Erratum

Erratum to “Positive Solution to a Fractional Boundary Value Problem”

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In the paper entitled “Positive solution to a fractional boundary value problems,” the following problem (P1) is studied:

$${}^c D_{0+}^q u(t) = f(t, u(t), {}^c D_{0+}^\sigma u(t)), \quad 0 < t < 1, \quad (1.1)$$

$$u(0) = u''(0) = 0, \quad u'(1) = \alpha u''(1), \quad (1.2)$$

where $f: [0, 1] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is a given function, $2 < q < 3$, and $1 < \sigma < 2$. Remarking that all the calculuses in this paper are done for $0 < \sigma < 1$ and that if we take $1 < \sigma < 2$, then ${}^c D_{0+}^\sigma Tu = (1/\Gamma(2-\sigma)) \int_0^t ((Tu)''(s)/(t-s)^{\sigma-1}) ds$ and the second derivative with respect to t of $G(t, s)$ is discontinuous for $s = t$, consequently we cannot apply this method to establish the existence and positivity of solution. For this reason, we correct the study of problem (P1) by taking $0 < \sigma < 1$, and then the following corrections are needed.

(1) In page 3, in Lemma 2.3, we should correct ${}^c D_{0+}^\alpha t^{\beta-1} = (\Gamma(\beta)/\Gamma(\beta-\alpha))t^{\beta-\alpha-1}$, $\beta > n$.

(2) Equation (2.6) must be

$$u(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(t, s) y(s) ds. \quad (2.6)$$

The Green function in (2.7) is

$$G(t, s) = \begin{cases} \frac{(1-s)^{3-q}(t-s)^{q-1}}{(q-1)(q-2)} + \alpha t - \frac{t(1-s)}{q-2}, & s < t, \\ \alpha t - \frac{t(1-s)}{q-2}, & t \leq s. \end{cases} \quad (2.7)$$

(3) Equation (2.11) becomes

$$\begin{aligned} u(t) &= \frac{1}{\Gamma(q-2)} \\ &\times \int_0^t \left[\frac{(t-s)^{q-1}}{(q-1)(q-2)} + \frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \right] y(s) ds \\ &+ \frac{1}{\Gamma(q-2)} \int_t^1 \left[\frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \right] y(s) ds. \end{aligned} \quad (2.11)$$

Equation (2.12) must be

$$u(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(t, s) y(s) ds. \quad (2.12)$$

(4) Equation (3.1) must be

$$\begin{aligned} Tu(t) &= \frac{1}{\Gamma(q-2)} \\ &\times \int_0^1 \frac{1}{(1-s)^{3-q}} G(t, s) f(s, u(s), {}^c D_{0+}^\sigma u(s)) ds. \end{aligned} \quad (3.1)$$

In Theorem 3.2, the condition (3.5) must be

$$C_g + C_h < \frac{1}{2}, \quad A_g + A_h < \frac{\Gamma(2-\sigma)}{2}. \quad (3.5)$$

(5) Equation (3.12) must be

$$|Tu - Tv| < \frac{1}{2} \|u - v\|, \quad (3.12)$$

and (3.13) becomes

$${}^c D_{0^+}^\sigma Tu - {}^c D_{0^+}^\sigma Tv = \frac{1}{\Gamma(1-\sigma)} \int_0^t \frac{(Tu)'(s) - (Tv)'(s)}{(t-s)^\sigma} ds. \quad (3.13)$$

(6) Equation (3.14) is

$$G_1(t, s) = \frac{\partial G(t, s)}{\partial t} = \begin{cases} \frac{(1-s)^{3-q}(t-s)^{q-2}}{(q-2)} + \alpha - \frac{(1-s)}{q-2}, & s < t, \\ \alpha - \frac{(1-s)}{q-2}, & t \leq s. \end{cases} \quad (3.14)$$

(7) Equation (3.15) is as follows:

$$\begin{aligned} & {}^c D_{0^+}^\sigma Tu - {}^c D_{0^+}^\sigma Tv \\ &= \frac{1}{\Gamma(q-2)\Gamma(1-\sigma)} \\ & \times \int_0^t \int_0^1 (t-s)^{-\sigma} \frac{1}{(1-r)^{3-q}} G_1(s, r) \\ & \times (f(r, u(r), {}^c D_{0^+}^\sigma u(r)) \\ & \quad - f(r, v(r), {}^c D_{0^+}^\sigma v(r))) dr ds. \end{aligned} \quad (3.15)$$

