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Research Article

Opportunistic Communication (Cognitive Radio) over Primary Discarded Subchannels by Applying a Double Power Distribution

G. A. Medina-Acosta and José A. Delgado-Penín

Signal Theory and Communications Department, Technical University of Catalonia, Building D4 Campus Nord, Jordi Girona, 1-3, 08034 Barcelona, Spain

Correspondence should be addressed to G. A. Medina-Acosta, agni_medina@tsc.upc.edu

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This paper proposes the establishment of a simultaneous cognitive radio communication based on a subdistribution of power made over unselected subchannels which were discarded by the primary user through an initial optimal power allotment. The aim of this work is to show the possibility of introducing an opportunistic communication into a licensed transmission where the total power constraint is shared. The analysis of the proposed transmission scheme was performed by considering 128 and 2048 independent subchannels affected by *Rayleigh* fading, over 10,000 channel realizations, and three different signal-to-noise ratios (8 dB, 16 dB, and 24 dB). From the system evaluation it was possible to find the optimal power allotment for the primary user, the subdistribution of power for the secondary user, as well as the attenuation and the capacity per subchannel for every channel realization. Moreover, the *PDF* and *CDF* of the total obtained capacities, as well as the generation of empirical capacity regions, were estimated as complementary results.

1. Introduction

The radio signals propagating through the environment are associated to a specific operation frequency belonging to one of the many wireless communications systems (i.e., LTE, WiMAX, etc.) existing today, which are strictly allocated by government agencies (i.e., FCC) or international organizations (i.e., ITU) [1, 2]. However, due to the continuous growing and development of the wireless industry, the current static frequency allocation has led to a problem related with spectrum scarcity [3, 4]. Nevertheless, recent worldwide measurement studies have revealed that most of the license spectrum experiences low utilization efficiency [5–7], which means that there exists the possibility of exploiting the underutilized spectrum in an opportunistic manner. According to this, an emerging technology that is able to reliable sense the spectral environment over a wide band, detect the presence/absence of licensed users (primary users), and use the spectrum only if the communication does not interfere with primary users is defined by the term cognitive radio (CR) [8, 9]. So, the spectrum utilization can be improved by making a secondary user access into the spectrum holes or spectrum portions that in a particular location and time are not being used by a primary user. In this regard, according to the current proposals of the CR protocol, the device is constantly aware of its wireless environment in order to determine (at least in space and time) which part of the spectrum is not being occupied by making use of spectrum sensing techniques [10, 11] to later on adapt its signal to fill those spectrum gaps. On the other hand, recent studies have opened the possibility of allowing secondary users to transmit simultaneously with the primary users over the same frequency band [12, 13]. In [14], a proposal that is closely related to the one here described in terms of power allocation involving a primary and a secondary user is presented. Nevertheless, between the main differences it is possible to highlight that the channel characteristics on that research work are considered to be nearly the same for both the primary and the secondary users, while the idea behind the establishment of a simultaneous cognitive radio transmission is based on the assumption that the primary user in any case will not

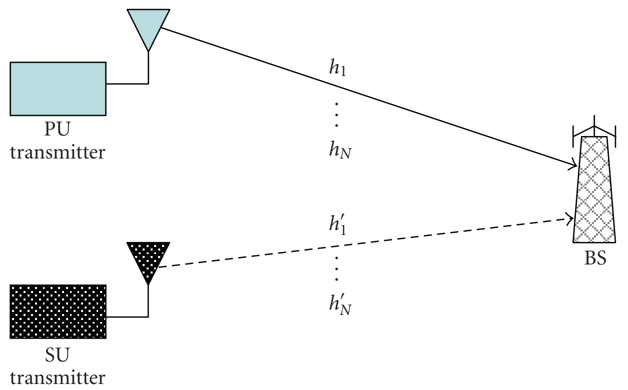

Figure 1: Proposed transmission scheme for establishing a simultaneous low-priority communication with a primary user.

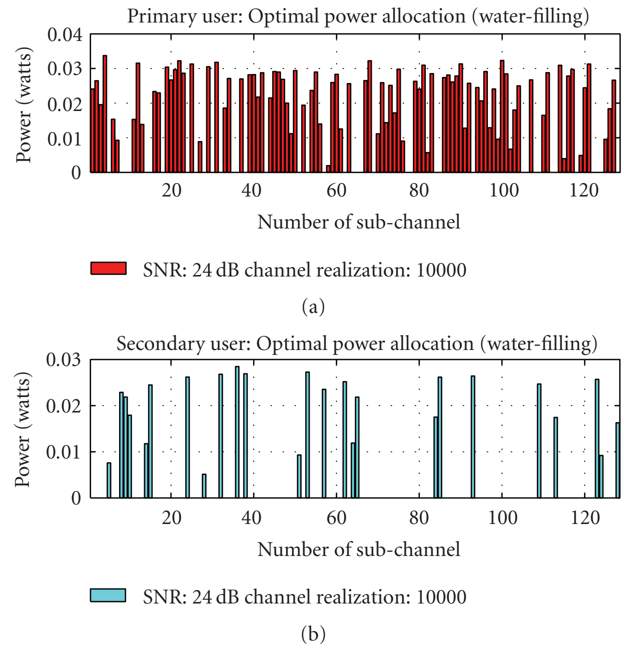

Figure 2: Optimal distribution and subdistribution of power (primary/secondary user), for channel realization number 10,000 with an SNR =24 dB.

need all its achievable rate; reason why the use of a virtual threshold (target rate) which overestimates the ambient noise is proposed by the authors.

