Erratum

Erratum to “Positive Solution to a Fractional Boundary Value Problem”

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In the paper entitled “Positive solution to a fractional boundary value problems,” the following problem (P1) is studied:

\[\begin{align*}
\frac{d^q}{dt^q} u(t) &= f\left(t, u(t), \frac{d^\sigma}{dt^\sigma} u(t)\right), \quad 0 < t < 1, \\
u(0) &= u''(0) = 0, \quad u'(1) = \alpha u''(1),
\end{align*}\]  

(1.1) (1.2)

where \(f: [0,1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}\) is a given function, \(2 < q < 3\), and \(1 < \sigma < 2\). Remarking that all the calculations in this paper are done for \(0 < \sigma < 1\) and that if we take \(1 < \sigma < 2\), then \(D_0^\sigma T u = (1/\Gamma(2 - \sigma)) \int_0^1 (Tu)'(s)/(t-s)^{\sigma-1})\,ds\) and the second derivative with respect to \(t\) of \(G(t, s)\) is discontinuous for \(s = t\), consequently we cannot apply this method to establish the existence and positivity of solution. For this reason, we correct the study of problem (P1) by taking \(0 < \sigma < 1\), and then the following corrections are needed.

1. In page 3, in Lemma 2.3, we should correct \(D_0^\alpha t^{\beta-1} = (\Gamma(\beta)/\Gamma(\beta-\alpha))t^{\beta-\alpha-1}, \beta > n\).

2. Equation (2.6) must be

\[u(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(t,s) y(s)\,ds.\]  

(2.6)

The Green function in (2.7) is

\[G(t,s) = \begin{cases} 
(1-s)^{3-q}(t-s)^{q-1} & s < t, \\
\frac{at - t(1-s)}{q-2} & t \leq s.
\end{cases}\]  

(2.7)

(3) Equation (2.11) becomes

\[u(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{(t-s)^{q-1}}{(q-1)(q-2)} + \frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \] 
\[\times y(s)\,ds + \frac{1}{\Gamma(q-2)} \int_0^1 \frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \] 
\[\times y(s)\,ds.\]  

(2.11)

Equation (2.12) must be

\[u(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(t,s) y(s)\,ds.\]  

(2.12)

(4) Equation (3.1) must be

\[Tu(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(t,s) f\left(s, u(s), \frac{d^\sigma}{dt^\sigma} u(s)\right)\,ds.\]  

(3.1)

In Theorem 3.2, the condition (3.5) must be

\[C_g + C_h < \frac{1}{2}, \quad A_g + A_h < \frac{\Gamma(2-\alpha)}{2}.\]  

(3.5)
(5) Equation (3.12) must be
\[ |Tu - Tv| < \frac{1}{2} \|u - v\|, \quad (3.12) \]
and (3.13) becomes
\[ cD_0^\sigma Tu - cD_0^\sigma Tv = \frac{1}{\Gamma(1 - \sigma)} \int_{0}^{t} \frac{(Tu)'(s) - (Tv)'(s)}{(t - s)\eta} ds. \quad (3.13) \]

(6) Equation (3.14) is
\[ G_1(t, s) = \frac{\partial G(t, s)}{\partial t} = \begin{cases} \frac{(1 - s)^{\alpha-q}(t - s)^{\alpha-2}}{(q - 2)} + \alpha - \frac{(1 - s)}{q - 2}, & s < t, \\ \alpha - \frac{(1 - s)}{q - 2}, & t \leq s. \end{cases} \quad (3.14) \]

(7) Equation (3.15) is as follows:
\[ cD_0^\sigma Tu - cD_0^\sigma Tv = \frac{1}{\Gamma(q - 2)\Gamma(1 - \sigma)} \int_{0}^{t} (t - s)^{-\sigma} \frac{1}{(1 - r)^{3-q}} G_1(s, r) \times (f(r, u(r), cD_0^\sigma u(r)) - f(r, v(r), cD_0^\sigma v(r))) dr ds. \quad (3.15) \]

