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Research Article

Location Discovery Based on Fuzzy Geometry in Passive Sensor Networks

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Location discovery with uncertainty using passive sensor networks in the nation's power grid is known to be challenging, due to the massive scale and inherent complexity. For bearings-only target localization in passive sensor networks, the approach of fuzzy geometry is introduced to investigate the fuzzy measurability for a moving target in R^2 space. The fuzzy analytical bias expressions and the geometrical constraints are derived for bearings-only target localization. The interplay between fuzzy geometry of target localization and the fuzzy estimation bias for the case of fuzzy linear observer trajectory is analyzed in detail in sensor networks, which can realize the 3-dimensional localization including fuzzy estimate position and velocity of the target by measuring the fuzzy azimuth angles at intervals of fixed time. Simulation results show that the resulting estimate position outperforms the traditional least squares approach for localization with uncertainty.

1. Introduction

Wireless sensor network localization in smart grid is an important area that attracted significant research interest. As a national smart grid constructed, it is important for developers to consider target localization problems to ensure both the smart grid operation efficiently. The objective of location discovery in sensor networks for smart grid is to estimate the location of a target from measurements collected by a single moving sensor or several fixed sensors at distinct and known locations.

For passive bearings-only localization, the sensor node detects the signals transmitted by a target to generate directional information in the form of bearing measurements. These measurements are triangulated to estimate the target location. While triangulation yields a unique intersection point for bearing lines in the absence of measurement errors, the noise present in bearing and observer measurements requires an optimal solution to be formulated based on noisy measurements; hence, statistical techniques for bearings-only target localization is introduced.

The pioneering work of Stansfield [1] provided a closed-form small error approximation of the maximum likelihood estimator in 1947. It is shown in [2] that the Stansfield estimator is asymptotically biased, where the traditional maximum likelihood (TML) formulation is examined in detail including a bias and variance analysis. A linearized least squares approach to bearings-based localization is given in [3]. The linearized and iterative algorithms typically require an initial estimate of the target location [2–5]. Liu et al. [6] proposed a vertical localization method using Euclidean geometry theory.

The practical passive target localization in sensor networks is characterized by a certain degree of uncertainty, which may result from approximate definition of the measurand, limited knowledge of the real environment, variability of influence quantities, inexact values of reference standards or parameters used in the model, background noise of the electronic devices, and so on. Especially, there exists uncertainty (fuzziness) with sensor locations and measurements, which can be studied based on fuzzy geometry. Rosenfeld [7] first discussed some concepts and properties of fuzzy plane geometry. Buckley and Eslami [8, 9] proposed another theory of fuzzy plane geometry, where the distances between fuzzy points, fuzzy area, and fuzzy circumference is considered as fuzzy numbers. Inspired by the authors in [7–9], the

approach of fuzzy geometry is introduced to investigate the passive bearings-only target localization for the case of fuzzy linear observer trajectory in passive sensor networks.

The paper is organized as follows. The concept of fuzzy geometry is provided in Section 2. A novel analysis approach of passive bearings-only target localization based on fuzzy geometry theory for the case of fuzzy linear observer trajectory is proposed in Section 3. Simulation examples are presented in Section 4 to validate the theoretical findings of the paper. Section 5 concludes the paper.

2. Fuzzy Geometry

In this section, fuzzy points and fuzzy lines in fuzzy geometry are introduced. Place a "bar" over a capital letter to denote a fuzzy subset of R^n $(n = 1, 2, 3)$ such as, $\overline{X}, \overline{Y}, \overline{A}$, and $\overline{B}$. Any fuzzy set is defined by its membership function. If $\overline{A}$ is a fuzzy subset of R^n $(n = 1, 2, 3)$, we write its membership function with $\mu((x_1, \ldots, x_n) \mid \overline{A})$ in $[0, 1]$ for all x. The α-cut of any fuzzy set $\overline{X}$ of R^n, $\overline{X}(\alpha)$, is defined as $\{x : \mu((x_1, \ldots, x_n) \mid \overline{A}) \geq \alpha\}$, $0 < \alpha \leq 1$, and $\overline{X}(0)$ is the closure of the union of $\overline{X}(\alpha)$, $0 < \alpha \leq 1$. $\vec{\overline{X}}$ is denoted as a fuzzy vector, and $\vec{x}$ is denoted as a traditional vector.

