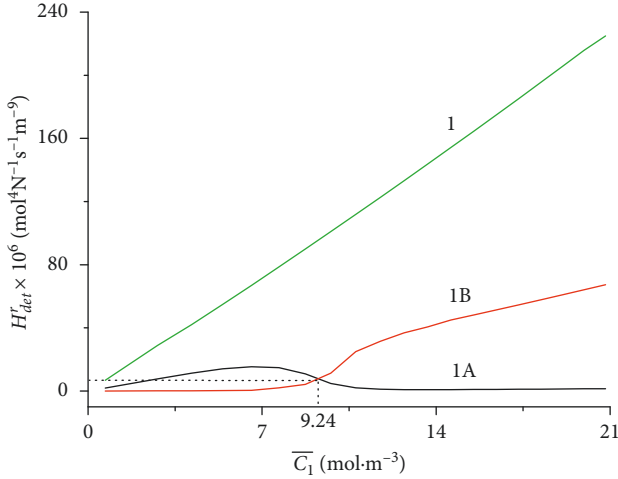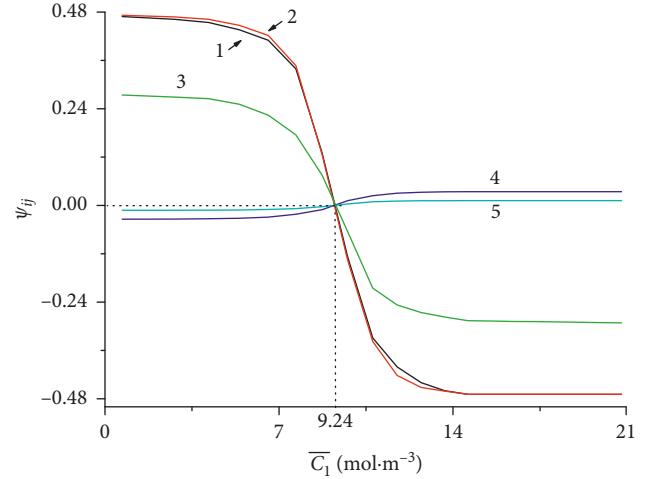
FIGURE 9: The graphic illustration of the dependence $H^r_{det} = f(\overline{C}_1,$ $\overline{C}_2 = 37.71 \text{ mol·m}^{-3})$ $(r = A, B)$ for the glucose in aqueous ethanol solution in conditions of concentration polarization for configurations A ($H^A_{det} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, curve 1A) and B ($H^B_{det} = f(\overline{C}_1,$ $\overline{C}_2 = \text{const.})$, curve 1B) of the membrane system. Line 1 illustrates the dependence $H_{det} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ in conditions of homogeneity of solutions.

The third group of characteristics includes the concentration characteristics of coefficients $H^r_{22}$, $H_{22}$, $H^r_{32}$, $H_{32}$, $H^r_{det}$, and $H_{det}$. Values of coefficients $H^r_{det}$ and $H_{det}$ are expressed in units other than $H^r_{22}$, $H_{22}$, $H^r_{32}$, and $H_{32}$. In addition, the values of these coefficients differ from each other by several orders of magnitude: $0.02 \times 10^{-9}$ $\text{mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3} \leq H^r_{22}$, $H_{22} \leq 17.33 \times 10^{-9} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1}$. $\text{m}^{-3}$, $0.03 \times 10^{-12} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-3} \leq H^r_{32}$, $H_{32} \leq 35.32 \times 10^{-12} \text{ mol}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-3}$, and $0.03 \times 10^{-6} \text{ mol}^4 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-9} \leq$ $H^r_{det}$, $H_{det} \leq 236.09 \times 10^{-6} \text{ mol}^4 \cdot \text{N}^{-1} \cdot \text{s}^{-1} \cdot \text{m}^{-9}$.