Equation (3.16) is as follows:

$$\begin{aligned} & |{}^c D_{0^+}^\sigma Tu - {}^c D_{0^+}^\sigma Tv| \\ & \leq \frac{\max |u - v|}{\Gamma(q-2)\Gamma(1-\sigma)} \\ & \times \int_0^t \int_0^1 (t-s)^{-\sigma} \frac{1}{(1-r)^{3-q}} |G_1(s, r)| g(r) dr ds \\ & + \frac{\max |{}^c D_{0^+}^\sigma u - {}^c D_{0^+}^\sigma v|}{\Gamma(q-2)\Gamma(1-\sigma)} \\ & \times \int_0^t \int_0^1 (t-s)^{-\sigma} \frac{1}{(1-r)^{3-q}} |G_1(s, r)| h(r) dr ds. \end{aligned} \quad (3.16)$$

Equation (3.17) is as follows:

$$\begin{aligned} & \int_0^1 \frac{1}{(1-r)^{3-q}} |G_1(s, r)| g(r) dr \\ & \leq \Gamma(q-2) (2I_{0^+}^{q-1} g(1) + |\alpha| I_{0^+}^{q-2} g(1)). \end{aligned} \quad (3.17)$$

(8) Equation (3.18) is as follows:

$$|{}^c D_{0^+}^\sigma Tu - {}^c D_{0^+}^\sigma Tv| \leq \|u - v\| \frac{1}{\Gamma(2-\sigma)} (A_g + A_h). \quad (3.18)$$

Equation (3.19) becomes

$$|{}^c D_{0^+}^\sigma Tu - {}^c D_{0^+}^\sigma Tv| \leq \frac{1}{2} \|u - v\|. \quad (3.19)$$

Equation (3.21) is as follows:

$$\text{a.e } (t, x, \bar{x}) \in [0, 1] \times \mathbb{R}^2. \quad (3.21)$$

Equation (3.22) is as follows:

$$(\psi(r) + \phi(r) + 1) \left(\frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)} \right) < r. \quad (3.22)$$

Equation (3.29) becomes

$$|{}^c D_{0^+}^\sigma Tu| \leq \frac{C_2}{\Gamma(2-\sigma)} (\psi(r) + \phi(r) + 1). \quad (3.29)$$

Equation (3.30) is

$$\|Tu\| = (\psi(r) + \phi(r) + 1) \left(\frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)} \right). \quad (3.30)$$

Equation (3.32) is as follows:

$$\begin{aligned} & |Tu(t_1) - Tu(t_2)| \\ & \leq \frac{C}{\Gamma(q-2)} \\ & \times \int_0^{t_1} \frac{(t_2-s)^{q-1} - (t_1-s)^{q-1}}{(q-1)(q-2)} \\ & \quad + (t_2-t_1) \left(\frac{|\alpha|}{(1-s)^{3-q}} - \frac{(1-s)^{q-2}}{q-2} \right) ds \\ & + \int_{t_1}^{t_2} \frac{(t_2-s)^{q-1}}{(q-1)(q-2)} \\ & \quad + (t_2-t_1) \left(\frac{|\alpha|}{(1-s)^{3-q}} - \frac{(1-s)^{q-2}}{q-2} \right) ds \\ & + \int_{t_2}^1 (t_1-t_2) \left(\frac{|\alpha|}{(1-s)^{3-q}} - \frac{(1-s)^{q-2}}{q-2} \right) ds. \end{aligned} \quad (3.32)$$

Equation (3.33) is as follows:

$$\begin{aligned}
 & |Tu(t_1) - Tu(t_2)| \\
 & \leq \frac{C}{\Gamma(q-2)} \\
 & \quad \times \left[(t_2 - t_1) \right. \\
 & \quad \times \int_0^{t_1} \frac{1}{(q-2)} \\
 & \quad \quad + \left(\frac{|\alpha|}{(1-s)^{3-q}} + \frac{(1-s)^{q-2}}{q-2} \right) ds \\
 & \quad + \int_{t_1}^{t_2} \frac{(t_2-s)^{q-1}}{(q-2)(q-1)} \\
 & \quad \quad + (t_2-t_1) \left(\frac{|\alpha|}{(1-s)^{3-q}} + \frac{(1-s)^{q-2}}{q-2} \right) ds \\
 & \quad \left. + (t_1-t_2) \int_{t_2}^1 \left(\frac{|\alpha|}{(1-s)^{3-q}} + \frac{(1-s)^{q-2}}{q-2} \right) ds \right]. \quad (3.33)
 \end{aligned}$$