2.1. Fuzzy Points

Definition 1. A fuzzy point at $p = (a_1, a_2, \ldots, a_n)$ in R^n $(n = 1, 2, 3)$, written $\overline{P}(a_1, \ldots, a_n)$, is defined by its membership function:

(1) $\mu((x_1, \ldots, x_n) \mid \overline{P}(a_1, \ldots, a_n))$ is upper semicontinuous;

(2) $\mu((x_1, \ldots, x_n) \mid \overline{P}(a_1, \ldots, a_n)) = 1$, if and only if $(x_1, \ldots, x_n) = (a_1, \ldots, a_n)$;

(3) $\overline{P}(\alpha)$ is a compact, convex, subset of R^n for all α, $0 < \alpha \leq 1$.

Next; we define the fuzzy distance between fuzzy points. Let $d(u, v)$ be the usual Euclidean distance metric between points u and v in R^n; we define the fuzzy distance $\overline{D}$ between two fuzzy points $\overline{P_1} = \overline{P}(a_{11}, \ldots, a_{n1})$, $\overline{P_2} = \overline{P}(a_{12}, \ldots, a_{n2})$.

Definition 2. Consider $\Omega(\alpha) = \{d(u, v) : u$ is in $\overline{P}(a_1, b_1)(\alpha)$ and v is in $\overline{P}(a_2, b_2)(\alpha)\}$, $0 \leq \alpha \leq 1$, then, $\mu(d \mid \overline{D}) = \sup\{\alpha : d \in \Omega(\alpha)\}$.

Theorem 1. *One has $\overline{D}(\alpha) = \Omega(\alpha), 0 \leq \alpha \leq 1$, and $\overline{D}$ is a real fuzzy number.*

Definition 3. A fuzzy metric $\overline{M}$ is a mapping from pairs of fuzzy points $(\overline{P_1}, \overline{P_2})$ into fuzzy numbers so that

(1) $\overline{M}(\overline{P_1}, \overline{P_2}) = \overline{M}(\overline{P_2}, \overline{P_1})$,

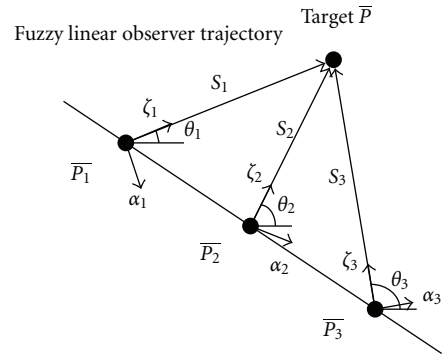

Figure 1: Fuzzy geometry for bearings-only target localization (3 sensors along fuzzy linear observer trajectory) in R^2 space.

(2) $\overline{M}(\overline{P_1}, \overline{P_2}) = \overline{0}$, if and only if $\overline{P_1}, \overline{P_2}$ are both fuzzy points at $(a_1, a_2, \ldots, a_n)$,

(3) $\overline{M}(\overline{P_1}, \overline{P_2}) \leq \overline{M}(\overline{P_1}, \overline{P_3}) + \overline{M}(\overline{P_3}, \overline{P_2})$ for any fuzzy points $\overline{P_1}, \overline{P_2}$, and $\overline{P_3}$.

2.2. Fuzzy Lines

Definition 4 (Two-point form). Let $\overline{P_1}, \overline{P_2}$ be two fuzzy points in R^n space $(n = 2, 3)$. Define

$$\Omega(\alpha) = \left\{ \begin{array}{c} (x_1, x_2, \ldots, x_n) : \dfrac{x_1 - b_1}{a_1 - b_1} = \dfrac{x_2 - b_2}{a_2 - b_2} = \cdots = \dfrac{x_n - b_n}{a_n - b_n}, \\ (a_1, a_2, \ldots, a_n) \in \overline{P_1}(\alpha), (b_1, b_2, \ldots, b_n) \in \overline{P_2}(\alpha) \end{array} \right\}, \quad 0 \leq \alpha \leq 1. \tag{1}$$

Then the fuzzy line $\overline{L}$ is

$$\mu\left((x_1, \ldots, x_n) \mid \overline{L}\right) = \sup\{\alpha : (x_1, \ldots, x_n) \in \Omega(\alpha)\}. \tag{2}$$

3. Fuzzy Geometry Analysis for Bearings-Only Target Localization in R^2 Space

Based on fuzzy geometry, a detailed analysis of the interplay between the target localization geometry and the fuzzy estimation bias for the case of fuzzy linear observer trajectory in R^2 space is provided, which can realize the 3-dimensional localization including fuzzy estimate coordinate and target velocity by measuring the fuzzy azimuth angles at intervals of fixed time.