The graphs 1–5 shown in Figure 10 illustrate the dependencies $\psi_{22} = \psi_{23} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $\psi_{32} = \psi_{33} = f(\overline{C}_1,$ $\overline{C}_2 = \text{const.})$, $\psi_{det} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $\psi_{13} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $\psi_{12} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, respectively. These coefficients fulfill the following conditions: for $\overline{C}_1 = 9.24 \text{ mol·m}^{-3}$, $\psi_{22} = \psi_{23} = \psi_{32} = \psi_{det} = \psi_{13} = \psi_{12} = 0$, for $\overline{C}_1 < 9.24 \text{ mol·m}^{-3}$, $\psi_{22} = \psi_{23} \approx \psi_{32} = \psi_{33} > \psi_{det} > 0$ and $\psi_{12} > \psi_{13} < 0$, and for $\overline{C}_1 > 9.24 \text{ mol·m}^{-3}$, $\psi_{22} = \psi_{23} \approx \psi_{32} = \psi_{33} < \psi_{det} < 0$ and, $\psi_{12} > \psi_{13} > 0$. Besides, throughout the range of solution concentrations $\psi_{11} = \psi_{21} = \psi_{31} = 0$.

For example, we will consider equation $\psi_{22} = \zeta^A_1 - \zeta^B_1$ and dependencies $\zeta^r_1 = f(\overline{C}_1, \overline{C}_2 = \text{const.})$ $(r = A, B)$ presented in Figure 2. It is drawn from the equation and Figure 2 that if $\psi_{22} = 0$, then $\zeta^A_1 = \zeta^B_1 = 0.234$. Moreover, by taking expressions $\zeta^A_1 = D_1$ $(D_1 + 2RT\omega_{11}\delta^A)^{-1}$ and $\zeta^B_1 = D_1$ $(D_1 + 2RT\omega_{11}\delta^B)^{-1}$ into account in equation (15), one can show that $\psi_{22} = 2RT\omega_{11}(\delta^A - \delta^B)D^{-1}_{11}$. From the equation, it becomes apparent that if $\psi_{22} = 0$, then $\delta^A = \delta^B$. Moreover, using equations $\delta^r = D_{ks}(1 - \zeta^r_i)(2RT\omega_{ks}\zeta^r_i)^{-1}$ and $\rho_h - \rho_l =$ $(\partial\rho/\partial C_1)(C_{1h} - C_{1l}) + (\partial\rho/\partial C_2)(C_{2h} - C_{2l})$, where $(\partial\rho/\partial C_1) =$ $0.06 \text{ kg·mol}^{-1}$, $(\partial\rho/\partial C_2) = -0.0095 \text{ kg·mol}^{-1}$, we can show that, if $\psi_{22} = 0$, it is for $\overline{C}_1 = 9.24 \text{ mol·m}^{-3}$ ($C_{1h} =$ $33.44 \text{ mol·m}^{-3}$ and $C_{1l} = 1 \text{ mol·m}^{-3}$) and $\overline{C}_2 = 37.71 \text{ m}^{-3}$ ($C_{2h} = 201 \text{ mol·m}^{-3}$ and $C_{2l} = 1 \text{ mol·m}^{-3}$) we get $\rho_h - \rho_l =$ $0.046 \text{ kg·m}^{-3}$ and $\delta^A = \delta^B \approx 1.4 \times 10^{-3} \text{ m}$.



1: $\psi_{22} = \psi_{23} = f(C_1, C_2)$　　　4: $\psi_{13} = f(C_1, C_2)$
2: $\psi_{32} = \psi_{33} = f(C_1, C_2)$　　　5: $\psi_{12} = f(C_1, C_2)$
3: $\psi_{det} = f(C_1, C_2)$

FIGURE 10: The graphic illustration of the dependence $\psi_{ij} = f(\overline{C}_1, \overline{C}_2 = 37.71 \text{ mol·m}^{-3})$ $(i \neq j, r = A, B)$ for the glucose in aqueous ethanol solution in conditions of concentration polarization of the membrane system, respectively: $\psi_{22} = \psi_{23} = f(\overline{C}_1,$ $\overline{C}_2 = \text{const.})$—curve 1; $\psi_{32} = \psi_{33} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—curve 2; $\psi_{det} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—curve 3; $\psi_{13} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—curve 4; and $\psi_{12} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$—curve 5.