Equation (3.34) is as follows:

$$\begin{aligned}
 & |Tu(t_1) - Tu(t_2)| \\
 & \leq \frac{C(t_2-t_1)}{\Gamma(q-2)} \frac{4+3|\alpha|}{(q-2)} \\
 & \quad + \frac{C}{\Gamma(q-2)} \int_{t_1}^{t_2} \frac{(t_2-s)^{q-1}}{(q-2)(q-1)} ds. \quad (3.34)
 \end{aligned}$$

Equation (3.35) is as follows:

$$\begin{aligned}
 & \left| {}^c D_{0+}^\sigma Tu(t_1) - {}^c D_{0+}^\sigma Tu(t_2) \right| \\
 & \leq \frac{1}{\Gamma(1-\sigma)} \int_0^{t_1} \left((t_1-s)^{-\sigma} - (t_2-s)^{-\sigma} \right) \\
 & \quad \times |(Tu(s))'| ds \\
 & \quad + \frac{1}{\Gamma(1-\sigma)} \int_{t_1}^{t_2} (t_2-s)^{-\sigma} |(Tu(s))'| ds. \quad (3.35)
 \end{aligned}$$

Equation (3.37) is as follows:

$$\begin{aligned}
 & \left| {}^c D_{0+}^\sigma Tu(t_1) - {}^c D_{0+}^\sigma Tu(t_2) \right| \\
 & \leq \left([\psi(r) + \phi(r) + 1] \right. \\
 & \quad \times C_2 \left[2(t_2-t_1)^{1-\sigma} + t_2^{1-\sigma} - t_1^{1-\sigma} \right] \\
 & \quad \left. \times (\Gamma(2-\sigma))^{-1} \right). \quad (3.37)
 \end{aligned}$$

Equation (3.39) is as follows:

$$\left| {}^c D_{0+}^\sigma u(t) \right| \leq \frac{C_2}{\Gamma(2-\sigma)} (\psi(r) + \phi(r) + 1). \quad (3.39)$$

Equation (3.40) is as follows:

$$\|u\| \leq (\psi(r) + \phi(r) + 1) \left(\frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)} \right) < r. \quad (3.40)$$

$$(9) \text{ (H2)} \quad 0 < \int_0^1 (1/(1-s)^{3-q}) G(s,s) a(s) ds < \infty.$$

In Lemma 4.1, we have $\alpha \geq 1/(q-2)$ and (4.1) becomes:

If $t, s \in [\tau, 1]$, $\tau > 0$, then

$$\begin{aligned}
 0 < \tau G(s,s) & \leq G(t,s) \leq \frac{2}{\tau^2} G(s,s), \\
 0 < G_1(s,s) & \leq G_1(t,s) \leq \frac{2}{\tau} G_1(s,s). \quad (4.1)
 \end{aligned}$$

Equation (4.2) should be

$$\begin{aligned}
 & \frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \\
 & = \frac{t}{(q-2)(1-s)^{3-q}} [(q-2)\alpha - 1 + s] \quad (4.2)
 \end{aligned}$$

which is positive if $\alpha \geq 1/(q-2)$.

Equation (4.3): let $t, s \in [\tau, 1]$; it is easy to see that $G(s,s) \neq 0$, and then we have

$$\begin{aligned}
 \frac{G(t,s)}{G(s,s)} & = \frac{(t-s)^{q-1}(1-s)^{3-q}}{(q-1)s[(q-2)\alpha - 1 + s]} + \frac{t}{s} \\
 & \leq \frac{1+(1-s)^2}{s^2} \\
 & \leq \frac{2}{\tau^2}, \quad 0 < \tau \leq s \leq t \leq 1, \quad (4.3)
 \end{aligned}$$

$$\frac{G(t,s)}{G(s,s)} = \frac{t}{s} \leq \frac{2}{\tau} \leq \frac{2}{\tau^2}, \quad 0 < \tau \leq t \leq s \leq 1.$$

Finally, since $G(s,s)$ is nonnegative, we obtain $0 < \tau G(s,s) \leq G(t,s) \leq (2/\tau^2)G(s,s)$.