3.1. Fuzzy Geometry for Bearings-Only Target Localization in R^2 Space. Fuzzy geometry for bearings-only target localization in R^2 space is discussed. The fuzzy geometric relationship between one-sensor locations $\overline{P_i}$, and the target is shown in Figure 1. Assume a fuzzy linear observer trajectory

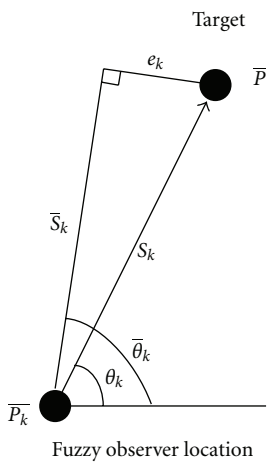

Figure 2: Fuzzy geometric relationship between a sensor and a target.

with three bearing measurements θ_1, θ_2, and θ_3 collected by three sensors at fuzzy observer locations $\overline{P_1}, \overline{P_2}$, and $\overline{P_3}$, respectively. The target localization is denoted as $\overline{P} = [x, y]$. In practical sensor networks, angle measurements are corrupted with some fuzzy factors.

Then define unit function α_k and ζ_k in Figure 1,

$$\alpha_k = \begin{bmatrix} \sin\theta_k \\ -\cos\theta_k \end{bmatrix}, \qquad \zeta_k = \begin{bmatrix} \cos\theta_k \\ \sin\theta_k \end{bmatrix}. \tag{3}$$

Using fuzzy points $\overline{P_i}$ in the plane, define

$$\text{Angle}_k(\alpha) = \left\{\theta_k \mid \theta_k \in \overline{P_i}(\alpha)\right\}, \quad 0 \le \alpha \le 1. \tag{4}$$

Then, the membership function of fuzzy angle $\overline{\Theta_k}$ is

$$\mu\left(\theta_k \mid \overline{\Theta_k}\right) = \sup\left\{\alpha : \theta_k \in \text{Angle}_k(\alpha)\right\}. \tag{5}$$

The target localization is related to the fuzzy observer locations through the fuzzy line equation. For fuzzy points $\overline{P_i}$ in the plane, by Definition 3, define

$$\Omega_{3i}(\alpha) = \left\{(x, y) : \frac{y - v_i}{x - u_i} = \frac{p_y - v_i}{p_x - u_i}, (u_i, v_i) \in \overline{P_i}(\alpha)\right\}, \quad 0 \le \alpha \le 1. \tag{6}$$

Then, the membership function of fuzzy line $\overline{P_iP}$ is

$$\mu\left((x, y) \mid \overline{P_iP}\right) = \sup\{\alpha : (x, y) \in \Omega_{3i}(\alpha)\}. \tag{7}$$

Define

$$\Omega_{3\overline{P_1P_2P_3}}(\alpha) = \left\{\begin{matrix} (x, y) : \dfrac{y - v_1}{x - u_1} = \dfrac{v_2 - v_1}{u_2 - u_1} = \dfrac{v_3 - v_1}{u_3 - u_1}, (u_1, v_1) \in \overline{P_1}(\alpha), \\ (u_2, v_2) \in \overline{P_2}(\alpha), (u_3, v_3) \in \overline{P_3}(\alpha) \end{matrix}\right\} \quad 0 \le \alpha \le 1. \tag{8}$$

Then, the membership function of fuzzy line $\overline{P_1P_2P_3}$ is

$$\mu\left((x, y) \mid \overline{P_1P_2P_3}\right) = \sup\left\{\alpha : (x, y) \in \Omega_{3\overline{P_1P_2P_3}}(\alpha)\right\}. \tag{9}$$

$\overline{P_k}$ fuzzy observer location of the sensor in the plane is illustrated in Figure 2.