Equation (3.16) is as follows:
\[ cD_0^\sigma Tu - cD_0^\sigma Tv \leq \max \left\{ \frac{\|u - v\|}{\Gamma(q - 2)\Gamma(1 - \sigma)} \right\} \frac{1}{\Gamma(q - 2)\Gamma(1 - \sigma)} \int_{0}^{t} (t - s)^{-\sigma} \frac{1}{(1 - r)^{3-q}} G_1(s, r) \times (f(r, u(r), cD_0^\sigma u(r)) - f(r, v(r), cD_0^\sigma v(r))) dr ds. \quad (3.16) \]

Equation (3.17) is as follows:
\[ \int_{0}^{1} \frac{1}{(1 - r)^{3-q}} G_1(s, r) f(r) dr \leq \Gamma(q - 2) \left( 2L_0^{q-1} g(1) + |\alpha| L_0^{q-2} g(1) \right). \quad (3.17) \]

(8) Equation (3.18) is as follows:
\[ \left\| cD_0^\sigma Tu - cD_0^\sigma Tv \right\| \leq \frac{1}{\Gamma(2 - \sigma)} \left( A_g + A_h \right). \quad (3.18) \]

Equation (3.19) becomes
\[ \left\| cD_0^\sigma Tu - cD_0^\sigma Tv \right\| \leq \frac{1}{2} \|u - v\|. \quad (3.19) \]

Equation (3.20) is as follows:
\[ a.e. \ (t, x, \xi) \in [0, 1] \times \mathbb{R}^2. \quad (3.21) \]

Equation (3.21) is as follows:
\[ (\psi(r) + \phi(r) + 1) \left( \frac{C_1}{\Gamma(q - 2)} + \frac{C_2}{\Gamma(2 - \sigma)} \right) < r. \quad (3.22) \]

Equation (3.22) is as follows:
\[ \|Tu\| = (\psi(r) + \phi(r) + 1) \left( \frac{C_1}{\Gamma(q - 2)} + \frac{C_2}{\Gamma(2 - \sigma)} \right). \quad (3.23) \]

Equation (3.23) is as follows:
\[ \|Tu(t_1) - Tu(t_2)\| \leq \frac{C}{\Gamma(q - 2)} \int_{0}^{t_1} (t_2 - s)^{-q-1} \frac{(t_2 - s)^{-q-1}}{(q - 1)(q - 2)} ds + \left( t_2 - t_1 \right) \left( \frac{|\alpha|}{(1 - s)^{3-q}} - \frac{(1 - s)^{q-2}}{q - 2} \right) ds \quad (3.24) \]

Equation (3.24) is as follows:
\[ \int_{0}^{t_1} (t_1 - t)^{-q-1} \frac{(t_1 - t)^{-q-1}}{(q - 1)(q - 2)} ds \quad (3.25) \]

Equation (3.25) is as follows:
\[ \int_{0}^{t_1} (t_1 - t)^{-q-1} \frac{(t_1 - t)^{-q-1}}{(q - 1)(q - 2)} ds \quad (3.26) \]

Equation (3.26) is as follows:
\[ \int_{0}^{t_1} (t_1 - t)^{-q-1} \frac{(t_1 - t)^{-q-1}}{(q - 1)(q - 2)} ds \quad (3.27) \]
Equation (3.33) is as follows:
\[
\left| Tu(t_1) - Tu(t_2) \right| \\
\leq \frac{C}{\Gamma(q-2)} \\
\times \left[ (t_2 - t_1) \\
\times \int_0^{t_1} \frac{1}{(q-2)} \\
\times \left( \frac{|a|}{(1-s)^{3-q}} + \frac{(1-s)^{-q}}{q-2} \right) ds \\
+ \int_{t_1}^{t_2} \frac{(t_2 - s)^{q-1}}{q-2} (q - 1) ds \\
\times \left( \frac{|a|}{(1-s)^{3-q}} + \frac{(1-s)^{-q}}{q-2} \right) ds \\
+ (t_2 - t_1) \int_0^{t_1} \frac{1}{(1-s)^{3-q}} + \frac{(1-s)^{-q}}{q-2} \right] ds.
\]