(10) In Lemma 4.3, put $\alpha \geq 1/(q-2)$ and inequality (4.5) becomes

$$\min_{t \in [\tau, 1]} \left(u(t) + {}^c D_{0+}^\sigma u(t) \right) \geq \frac{\tau^3}{2} \|u\|. \quad (4.5)$$

Equation (4.7) is as follows:

$$\begin{aligned}
 u(t) & \leq \frac{2}{\tau^2 \Gamma(q-2)} \\
 & \times \int_0^1 \frac{1}{(1-s)^{3-q}} G(s,s) a(s) f_1(u(s), {}^c D_{0+}^\sigma u(s)) ds. \quad (4.7)
 \end{aligned}$$

Equation (4.8) is as follows:

$$\begin{aligned}
 {}^c D_{0^+}^\sigma u(t) &= \frac{1}{\Gamma(q-2)\Gamma(1-\sigma)} \\
 &\quad \times \int_0^t \int_0^1 (t-s)^{-\sigma} \frac{1}{(1-r)^{3-q}} G_1(s, r) \\
 &\quad \times a(r) f_1(u(r), {}^c D_{0^+}^\sigma u(r)) ds dr \\
 &\leq \frac{2}{\tau \Gamma(q-2)\Gamma(2-\sigma)} \\
 &\quad \times \int_0^1 \frac{1}{(1-r)^{3-q}} G_1(r, r) a(r) \\
 &\quad \times f_1(u(r), {}^c D_{0^+}^\sigma u(r)) dr.
 \end{aligned} \tag{4.8}$$

Equation (4.9) is as follows:

$$\begin{aligned}
 \|u\| &\leq \frac{2}{\tau^2 \Gamma(q-2)} \\
 &\quad \times \int_0^1 \frac{1}{(1-s)^{3-q}} \left[G(s, s) + \frac{G_1(s, s)}{\Gamma(2-\sigma)} \right] \\
 &\quad \times a(s) f_1(u(s), {}^c D_{0^+}^\sigma u(s)) ds.
 \end{aligned} \tag{4.9}$$

Equation (4.10) is as follows:

$$\begin{aligned}
 &\int_0^1 \frac{1}{(1-s)^{3-q}} \left[G(s, s) + \frac{G_1(s, s)}{\Gamma(2-\sigma)} \right] \\
 &\quad \times a(s) f_1(u(s), {}^c D_{0^+}^\sigma u(s)) ds \\
 &\geq \frac{\tau^2 \Gamma(q-2)}{2} \|u\|.
 \end{aligned} \tag{4.10}$$

Equation (4.11): in view of the left hand side of (4.1), we obtain for all $t \in [\tau, 1]$

$$\begin{aligned}
 u(t) &\geq \frac{\tau}{\Gamma(q-2)} \\
 &\quad \times \int_0^1 \frac{1}{(1-s)^{3-q}} G(s, s) a(s) f_1(u(s), {}^c D_{0^+}^\sigma u(s)) ds.
 \end{aligned} \tag{4.11}$$

Equation (4.12) is as follows:

$$\begin{aligned}
 {}^c D_{0^+}^\sigma u(t) &\geq \frac{1}{\Gamma(q-2)\Gamma(2-\sigma)} \\
 &\quad \times \int_0^1 \frac{1}{(1-r)^{3-q}} G_1(r, r) a(r) \\
 &\quad \times f_1(u(r), {}^c D_{0^+}^\sigma u(r)) dr.
 \end{aligned} \tag{4.12}$$

Equation (4.13) is as follows:

$$\begin{aligned}
 \min_{t \in [\tau, 1]} (u(t) + {}^c D_{0^+}^\sigma u(t)) &\geq \frac{\tau}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} \\
 &\quad \times \left[G(s, s) + \frac{G_1(s, s)}{\Gamma(2-\sigma)} \right] \\
 &\quad \times a(s) f_1(u(s), {}^c D_{0^+}^\sigma u(s)) ds.
 \end{aligned} \tag{4.13}$$

Equation (4.14) is as follows:

$$\min_{t \in [\tau, 1]} (u(t) + {}^c D_{0^+}^\sigma u(t)) \geq \frac{\tau^3}{2} \|u\|. \tag{4.14}$$

Equation (4.17) is as follows:

$$K = \left\{ u \in E^+, \min_{t \in [\tau, 1]} (u(t) + {}^c D_{0^+}^\sigma u(t)) \geq \frac{\tau^3}{2} \|u\| \right\}. \tag{4.17}$$

Equation (4.19) is as follows:

$$\begin{aligned}
 Tu(t) &= \frac{1}{\Gamma(q-2)} \\
 &\quad \times \int_0^1 \frac{1}{(1-s)^{3-q}} \\
 &\quad \times G(t, s) a(s) f_1(u(s), {}^c D_{0^+}^\sigma u(s)) ds \\
 &\leq \frac{2\varepsilon \|u\|}{\tau^2 \Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(s, s) a(s) ds.
 \end{aligned} \tag{4.19}$$