So, by following this last research line, this paper proposes a methodology for establishing a simultaneous low-priority communication (in an opportunistic manner) where the total power constraint is shared. In which, a subdistribution of power is made over small transmission bandwidths (subchannels) that were identified as unselected once an initial optimal power allotment for the primary user (PU) took place. For simulation purposes it was considered that the operation frequency belonging to the primary user was divided in 128 and 2048 independent subchannels of 1 Hz each, affected by *Rayleigh* fading over 10,000 channel realizations and three different signal-to-noise ratios (8 dB, 16 dB, and 24 dB), having destined 20% of the total available power to the secondary user (SU). Among the obtained results, the optimal power allotment for the primary user, the subdistribution of power for the secondary user, as well as the computation corresponding to the attenuation and the capacity per subchannel for one of the channel realizations are shown. Being computed, in addition are the PDF, CDF, and a set of empirically generated capacity regions.

2. Transmission Scheme

The context of this proposal has to do with OFDM systems, being the particular case of a single user OFDM system the candidate scenario for the application of this methodology [15, 16]. So, the PU and SU are assumed to utilize exactly the same harmonic related frequencies (which are carefully chosen to be orthogonal), being the total available transmission bandwidth equally divided amongst N narrowband subcarriers [17–19]. In Figure 1, an uplink considering the presence of a PU and an SU communicating with a base station (BS) over the same frequency band is shown.

Figure 1 is possible to observe a simultaneous communication carried out by two users and a single base station (which is assumed to be equipped with cognitive radio capabilities), where PU is the high-priority user which undergoes several fadings ($h_1 \cdots h_N$) when it is considered that the frequency band over which it operates has been divided in N subbands, same bands that are used for establishing an opportunistic low-priority communication by a secondary user (SU) which undergoes its own fadings ($h'_1 \cdots h'_N$) due to its different spatial location.

So, by assuming that there are N subchannels, the proposal initially focuses on the primary communication over which the mathematical algorithm known as *waterfilling* is applied [20]. The algorithm allows distributing the total available power in an optimal way, which originates that sometimes such as optimization concentrates the power only over certain subchannels (those with the lowest noise levels) leaving the others without using, which corresponds to frequency subbands that can be utilized to transmit low-priority information. Thus, in order to use the spectrum in a more efficient way what is proposed here is to destine a certain percentage (i.e., 20%) of the total available power to a secondary user, so that once the optimal power allotment for the primary user was applied, such information can be used to perform a second power distribution (a new application of the waterfilling algorithm) only over the unused subchannels. Which in agreement with what is presented here turns out to be more efficient in terms of spectrum usage, increasing the total system capacity with respect to the total obtained capacity when the 100% of the total available power is destined to the licensed user.

3. Optimal Power Allotment

The mathematical algorithm known as *waterfilling*, as it was stated before, allows distributing the total available power maximizing the capacity [21, 22]. The algorithm uses the

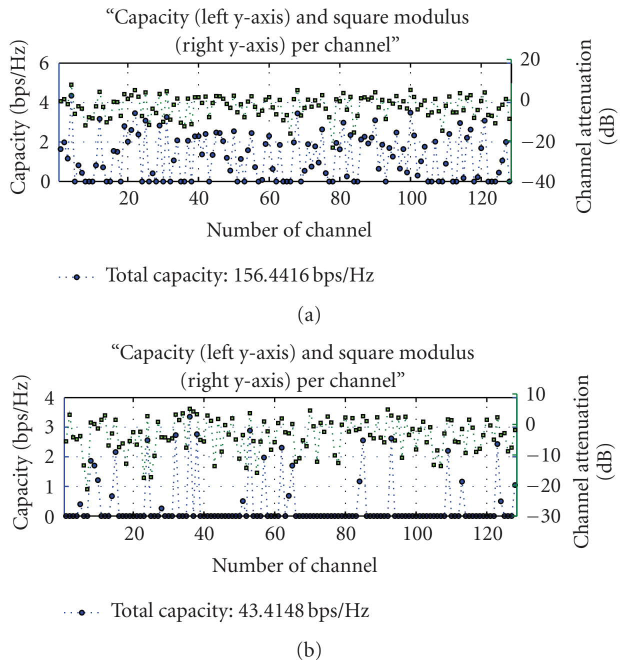

FIGURE 3: Attenuation and capacity per subchannel (primary/secondary user), for the channel realization number 10,000 with SNR = 24 dB.

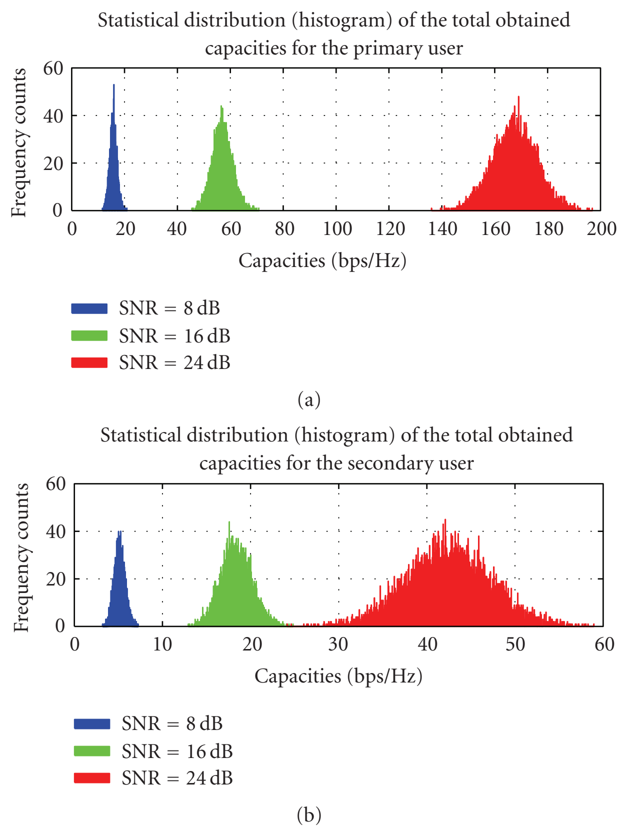

FIGURE 4: Histograms of the total obtained capacities (primary/secondary user), SNR = 8 dB, 16 dB, and 24 dB.