Equation (3.34) is as follows:
\[
\left| Tu(t_1) - Tu(t_2) \right| \\
\leq \frac{C}{\Gamma(q-2)} \frac{4 + 3|a|}{(q-2)} \\
\times \left[ \left( \frac{|a|}{(1-s)^{3-q}} + \frac{(1-s)^{-q}}{q-2} \right) ds \\
+ \int_{t_1}^{t_2} \frac{(t_2 - s)^{q-1}}{q-2} (q - 1) ds \\
\times \left( \frac{|a|}{(1-s)^{3-q}} + \frac{(1-s)^{-q}}{q-2} \right) ds \\
+ (t_2 - t_1) \int_0^{t_1} \frac{1}{(1-s)^{3-q}} + \frac{(1-s)^{-q}}{q-2} \right] ds.
\]

Equation (3.35) is as follows:
\[
\left| \frac{c}{\alpha} D_0^\sigma Tu(t_1) - \frac{c}{\alpha} D_0^\sigma Tu(t_2) \right| \\
\leq \frac{1}{\Gamma(1-\sigma)} \int_0^{t_1} \left( (t_1 - s)^{1-\sigma} - (t_2 - s)^{1-\sigma} \right) ds \\
\times \left| Tu(s) \right| ds \\
\times \left| Tu(s) \right| ds.
\]

Equation (3.36) is as follows:
\[
\left| \frac{c}{\alpha} D_0^\sigma Tu(t_1) - \frac{c}{\alpha} D_0^\sigma Tu(t_2) \right| \\
\leq \left( \psi(r) + \phi(r) + 1 \right) \\
\times C_2 \left[ 2(t_2 - t_1)^{1-\sigma} + \left( t_2^{1-\sigma} - t_1^{1-\sigma} \right) \right] \\
\times (\Gamma(2-\sigma))^{-1}.
\]

Equation (3.37) is as follows:
\[
\left| \frac{c}{\alpha} D_0^\sigma Tu(t) \right| \\
\leq \frac{C_2}{\Gamma(2-\sigma)} \left( \psi(r) + \phi(r) + 1 \right).
\]

Equation (3.38) is as follows:
\[
\| u \| \leq (\psi(r) + \phi(r) + 1) \left( \frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)} \right) < r.
\]

Equation (3.39) is as follows:
\[
\left| \frac{c}{\alpha} D_0^\sigma u(t) \right| \\
\leq \frac{2}{\Gamma(2-\sigma)} \left( \psi(r) + \phi(r) + 1 \right) \\
\times \int_0^1 \frac{1}{(1-s)^{3-q}} G(s, s) a(s) f_1(u(s), \frac{c}{\alpha} D_0^\sigma u(s)) ds.
\]

Equation (3.40) is as follows:
\[
\| u \| \leq (\psi(r) + \phi(r) + 1) \left( \frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)} \right) < r.
\]
Equation (4.8) is as follows:
\[
\frac{\sigma_0 + u(t)}{\Gamma(q-2)} \leq \frac{2}{\tau \Gamma(q-2) \Gamma(2-\sigma)} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds.
\]

Equation (4.9) is as follows:
\[
\|u\| \leq \frac{2}{\tau^2 \Gamma(q-2)} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds.
\]

Equation (4.10) is as follows:
\[
\int_0^1 \frac{1}{(1-s)^{3-q}} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds \geq \frac{2}{\tau^2 \Gamma(q-2)} \|u\|.
\]

Equation (4.11): in view of the left hand side of (4.1), we obtain for all \( t \in [\tau, 1] \)
\[
u(t) \geq \frac{\tau}{\Gamma(q-2)} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds.
\]

Equation (4.12) is as follows:
\[
\sigma_0 + u(t) \geq \frac{1}{\Gamma(q-2) \Gamma(2-\sigma)} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds.
\]

Equation (4.13) is as follows:
\[
\min_{t \in [\tau, 1]} \left( u(t) + \frac{\sigma_0 + u(t)}{\Gamma(q-2)} \right) \geq \frac{\tau}{\Gamma(q-2) \Gamma(2-\sigma)} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds.
\]