It is shown from Figure 2 that the fuzzy error e_k is obtained by

$$e_k = s_k \sin\left(\overline{\theta}_k - \theta_k\right), \tag{10}$$

where s_k is fuzzy distance between $\overline{P_k}$ and $\overline{P}$ which satisfies Definition 2.

Define

$$\Omega(\alpha) = \left\{\begin{matrix} x = \overline{x_1} + d\left(\overline{P_1}, \overline{P_2}\right) \bullet \dfrac{\sin\left(\overline{\theta_2} + \psi\right)}{\sin\left(\left|\overline{\theta_2} - \overline{\theta_1}\right|\right)} \bullet \cos\left(\overline{\theta_1}\right) \\ y = \overline{y_1} + d\left(\overline{P_1}, \overline{P_2}\right) \bullet \dfrac{\sin\left(\overline{\theta_2} + \psi\right)}{\sin\left(\left|\overline{\theta_2} - \overline{\theta_1}\right|\right)} \bullet \sin\left(\overline{\theta_1}\right) \\ \psi = \text{arctg}\left(\dfrac{\overline{y_2} - \overline{y_1}}{\overline{x_2} - \overline{x_1}}\right) \end{matrix} \middle| \left\{\begin{matrix} (\overline{x_1}, \overline{y_1}) \in \overline{P_1}(\alpha), \\ (\overline{x_2}, \overline{y_2}) \in \overline{P_2}(\alpha) \\ \overline{\theta_1} \in \left\{\theta_1 \mid \theta_1 \in \overline{P_1}(\alpha)\right\} \\ \overline{\theta_2} \in \left\{\theta_2 \mid \theta_2 \in \overline{P_2}(\alpha)\right\} \\ 0 \le \alpha \le 1 \end{matrix}\right\}\right\}. \tag{11}$$

Then, the membership function of fuzzy location (x, y) of the target is

$$\mu\left((x, y) \mid \overline{P}\right) = \sup\{\alpha : (x, y) \in \Omega(\alpha)\}, \tag{12}$$

where $d(\overline{P_1}, \overline{P_2})$ is fuzzy distance with fuzzy points $\overline{P_1}$ and $\overline{P_2}$ which satisfies Definition 2. By the formulas (3)–(12), for the case of N sensors deployed along using the fuzzy object programming, we have the fuzzy location of the target as follows:

$$\text{Min} \quad e = \sum_k w_k e_k,$$

$$\text{S.T.} \quad e_k = s_k \sin\left(\overline{\theta_k} - \theta_k\right), \quad k = 1, 2, \ldots, N,$$

$$\Omega_i(\alpha) = \left\{ \begin{array}{c} x = \overline{x_i} + d\left(\overline{P_i}, \overline{P_j}\right) \bullet \dfrac{\sin\left(\overline{\theta_j} + \psi\right)}{\sin\left(\left|\overline{\theta_j} - \overline{\theta_i}\right|\right)} \bullet \cos\left(\overline{\theta_i}\right) \\ y = \overline{y_i} + d\left(\overline{P_i}, \overline{P_j}\right) \bullet \dfrac{\sin\left(\overline{\theta_j} + \psi\right)}{\sin\left(\left|\overline{\theta_j} - \overline{\theta_i}\right|\right)} \bullet \sin\left(\overline{\theta_i}\right) \\ \psi = \operatorname{arctg}\left(\dfrac{\overline{y_j} - \overline{y_i}}{\overline{x_j} - \overline{x_i}}\right) \end{array} \middle| \left\{ \begin{array}{c} \left(\overline{x_i}, \overline{y_i}\right) \in \overline{P_i}(\alpha), \\ \left(\overline{x_j}, \overline{y_j}\right) \in \overline{P_j}(\alpha) \\ \overline{\theta_i} \in \left\{\theta_i \mid \theta_i \in \overline{P_i}(\alpha)\right\} \\ \overline{\theta_j} \in \left\{\theta_j \mid \theta_j \in \overline{P_j}(\alpha)\right\} \\ 0 \le \alpha \le 1 \end{array} \right\} \right\},$$