equation corresponding to the total capacity for the primary user (2) subject to a total power constraint (3), leading to an optimization problem that can be solved by using the *Lagrange* multipliers [23].

$$\begin{gathered}\nabla f(x) - \lambda \nabla g(x) = 0, \\ \nabla F(x) = 0, \\ \frac{\partial F(x)}{\partial x} = 0,\end{gathered} \tag{1}$$

where $f(x)$ is the function from which we want to find the extreme values subject to the constraint given by $g(x)$, while λ is known as *Lagrange* multiplier

$$C = \sum_{i=1}^{N} \log\left(1 + \frac{P_i |hi|^2}{\sigma^2}\right) \text{ (bps/Hz)}. \tag{2}$$

In this regard the above expression refers to $f(x)$, the total capacity of the primary user which we want to maximize by considering N subchannels, with σ^2 being the noise power, $|hi|^2$ the attenuation, and Pi the power of the *i-esim* subchannel, respectively. While the power constraint represents $g(x)$, which is defined as

$$\sum_{i=1}^{N} P_i = P_{\mathrm{PU}} \text{ (watts)}. \tag{3}$$

Which establishes that the sum of each of the powers assigned to the subchannels must be equal to a total given available power (in this case the one corresponding to the primary user P_{PU}). So, based on the two above equations and by using the *Lagrange* multipliers it is possible to write,

$$F(P_i) = \sum_{i=1}^{N} \log\left(1 + \frac{P_i |hi|^2}{\sigma^2}\right) - \lambda\left(\sum_{i=1}^{N} P_i - P_{\mathrm{PU}}\right). \tag{4}$$

The above expression corresponds to a function which, when is derived with respect to P_i, allows to find the optimal power distribution

$$\frac{1}{\lambda} - \frac{\sigma^2}{|hi|^2} = P_i \text{ (watts)}. \tag{5}$$

So, by following what is established in the above equation, in order to optimally allocate the power for every subchannel it is necessary to previously determine the value for the constant $1/\lambda$, which corresponds to a level of power that acts as a threshold and that is defined by,

$$\frac{1}{\lambda} = \frac{P_{\mathrm{PU}} + \sum_{i=1}^{N} \sigma^2 / |hi|^2}{N}. \tag{6}$$

Which can be found by substituting the result obtained in (5) directly in (3). On the other hand, because the algorithm could assign negative powers, it is necessary to

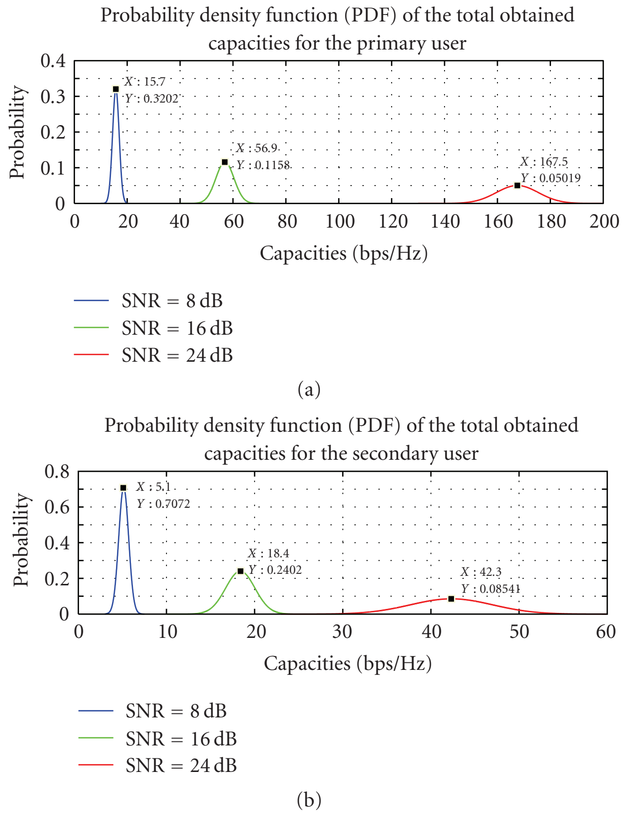

Figure 5: PDFs of the total obtained capacities (primary/secondary user), SNR = 8 dB, 16 dB, and 24 dB.

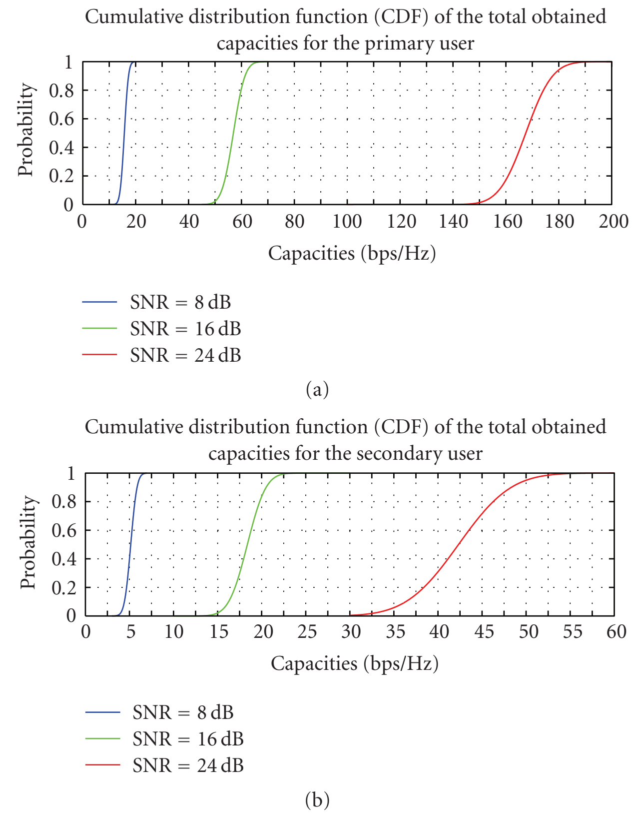

Figure 6: CDFs of the total obtained capacities (primary/secondary user), SNR = 8 dB, 16 dB, and 24 dB.