Taking these data into consideration as well as $D_{11} = 0.69 \times 10^{-9} \text{ m}^2 \cdot \text{s}^{-1}$, $g = 9.81 \text{ m·s}^{-2}$, $\omega_{11} = 0.8 \times 10^{-9}$ $\text{mol·N}^{-1}$ $\text{s}^{-1}$, $\nu = 1.063 \times 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$, $\rho_l = 998.3 \text{ kg·m}^{-3}$, $\zeta^A_1 = \zeta^B_1 = \zeta = 0.234$, and $\delta^A = \delta^B = \delta = 1.4 \times 10^{-3} \text{ m}$ in the expression for the concentration Rayleigh number $R_C =$ $[g(\rho_h - \rho_l)(\delta)^3](\rho_h\nu_hD_{11})^{-1} = [g(\rho_h - \rho_l)(1 - \zeta D_{11}^2)^3]$ $[8\rho_h$ $\nu_hD_1^4(\zeta RT\omega_{11})^3]^{-1}$ [31, 34], we get $R_C = 1691.09$. The value is similar to the classical Rayleigh number critical value for Bernard problem $R_C = 1707.76$ [38].

## 4. Conclusions

(1) The Kedem–Katchalsky equations in matrix form for nonhomogeneous ternary nonelectrolyte solutions were applied for interpretation of transport through the membrane mounted in horizontal plane. Coefficients $H^r_{ij}$ and $H^r_{det} = \det [H^r]$ (for nonhomogeneous solutions), $H_{ij}$ and $H_{det} = \det [H]$ (for homogeneous solutions) $(i, j \in \{1, 2, 3\}, r = A, B)$, $\psi_{ij} = (H^A_{ij} - H^B_{ij})/H_{ij}$, and $\psi_{det} = (H^A_{det} - H^B_{det}])/H_{det}$ were calculated on the basis of experimentally determined coefficients ($L_p$, $\sigma_1$, $\sigma_2$ $\omega_{11}$, $\omega_{22}$, $\omega_{21}$, $\omega_{12}$, $\zeta^r_1$, and $\zeta^r_2$) for glucose in aqueous ethanol solutions and two configurations of the membrane system.

(2) We can conclude that Peusner's Network Thermodynamics (PNT) is an alternative manner of description of membrane transport both for homogeneity of solutions separated by a membrane and in conditions of concentration polarization.

(3) The values of coefficients $H^r_{12}$, $H^r_{13}$, $H^r_{22}$, $H^r_{23}$, $H^r_{32}$, $H^r_{33}$, and $H^r_{det}$ depend nonlinearly on solution

concentration as well as on a configuration of membrane system. The values of these coefficients in the convective state are greater then their values in the nonconvective state.

(4) The values of coefficients $H_{21}^r$, $H_{12}$, $H_{21}$, $H_{22}$, $H_{33}^r$, and $H_{det}$ depend linearly on solution concentration. The value of coefficients $H_{13}$, $H_{23}$, and $H_{33}$ does not depend on solution concentration.