Equation (4.20) is as follows:

$$\begin{aligned}
 {}^c D_{0^+}^\sigma Tu(t) &= \frac{1}{\Gamma(q-2)\Gamma(1-\sigma)} \\
 &\quad \times \int_0^t \int_0^1 \frac{1}{(1-r)^{3-q}} (t-s)^{-\sigma} G_1(s, r) \\
 &\quad \times a(r) f_1(u(r), {}^c D_{0^+}^\sigma u(r)) ds dr \\
 &\leq \frac{2}{\tau \Gamma(q-2)\Gamma(2-\sigma)} \\
 &\quad \times \int_0^1 \frac{1}{(1-r)^{3-q}} G_1(r, r) a(r) \\
 &\quad \times (|u(r)| + |{}^c D_{0^+}^\sigma u(r)|) dr \\
 &\leq \frac{2\varepsilon \|u\|}{\tau^2 \Gamma(q-2)\Gamma(2-\sigma)} \int_0^1 \frac{1}{(1-s)^{3-q}} G_1(s, s) a(s) ds.
 \end{aligned} \tag{4.20}$$

Equation (4.21) is as follows:

$$\|Tu\| \leq \frac{2\varepsilon \|u\|}{\tau^2 \Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} \times \left[G(s, s) + \frac{G_1(s, s)}{\Gamma(2-\sigma)} \right] a(s) ds. \quad (4.21)$$

Equation (4.22) is as follows:

$$\varepsilon \leq \tau^2 \Gamma(q-2) \times \left(2 \int_0^1 \frac{1}{(1-s)^{3-q}} \times \left[G(s, s) + \frac{G_1(s, s)}{\Gamma(2-\sigma)} \right] a(s) ds \right)^{-1}. \quad (4.22)$$

Let

$$R = \max \left\{ 2R_1, \frac{2R_2}{\tau^3} \right\}. \quad (1)$$

Equation (4.23) is as follows:

$$\min_{t \in [\tau, 1]} \left(u(t) + {}^c D_{0^+}^\sigma u(t) \right) \geq \frac{\tau^3}{2} \|u\| = \frac{\tau^3}{2} R \geq R_2. \quad (4.23)$$

Using the left hand side of (4.1) and Lemma 4.1, we obtain (4.24):

$$Tu(t) \geq \frac{\tau^4 M \|u\|}{2\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(s, s) a(s) ds. \quad (4.24)$$

Equation (4.25) is as follows:

$${}^c D_{0^+}^\sigma Tu(t) \geq \frac{\tau^3 M \|u\|}{2\Gamma(q-2) \Gamma(2-\sigma)} \times \int_0^1 \frac{1}{(1-s)^{3-q}} G_1(s, s) a(s) ds. \quad (4.25)$$

Equation (4.26) is as follows:

$$Tu(t) + {}^c D_{0^+}^\sigma Tu(t) \geq \frac{\tau^4 M \|u\|}{2\Gamma(q-2)} \times \int_0^1 \frac{1}{(1-s)^{3-q}} \left[G(s, s) + \frac{G_1(s, s)}{\Gamma(2-\sigma)} \right] \times a(s) ds. \quad (4.26)$$

Equation (4.27) is as follows:

$$M \geq 2\Gamma(q-2) \times \left(\tau^4 \int_0^1 \left[\left(\frac{1}{(1-s)^{3-q}} \right) G(s, s) + G_1(s, s) / \Gamma(2-\sigma) \right] a(s) ds \right)^{-1}. \quad (4.27)$$

(11) In Example 4.6, if we choose $\sigma = 1/4 < 1$; then we get the same results with

$$C_g + C_h = 0.49821 < \frac{1}{2}, \quad (2)$$

$$A_g + A_h = 0.42552 < \frac{\Gamma(2-\sigma)}{2} = 0.459.$$

In Example 4.7, choose $\sigma = 1/5$, $\psi(x) = (x/10)^2 + 1$, and $\phi(\bar{x}) = \ln(1 + \bar{x}^2)/9 + 1$; then we get the same results.

Remark 1. One can study the problem (P1) for $1 < \sigma < 2$ and the function f depending only on t and u instead of $f(t, u(t), {}^c D_{0^+}^\sigma u(t))$.