apply the conditions known as *Kuhn-Tucker* [24]

$$[x]^+ = \begin{cases} x & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases} \tag{7}$$

In this context, after considering that P_i takes the value of x, the imposed conditions are established to assign a zero whenever a negative power is obtained. Consequently it is necessary to find a new threshold and to recalculate the power allotment by only taking into account those channels for which we previously had obtained positive powers, discarding therefore the rest. This concludes the description of the methodology that allows optimally allocating the available power for the primary user. Hence, the following step consists in describing the insertion of a secondary user into the system by considering the implementation of a strategy related with a subdistribution of power.

4. Subdistribution of Power

According to what is proposed here, a certain percentage of the total system's available power (P) would be destined to a secondary user, therefore $P = P_{\text{PU}} + P_{\text{SU}}$. Being necessary to take into account that the primary user must not be affected by the presence of the low-priority user is the reason why such communication would not have to notice the establishment of a secondary communication in

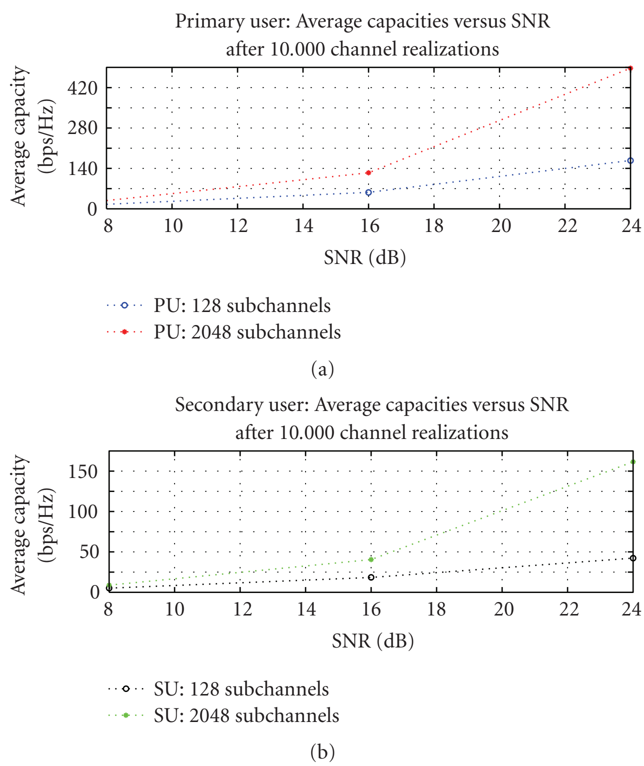

Figure 7: Average Capacities comparison for 128 and 2048 subchannels (primary/secondary user), SNR = 8 dB, 16 dB, and 24 dB.