(5) We can distinguish three groups of characteristics $H_{ij}^r = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $H_{ij} = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, and $H_{det}^r = f(\overline{C}_1, \overline{C}_2 = \text{const.})$, $(i, j \in \{1, 2, 3\}$ and $r = $ A, B). In the case of group 1, which includes concentration characteristics of coefficients $H_{12}^r$, $H_{12}$, $H_{21}^r$, and $H_{21}$ $(r = $ A, B), these coefficients are expressed in the same units and their values are in the range $-21.62$ mol·m$^{-3} \div 20.19$ mol·m$^{-3}$. In the case of the second group of characteristics, the shape of the concentration characteristics of the coefficients $H_{13}^r$, $H_{13}$, $H_{23}^r$, $H_{23}$, $H_{33}^r$, and $H_{33}$ $(r = $ A, B) is very similar. The values of coefficients $H_{13}^r$ and $H_{13}$ are expressed in different units than coefficients $H_{23}^r$, $H_{23}$, $H_{33}^r$, and $H_{33}$. The third group of characteristics includes the concentration characteristics of coefficients $H_{22}^r$, $H_{22}$, $H_{32}^r$, $H_{32}$, $H_{det}^r$, and $H_{det}$. Values of coefficients $H_{det}^r$ and $H_{det}$ are expressed in units other than $H_{22}^r$, $H_{22}$, $H_{32}^r$, and $H_{32}$.

(6) There is a threshold value of concentration $\overline{C}_1 \approx 9.24$ mol·m$^{-3}$ above which the values of coefficients $\psi_{12}$, $\psi_{13}$, $\psi_{22} = \psi_{23}$, $\psi_{32} = \psi_{33}$, and $\psi_{det}$ are equal to zero. For $\overline{C}_1 < 9.24$ mol·m$^{-3}$, the values of coefficients $\psi_{12}$ and $\psi_{13}$ are negative and fulfill the conditions $\psi_{22} = \psi_{23} \approx \psi_{32} = \psi_{33} > \psi_{det} > 0$ and $\psi_{12} > \psi_{13} < 0$. For $\overline{C}_1 > 9.23$ mol·m$^{-3}$ values of coefficients $\psi_{12}$ and $\psi_{13}$ are positive and fulfill the conditions $\psi_{22} = \psi_{23} \approx \psi_{32} = \psi_{33} < \psi_{det} < 0$ and, $\psi_{12} > \psi_{13} > 0$. In contrast, the values of coefficients $\psi_{22} = \psi_{23}$, $\psi_{32} = \psi_{33}$, and $\psi_{det}$ for $\overline{C}_1 < 9.24$ mol m$^{-3}$ are positive and for $\overline{C}_1 > 9.24$ mol·m$^{-3}$ negative. Besides, throughout the range of solution concentrations, $\psi_{11} = \psi_{21} = \psi_{31} = 0$.

(7) If $\psi_{22} = 0$, then $\zeta_1^A = \zeta_1^B = 0.234$ and it is conjugated with critical value of concentration Rayleigh number $R_C = [g(\rho_h - \rho_l)(\delta)^3](\rho_h \nu_h D_{11})^{-1} = [g(\rho_h - \rho_l)(1 - \zeta D_{11}^2)^3][8\rho_h \nu_h D_1^4(\zeta RT\omega_{11})^3]^{-1} = 1691.09$. The value is similar to the classical Rayleigh number critical value for Bernard problem $R_C = 1707.76$ [38].

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

## References

[1] M. A. Shannon, P. W. Bohn, M. Elimelech, J. G. Georgiadis, B. J. Mariñas, and A. M. Mayes, "Science and technology for water purification in the coming decades," *Nature*, vol. 452, no. 7185, pp. 301–310, 2008.

[2] T. Uragami, *Science and Technology of Separation Membranes*, Wiley, Chichester, UK, 2017.

[3] L. r. Bocquet and J.-L. Barrat, "Flow boundary conditions from nano- to micro-scales," *Soft Matter*, vol. 3, no. 6, pp. 685–693, 2007.

[4] P. Bacchin, "Colloid-interface interactions initiate osmotic flow dynamics," *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, vol. 533, pp. 147–158, 2017.

[5] J. Titze, "Interstitial fluid homeostasis and pressure: news from the black box," *Kidney International*, vol. 84, no. 5, pp. 869–871, 2013.