Equation (4.14) is as follows:
\[
\min_{t \in [\tau, 1]} \left( u(t) + \frac{\sigma_0 + u(t)}{\Gamma(q-2)} \right) \geq \frac{\tau^3}{2} \|u\|.
\]

Equation (4.17) is as follows:
\[
K = \left\{ u \in E^+, \min_{t \in [\tau, 1]} \left( u(t) + \frac{\sigma_0 + u(t)}{\Gamma(q-2)} \right) \geq \frac{\tau^3}{2} \|u\| \right\}.
\]

Equation (4.19) is as follows:
\[
Tu(t) = \frac{1}{\Gamma(q-2)} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds.
\]

Equation (4.20) is as follows:
\[
\sigma_0 + Tu(t) \leq \frac{2}{\tau^2 \Gamma(q-2) \Gamma(2-\sigma)} \left( G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right) \times f_1(u(s), \sigma_0 + u(s)) \, ds.
\]
Equation (4.21) is as follows:
\[ \|Tu\| \leq \frac{2\varepsilon \|u\|}{\tau^2 \Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} \times \left[ G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right] a(s) \, ds. \] (4.21)

Equation (4.22) is as follows:
\[ \varepsilon \leq \frac{\tau^2 \Gamma(q-2)}{2} \left( \int_0^1 \frac{1}{(1-s)^{3-q}} \times \left[ G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)} \right] a(s) \, ds \right)^{-1}. \] (4.22)

Let
\[ R = \max \left\{ 2R_1, \frac{2R_2}{\tau^2} \right\}. \] (1)

Equation (4.23) is as follows:
\[ \min_{t \in [\tau,1]} \left( u(t) + \int_0^t \left( \frac{\tau^3}{2} \right)^{\frac{3}{2}} \|u\| \right) \geq \frac{\tau^3}{2} \|u\| \geq \frac{\tau^3}{2} R \geq R_2. \] (4.23)

Using the left hand side of (4.1) and Lemma 4.1, we obtain (4.24):
\[ Tu(t) \geq \frac{\tau^4 M \|u\|}{2^{\Gamma(q-2)}} \int_0^1 \frac{1}{(1-s)^{3-q}} G(s,\phi) a(s) \, ds. \] (4.24)

Equation (4.25) is as follows:
\[ \int_0^1 \frac{1}{(1-s)^{3-q}} G_1(s,\phi) a(s) \, ds. \] (4.25)

Equation (4.26) is as follows:
\[ Tu(t) + \int_0^t \left( \frac{\tau^4}{2} \right)^{\frac{3}{2}} \|u\| \int_0^1 \frac{1}{(1-s)^{3-q}} \left[ G(s,\phi) + \frac{G_1(s,\phi)}{\Gamma(2-\sigma)} \right] a(s) \, ds. \] (4.26)

Equation (4.27) is as follows:
\[ M \geq 2^\Gamma(q-2) \times \left( \tau^2 \right)^{\frac{3}{2}} \left( \frac{1}{(1-s)^{3-q}} \right) G(s,\phi) + \frac{G_1(s,\phi)}{\Gamma(2-\sigma)} \right] a(s) \, ds \right)^{-1}. \] (4.27)

In Example 4.6, if we choose \( \sigma = 1/4 < 1 \); then we get the same results with
\[ C_g + C_h = 0.49821 < \frac{1}{2}, \] (2)
\[ A_g + A_h = 0.42552 < \frac{\Gamma(2-\sigma)}{2} = 0.459. \]

In Example 4.7, choose \( \sigma = 1/5, \psi(x) = (x/10)^2 + 1, \) and \( \phi(\overline{x}) = \ln(1 + \overline{x}^2)/9 + 1; \) then we get the same results.

Remark 1. One can study the problem (P1) for \( 1 < \sigma < 2 \) and the function \( f \) depending only on \( t \) and \( u \) instead of \( f(t, u(t), \int_0^t u(t)) \).
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