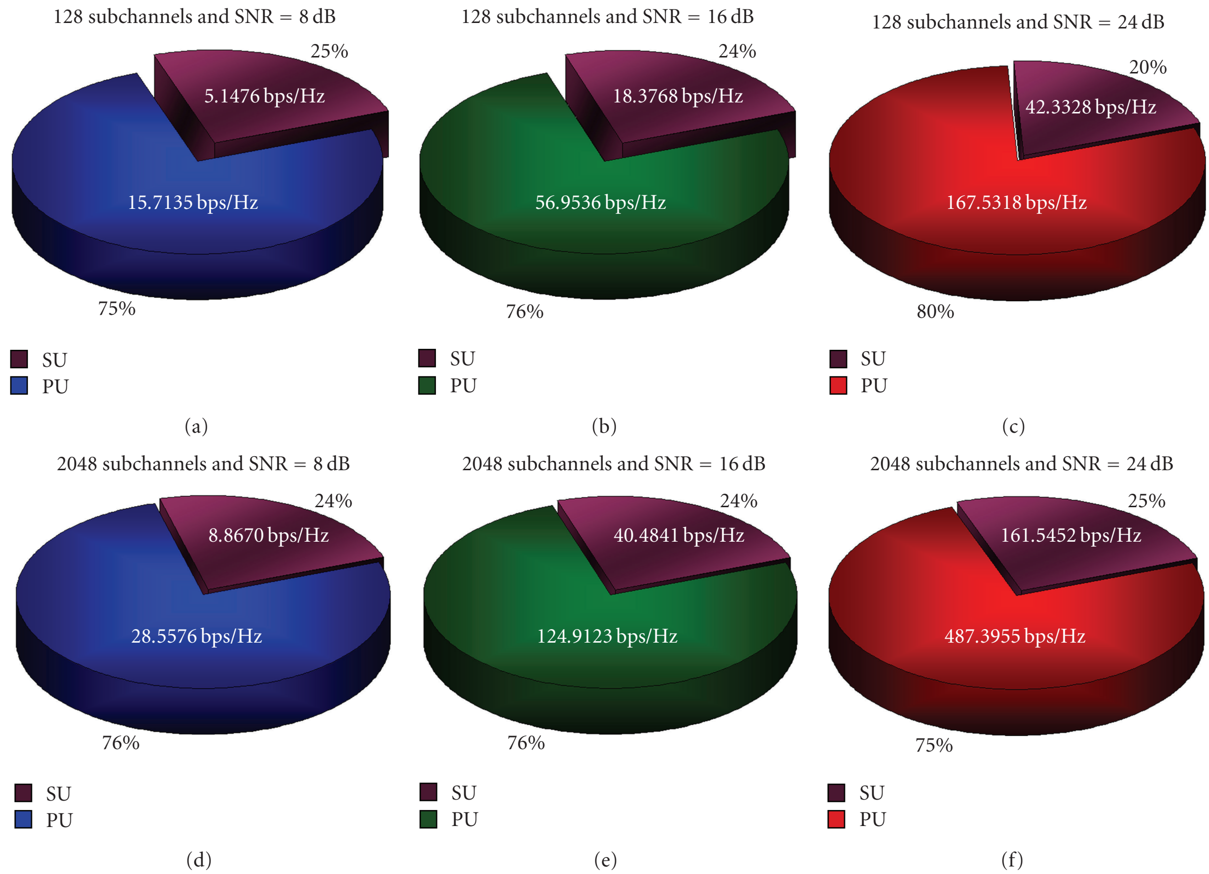

FIGURE 8: Percentage contribution (PU and SU) in terms of the average capacities (128/2048 subchannels), SNR = 8 dB, 16 dB, and 24 dB.

parallel, while this last one would perceive the high-priority communication as noise. This leads to the following equation corresponding to the total capacity for the secondary user:

$$C' = \sum_{i=1}^{N} \log\left(1 + \frac{P'_i |h'_i|^2}{P_i |h_i|^2 + \sigma^2}\right) \text{(bps/Hz)}, \tag{8}$$

where the terms $|h'_i|^2$ and P'_i correspond to the attenuation and the power of the *i-esim* subchannel for the secondary user, respectively. For its part, the power constraint in this case is defined as

$$\sum_{i=1}^{N} P'_i = P_{\text{SU}} \text{ (watts)}. \tag{9}$$

Which infers that the sum of the powers allocated to the subchannels must be equal to the percentage of power destined to the secondary user. So, based on the above expressions it is possible to apply once again the algorithm known as *waterfilling*, which together with the *Kuhn-Tucker* conditions produces as optimal solution the following:

$$\left[\frac{1}{\lambda} - \left(\frac{P_i |h_i|^2 + \sigma^2}{|h_i|'^2}\right)\right]^{+} = P'_i \text{ (watts)}. \tag{10}$$

It is once again necessary to determine the value corresponding to the constant $1/\lambda$, which can be found by substituting (10) in (9)

$$\frac{1}{\lambda} = \frac{P_{\text{SU}} + \sum_{i=1}^{N} \left(P_i |h_i|^2 + \sigma^2\right) / \left(|h_i|'^2\right)}{N}. \tag{11}$$

So, under these considerations it is possible to establish a simultaneous low-priority communication over the same operation band that originally is utilized only by the licensed user.

Summarizing the proposal, in general once the optimal power allotment for the primary user applied it was possible to know which subchannels were discarded by the optimization algorithm, or in other words to which of them was power not given. Thus, by making use of this information it is possible to apply a second optimization algorithm only over those subchannels which were previously indentified as available, leading to a subdistribution of power that concentrates P_{SU} exclusively on those subbands. Reason why the low-priority communication is established in an opportunistic manner over the unused subchannels is avoiding, this way, to interfere with the primary user, whereas at the same time the spectrum is utilized in a more efficient way.

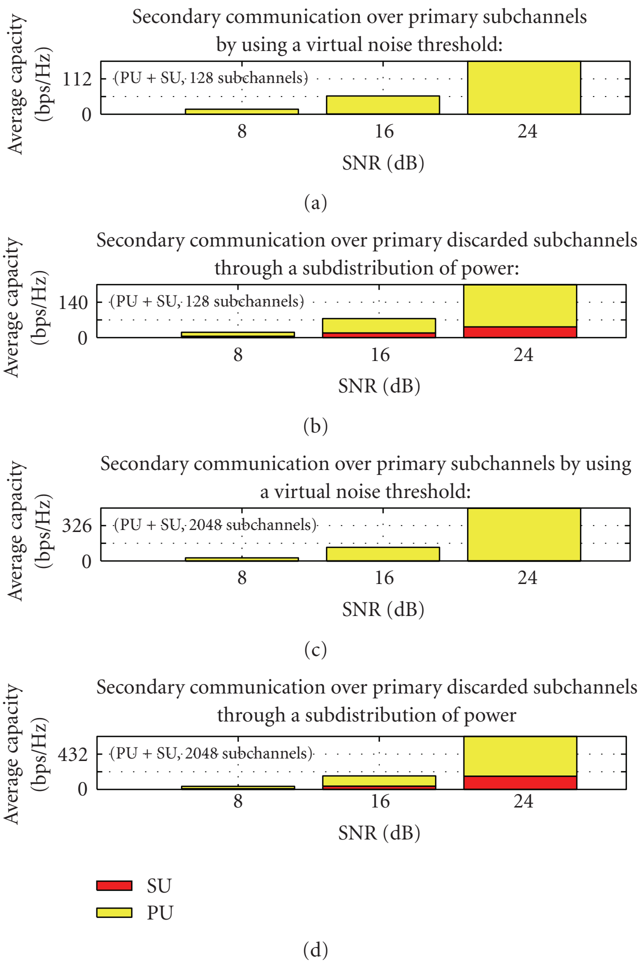

Figure 9: System's performance comparison (128/2048 subchannels, $P_{\mathrm{PU}} = 80\%$ & $P_{\mathrm{SU}} = 20\%$), SNR = 8 dB, 16 dB, and 24 dB.

5. Results

In this section, the obtained results after applying the methodology described before are discussed. In Figure 2, the optimal power allotment for the primary user (PU), as well as the subdistribution of power by the secondary user (SU), for one of the channel realizations (number 10,000/10,000) considering a signal-to-noise ratio equal to 24 dB and when the total transmission bandwidth is equally divided in 128 subchannels is shown.