[6] X. Cheng and P. M. Pinsky, "The balance of fluid and osmotic pressures across active biological membrane with application to the corneal endothelium," *PLoS One*, vol. 10, no. 12, Article ID e0145422, 2015.

[7] B. Huang, H. Wang, and B. Yang, "Water transport mediated by other membrane proteins," in *Advances in Experimental Medicine and Biology*, B. Young, Ed., vol. 969, pp. 251–261, Springer, Dordrecht, Netherlands, 2017.

[8] Z. Siwy and F. Fornasiero, "Improving on aquaporins," *Science*, vol. 357, no. 6353, p. 753, 2017.

[9] O. Kedem and A. Katchalsky, "Thermodynamic analysis of the permeability of biological membranes to non-electrolytes," *Biochimica et Biophysica Acta*, vol. 27, pp. 229–246, 1958.

[10] A. Katchalsky and P. F. Curran, *Nonequilibrium Thermodynamics in Biophysics*, Harvard University Press, Cambridge, UK, 1965.

[11] Y. Demirel, *Nonequilibrium Thermodynamics: Transport and Rate Processes in Physical, Chemical and Biological Systems*, Elsevier, Amsterdam, Netherlands, 2014.

[12] H. Y. Elmoazzen, J. A. W. Elliott, and L. E. McGann, "Osmotic transport across cell membranes in nondilute solutions: a new nondilute solute transport equation," *Biophysical Journal*, vol. 96, no. 7, pp. 2559–2571, 2009.

[13] J. G. Wijmans and R. W. Baker, "The solution-diffusion model: a review," *Journal of Membrane Science*, vol. 107, no. 1-2, pp. 1–21, 1995.

[14] M. A. Al-Obaidi, C. Kara-Zaitri, and I. M. Mujtaba, "Scope and limitations of the irreversible thermodynamics and the solution diffusion models for the separation of binary and multi-component systems in reverse osmosis process," *Computers & Chemical Engineering*, vol. 100, pp. 48–79, 2017.

[15] K. S. Spiegler, "Transport processes in ionic membranes," *Transactions of the Faraday Society*, vol. 54, pp. 1408–1428, 1958.

[16] O. Kedem and A. Katchalsky, "A physical interpretation of the phenomenological coefficients of membrane permeability," *Journal of General Physiology*, vol. 45, no. 1, pp. 143–179, 1961.

[17] A. Ślęzak, "A frictional interpretation of the phenomenological coefficients of membrane permeability for

multicomponent non-ionic solutions," *Journal of Biological Physics*, vol. 23, no. 4, pp. 239–250, 1997.

[18] M. H. Friedman and R. A. Meyer, "Transport across homoporous and heteroporous membranes in nonideal, nondilute solutions. I. Inequality of reflection coefficients for volume flow and solute flow," *Biophysical Journal*, vol. 34, no. 3, pp. 535–544, 1981.

[19] M. H. Friedman and R. A. Meyer, "Transport across homoporous and heteroporous membranes in nonideal, nondilute solutions. II. Inequality of phenomenological and tracer solute permeabilities," *Biophysical Journal*, vol. 34, no. 3, pp. 545–557, 1981.

[20] E. A. Mason and H. K. Lonsdale, "Statistical-mechanical theory of membrane transport," *Journal of Membrane Science*, vol. 51, no. 1-2, pp. 1–81, 1990.

[21] G. Oster, A. Perelson, and A. Katchalsky, "Network thermodynamics," *Nature*, vol. 234, no. 5329, pp. 239–399, 1971.

[22] L. Peusner, *Studies in Network Thermodynamics*, Elsevier, Amsterdam, Netherlands, 1986.

[23] K. M. Batko, I. Ślęzak-Prochazka, S. Grzegorczyn, and A. Ślęzak, "Membrane transport in concentration polarization conditions: network thermodynamics model equations," *Journal of Porous Media*, vol. 17, no. 7, pp. 573–586, 2014.