In Figure 2(a) the result corresponding to the application of the first optimization algorithm (*waterfilling*) is shown, where it is possible to observe that some of the subchannels were discarded or not taken into account for establishing the primary communication (there is no power allocated to these subchannels). Which, as can be observed in the low part of the figure were utilized for establishing a low-priority communication through an optimal subdistribution of power made exclusively on those subbands.

On the other hand, in Figure 3, for the same channel realization and the same signal-to-noise ratio, the attenuation and the obtained capacity for each of the subchannels are shown.

Figure 3 is possible to note that for the first case (primary user), the subchannels having the best channel conditions reached higher capacities while those that presented a more severe attenuation were discarded, this is the reason why its capacity is equal to zero (as it happens with the subchannels 5, 62, 105, and 122 to mention some). On the other hand, for the second case (secondary user) it is possible to verify that the unused frequency subbands by the primary user were seen as available by the secondary user, who undergoes different channel conditions and that after applying an optimal subdistribution of power was able to obtain (for most of the cases and depending on the attenuation) capacities different from zero on these subchannels, which allows establishing a low-priority communication in parallel. In Figure 4, the statistical distribution of the total obtained capacities for each of the signal-to-noise ratios considered in this analysis is shown.

By taking into account all the channel realizations and once the last one took place, it was possible to determine the statistical distribution (*Histogram*) of the total obtained capacities. Where it can be verified that the total obtained average capacities PU: 15.7135 bps/Hz, 56.9536 bps/Hz, and 167.5318 bps/Hz / SU: 5.1476 bps/Hz, 18.3768 bps/Hz, and 42.3328 bps/Hz are located (resp.) on the intervals corresponding to the highest point in each of the histograms. Additionally, by making use of maximum likelihood estimators [25–27], it was found that the data corresponding to the total obtained capacities fit better to normal distributions with parameters shown in Table 1.

In Figures 5 and 6, respectively, approximations for the probability density function (PDF) and cumulative distribution function (CDF) are shown.

This way, from the approximations made for the PDF and CDF it is possible to extract valuable information related with the probability of observing a certain capacity. For example when as SNR = 24 dB, for the PUs the probability of observing a capacity less than or equal to 165.6 bps/Hz is 0.404 while for the SU there is a probability equal to 0.4042 of observing a capacity less or equal to 41.2 bps/Hz.

On the other hand, aiming at extending the results an analogue procedure was followed in order to obtain the average capacities by considering now 2048 subchannels, which, in Figure 7, were compared to the case of 128 subchannels.

Figure 7 shows a comparison between the previously discussed average capacities for the case of 128 subchannels and the obtained ones for the case of considering 2048 subchannels (PU: 28.5576 bps/Hz, 124.9123 bps/Hz, and 487.3955 bps/Hz / SU: 8.8670 bps/Hz, 40.4841 bps/Hz, and 161.5452 bps/Hz). Where it is possible to observe that for most of the cases (considering both 128 & 2048 subchannels) the SU reaches average capacities about one third with respect to those ones obtained by the PU.

Moreover, when the primary user (PU) and the secondary user (SU) are seen as unique whole system (which can be assumed since they share the total available power), their percentage contributions in terms of average capacities for each of the SNRs considered before are shown through a pie chart in Figure 8.

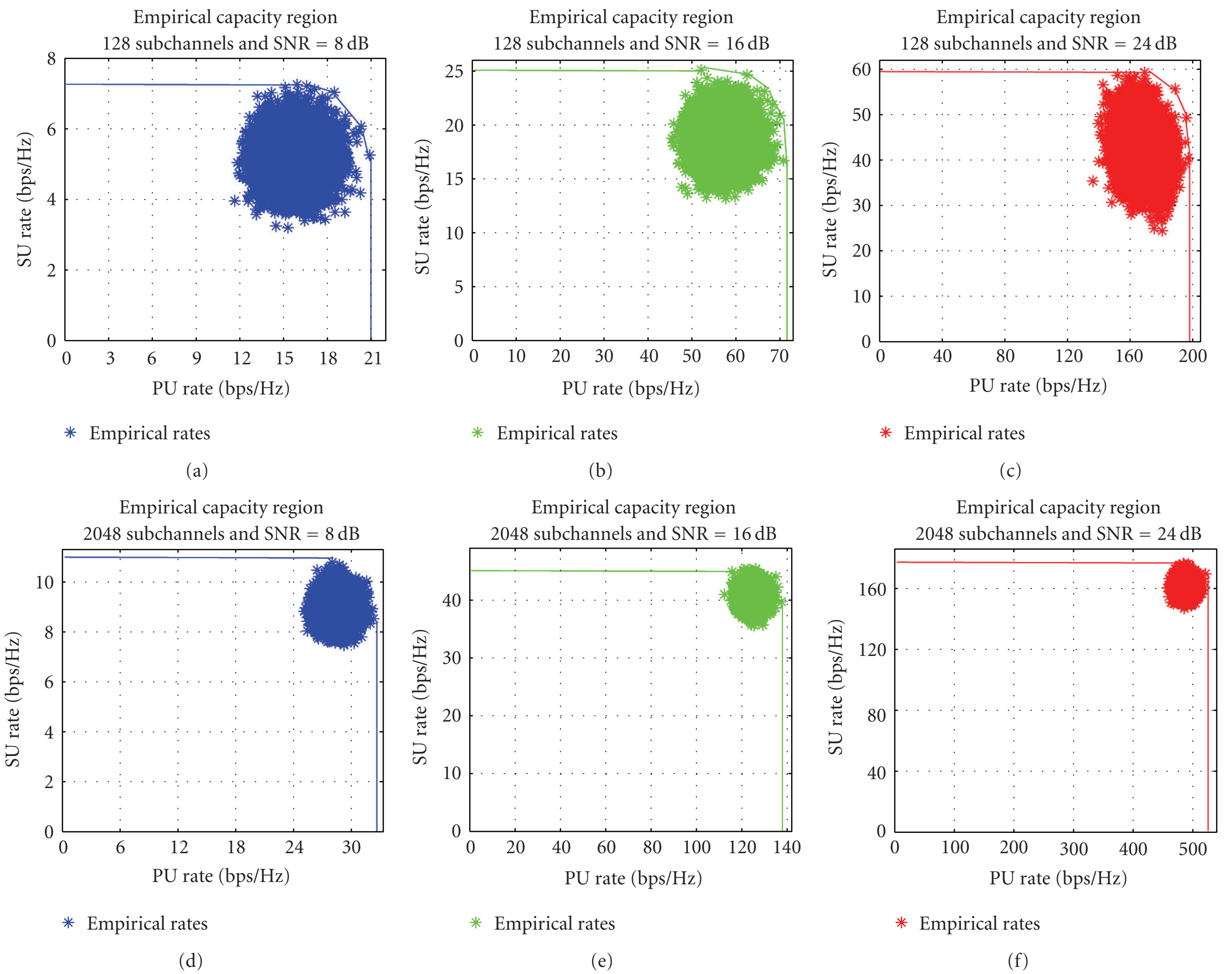

Figure 10: Empirical capacity regions (128/2048 subchannels, $P_{\mathrm{PU}} = 80\%$ & $P_{\mathrm{PU}} = 20\%$), SNR= 8 dB, 16 dB, and 24 dB.

Table 1: Parameters of the fitted curves.

SNR	PU	SU
8 dB	$\mu = 15.7135$ $\sigma^2 = 1.55186$	$\mu = 5.14764$ $\sigma^2 = 0.315983$
16 dB	$\mu = 56.9537$ $\sigma^2 = 11.8711$	$\mu = 18.3768$ $\sigma^2 = 2.75803$
24 dB	$\mu = 167.532$ $\sigma^2 = 63.1848$	$\mu = 42.3328$ $\sigma^2 = 21.8141$

Figure 8 allows visualizing the percentage contributions that the PU and SU make when they are considered as a single system. Under this assumption (PU+SU), the total average capacities supplied by the system turned out to be 20.8611 bps/Hz, 75.3304 bps/Hz, and 209.8646 bps/Hz & 37.4246 bps/Hz, 165.3964 bps/Hz, and 648.9407 bps/Hz for 128 and 2048 subchannels, respectively.

In this context, and in order to compare the obtained results, the principles stated in the proposal cited in [14], about considering the same channel impairments for PU & SU as well as the use of a given virtual noise threshold (chosen in this case arbitrarily to overestimate the ambient noise by 35%), were simulated under similar conditions (number of channel realizations, signal-to-noise ratios, and power sharing scheme) to those used in our analysis, whose results are shown in Figure 9.

After comparing the results, (for most of the signal-to-noise ratio levels considered) a low (even null) performance by part of the SU in the case of the proposal based on the virtual noise threshold can be noted, this due to the inherent differences between the foundations of the proposals, because while the first one originally treats the PU & SU as independent entities in terms of technical resources (i.e., no power sharing is contemplated, and therefore more power is required for its proper operation), the second one (our proposal) aims to maximize the use of the already available resources, which is fulfilled after observing the significant contribution made by the SU to the system's throughput.

On the other hand, when the same number of channel realizations is considered by only taking into account the primary user (that means by assigning all the available power to PU) the average capacities turned out to be 18.5231 bps/Hz, 65.7161 bps/Hz, and 187.6908 bps/Hz for the case of 128 subchannels & 34.3087 bps/Hz, 148.3475 bps/Hz, and 568.8980 bps/Hz for the case of 2048 subchannels, which do not differ in a significant way from the obtained capacities when the power is shared with a secondary user, and nevertheless if these last ones (secondary) are grouped with the primary ones (as we verified previously in the case of our proposal), they exceed all the total obtained capacities on average. This is reason why we use an opportunistic subdistribution of power over primary discarded subchannels for establishing a low-priority communication, which results in a more efficient usage of the available (natural & technical) resources.

Finally, bidimensional surfaces were generated in order to provide empirical approaches of the system's capacity region for each of the signal-to-noise ratios considered above, which are shown in Figure 10.

Figure 10 highlights the empirically obtained rate combinations (i.e., 128 subchannels & 8 dB: 20.89 bps/Hz, 5.255 bps/Hz / 2048 subchannels & 24 dB: 521.2 bps/Hz, 169.4 bps/Hz) for each of the 10,000 channel realizations, which correspond to points located inside a region representing the system's achievable rates (error-free) when it is considered that the PU & SU transmit simultaneously.

The capacity regions for Cognitive Radios have been theoretically studied and developed in [28–30], the results here being obtained nearer to be classified as an hybrid scheme that combines the overlay-interweave paradigm for Cognitive Radios since a power split is utilized, because the SU utilizes knowledge about the PU channel conditions (discarded subchannels) and because of the interference mitigation given by the orthogonal nature of this proposal. In concrete from the empirically obtained results, it can be verified that in spite of the rate reduction produced sometimes by having only a few available (primary discarded) subchannels, the SU was able to achieve (for each of the 10,000 channel realizations) opportunistic rates, in parallel, different from zero, optimizing, this way, the spectrum usage.