[24] K. M. Batko, I. Ślęzak-Prochazka, and A. Ślęzak, "Network hybrid form of the Kedem-Katchalsky equations for non-homogenous binary non-electrolyte solutions: evaluation of $P_{ij}{}^*$ Peusner's tensor coefficients," *Transport in Porous Media*, vol. 106, no. 1, pp. 1–20, 2015.

[25] I. Ślęzak-Prochazka, K. M. Batko, S. Wąsik, and A. Ślęzak, "$H^*$ Peusner's form of the Kedem-Katchalsky equations for non-homogeneous non-electrolyte binary solutions," *Transport in Porous Media*, vol. 111, no. 2, pp. 457–477, 2016.

[26] A. Ślęzak, S. Grzegorczyn, and K. Batko, "Resistance coefficients of polymer membrane with concentration polarization," *Transport in Porous Media*, vol. 95, no. 1, pp. 151–170, 2012.

[27] M. H. Friedman, *Principles and Models of Biological Transport*, Springer, New York, NY, USA, 2008.

[28] Y. Demirel and S. I. Sandler, "Thermodynamics and bioenergetics," *Biophysical Chemistry*, vol. 97, no. 2-3, pp. 87–111, 2002.

[29] M. Kargol and A. Kargol, "Mechanistic formalism for membrane transport generated by osmotic and mechanical pressure," *General Physiology and Biophysics*, vol. 22, no. 1, pp. 51–68, 2003.

[30] J. Jasik-Ślęzak, I. Ślęzak-Prochazka, and A. Ślęzak, "Evaluation of the Peusner's coefficients matrix for polymeric membrane and ternary non-electrolyte solutions," *Polymers in Medicine*, vol. 44, pp. 167–178, 2014.

[31] A. Ślęzak, K. Dworecki, I. Ślęzak, and S. Wąsik, "Permeability coefficient model equations of the complex: membrane-concentration boundary layers for ternary nonelectrolyte solutions," *Journal of Membrane Science*, vol. 267, no. 1-2, pp. 50–57, 2005.

[32] K. M. Batko, A. K. Kołodziej, and A. Ślęzak, "$R^r$ form of the Kedem-Katchalsky-Peusner equations for membrane transport of non-electrolyte ternary solutions under condition of concentration polarization," *Journal of Non-Equilibrium Thermodynamics*, 2018.

[33] A. Ślęzak, "Irreversible thermodynamic model equations of the transport across a horizontally mounted membrane," *Biophysical Chemistry*, vol. 34, no. 2, pp. 91–102, 1989.

[34] J. Jasik-Ślęzak, K. M. Olszówka, and A. Ślęzak, "Estimation of thickness of concentration boundary layers by osmotic

volume flux determination," *General Physiology and Biophysics*, vol. 30, no. 2, pp. 186–195, 2011.

[35] K. Dworecki, A. Ślęzak, B. Ornal-Wąsik, and S. Wąsik, "Effect of hydrodynamic instabilities on solute transport in a membrane system," *Journal of Membrane Science*, vol. 265, no. 1-2, pp. 94–100, 2005.

[36] A. Ślęzak, K. Dworecki, J. Jasik-Ślęzak, and J. Wąsik, "Method to determine the practical concentration Rayleigh number in isothermal passive membrane transport processes," *Desalination*, vol. 168, pp. 397–412, 2004.

[37] A. Ślęzak, S. Grzegorczyn, J. Jasik-Ślęzak, and K. Michalska-Małecka, "Natural convection as an asymmetrical factor of the transport through porous membrane," *Transport in Porous Media*, vol. 84, no. 3, pp. 685–698, 2010.

[38] C. Normand, Y. Pomeau, and M. G. Velarde, "Convective instability: a physicist's approach," *Reviews of Modern Physics*, vol. 49, no. 3, pp. 581–624, 1977.