6. Conclusions

In this paper, a strategy for establishing a secondary communication where the total system's available power is shared involving a double optimization procedure is proposed. On the basis of a scenario where the operation band belonging to the primary user is equally divided in several (128 & 2048) subbands, and once an optimal power distribution is applied by using the algorithm known as *waterfilling*, the proposed methodology (taking advantage of the system's orthogonality) consists in making use of those subchannels that were not used or discarded by the PU for applying a subdistribution of power leading to the establishment of a low-priority communication in parallel. From the analysis of the obtained results it was possible to find approximations for the PDF and CDF in both cases PU & SU and to later on verify that despite of having presence in an opportunistic way the secondary user could reach average capacities up to one third of the obtained ones by the primary user, which turns out to be useful if we consider that initially those resources are not used. Moreover, if the primary and the secondary users are seen as a unique system, they exceed the total average capacities with respect to those obtained when a conventional single primary transmission (using exactly the same resources) is considered. In addition, the rate combinations obtained for each of the 10,000 channel realizations were utilized in order to create an approach about the system's achievable rates through the generation of empirical capacity regions. On the other hand, in terms of scalability it can be inferred that the fact of using a methodology like this could allow incorporating a low-priority communication per every licensed user located in an existing system (i.e., single user OFDM system). So, in order to finalize it is possible to conclude that by destining a certain percentage of the total available power to a secondary user and by applying a strategy related with a double distribution of power as suggested here, it is possible to use the spectrum in more efficient way without interfering and modifying drastically the obtained capacities when the total available power is destined only to a primary user.

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References

[1] Federal Communications Commission (FCC), http://www.fcc.gov.

[2] The ITU Radio Communications Sector (ITU-R), http://www.itu.int/ITU-R.

[3] R. W. Brodersen, A. Wolisz, D. Cabric, S. M. Mishra, and D. Willkomm, "A Cognitive Radio Approach for Usage of Virtual Unlicensed Spectrum," UC Berkeley White Paper, July 2004.

[4] H. Tang, "Some physical layer issues of wide-band cognitive radio systems," in *Proceedings of the 1st IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN '05)*, pp. 151–159, November 2005.

[5] D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proceedings of the 38th Asilomar Conference on Signals, Systems and Computers*, pp. 772–776, November 2004.

[6] Shared Spectrum Company, "Recent Measurements," http://www.sharedspectrum.com/measurements/recent.html.

[7] M. H. Islam, L. K. Choo, W. O. Ser et al., "Spectrum survey in Singapore: occupancy measurements and analyses," in *Proceedings of the 3rd International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CrownCom '08)*, May 2008.

[8] D. Čabrić and R. W. Brodersen, "Physical layer design issues unique to cognitive radio systems," in *Proceedings of the 16th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '05)*, vol. 2, pp. 759–763, September 2005.

[9] FCC, "Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," ET Docket 03-108, December 2003.

[10] T. Yücek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communications Surveys and Tutorials*, vol. 11, no. 1, pp. 116–130, 2009.

[11] H. Arslan, *Cognitive Radio Software Defined Radio and Adaptive Wireless Systems*, Springer, Berlin, Germany, 2007.

[12] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Transactions on Information Theory*, vol. 52, no. 5, pp. 1813–1827, 2006.

[13] A. Jovičić and P. Viswanath, "Cognitive radio: an information-theoretic perspective," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT '06)*, pp. 2413–2417, Seattle, Wash, USA, July 2006.

[14] M. Haddad, A. M. Hayar, and M. Debbah, "Optimal power allocation for cognitive radio based on a virtual noise threshold," in *Proceedings of the 1st International Workshop on Cognitive Wireless Networks (CWNETS '07)*, August 2007.

[15] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 2, pp. 171–178, 2003.

[16] M. Saleh, "Adaptive resource allocation in multiuser OFDM systems," Literature Survey on Multidimensional Digital Signal Processing, Texas University, 2005.

[17] T. Hwang, C. Yang, G. Wu, S. Li, and G. Y. Li, "OFDM and its wireless applications: a survey," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 4, pp. 1673–1694, 2009.

[18] V. Chakravarthy, A. S. Nunez, J. P. Stephens, A. K. Shaw, and M. A. Temple, "TDCS, OFDM, and MC-CDMA: a brief tutorial," *IEEE Communications Magazine*, vol. 43, no. 9, pp. S11–S16, 2005.

[19] X. Huang and H.-C. Wu, "Intercarrier interference analysis for wireless OFDM in mobile channels," in *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC '06)*, vol. 4, pp. 1848–1853, April 2006.

[20] T. Cover and J. Thomas, *Elements of Information Theory*, John Wiley & Sons, New York, NY, USA, 1991.

[21] R. G. Gallager, *Information Theory and Reliable Communication*, John Wiley & Sons, New York, NY, USA, 1968.

[22] A. Lozano, A. M. Tulino, and S. Verdú, "Optimum power allocation for parallel gaussian channels with arbitrary input distributions," *IEEE Transactions on Information Theory*, vol. 52, no. 7, pp. 3033–3051, 2006.

[23] J. Stewart, *Multivariable Calculus: Concepts and Contexts*, Thompson, Boston, Mass, USA, 2004.

[24] S. Haykin, *Communications Systems*, John Wiley & Sons, New York, NY, USA, 1991.

[25] "Likelihood and method of Maximum Likelihood (AI-ACC)," http://www.aiaccess.net/English/Glossaries/GlosMod/e_gm_likelihood.htm.

[26] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice Hall, Upper Saddle River, NJ, USA, 1993.

[27] E. L. Lehmann and G. Casella, *Theory of Point Estimation*, Springer Texts in Statistics, Springer, New York, NY, USA, 2nd edition, 1998.

[28] N. Devroye, P. Mitran, and V. Tarokh, "Limits on communications in a cognitive radio channel," *IEEE Communications Magazine*, vol. 44, no. 6, pp. 44–49, 2006.

[29] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Transactions on Information Theory*, vol. 52, no. 5, pp. 1813–1827, 2006.

[30] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894–914, 2009.