$$i = 1, 2, \ldots, N-1, \; j = i+1, \; i+2, \ldots, N. \tag{13}$$

Then, the membership function of fuzzy location (x, y) of the target is

$$\mu\left((x, y) \mid \overline{P_i}\right) = \sup\{\alpha : (x, y) \in \Omega_i(\alpha)\}, \tag{14}$$

where, $d(\overline{P_i}, \overline{P_j})$ is fuzzy distance with fuzzy points $\overline{P_i}$ and $\overline{P_j}$ which satisfies on Definition 2. Define

$$\Omega_{3\overline{P_iP_{i+1}P_{i+2}}}(\alpha) = \left\{ \begin{array}{c} (x, y) : \dfrac{y - v_i}{x - u_i} = \dfrac{v_{i+1} - v_i}{u_{i+1} - u_i} = \dfrac{v_{i+2} - v_i}{u_{i+2} - u_i}, \\ (u_i, v_i) \in \overline{P_i}(\alpha), \; i = 1, \ldots, N-2 \end{array} \right\},$$

$$0 \le \alpha \le 1 \tag{15}$$

the membership function of fuzzy line $\overline{P_iP_{i+1}P_{i+2}}$ is

$$\mu\left((x, y) \mid \overline{P_iP_{i+1}P_{i+2}}\right) = \sup\left\{\alpha : (x, y) \in \Omega_{3\overline{P_iP_{i+1}P_{i+2}}}(\alpha)\right\}. \tag{16}$$

Next, the target velocity needs to be determined, which is based on the time-neighboring estimate locations and time intervals. Similarly, there also exists some fuzziness among target locations and time intervals. Let $\overline{P_i}(\alpha), \overline{P_{i+1}}(\alpha)$ be the estimate location at time t_i, t_{i+1}, respectively. The time interval $t_{i+1} - t_i$ is denoted as the fuzzy number $\overline{T_{i+1}} = (a/b/c)$; define

$$\Omega_{v_{i+1}}(\alpha) = \left\{ v_{i+1} = \frac{d(p_i, p_{i+1})}{\varepsilon_{i+1}} \middle| \; p_i \in \overline{P_i}(\alpha), p_{i+1} \in \overline{P_{i+1}}(\alpha), \varepsilon_{i+1} \in \overline{T_{i+1}} \right\}. \tag{17}$$

Then, the fuzzy target velocity $\overline{V_{i+1}}$ at time t_{i+1} is

$$\mu\left(v_{i+1} \mid \overline{V_{i+1}}\right) = \sup\{\alpha : v_{i+1} \in \Omega_{v_{i+1}}(\alpha)\}. \tag{18}$$

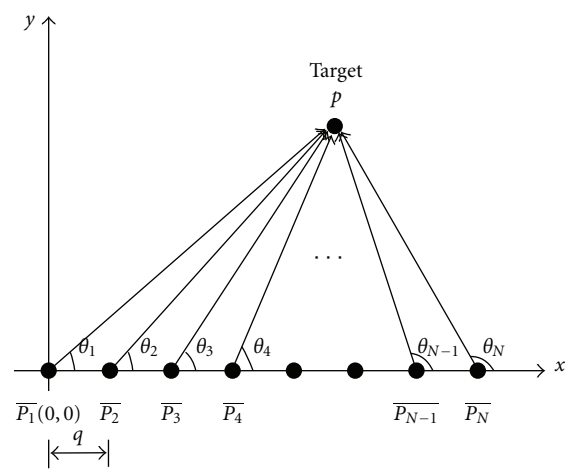

Figure 3: Fuzzy geometry of target localization for the case of fuzzy linear observer trajectory.

Based on the above formulas (13), (14), (17), (18), define

$$\Omega_{i+1}(\alpha) = \left\{ \begin{array}{c} (x_{i+1}, y_{i+1}) \\ v_{i+1} = \dfrac{d((x_i, y_i), (x_{i+1}, y_{i+1}))}{\varepsilon_{i+1}} \end{array} \middle| \begin{array}{c} (x_i, y_i) \in \overline{P_i}(\alpha), \\ (x_{i+1}, y_{i+1}) \in \overline{P_{i+1}}(\alpha), \\ \varepsilon_{i+1} \in \overline{T_{i+1}} \end{array} \right\}. \tag{19}$$

Then, in R^2 space, the 3-dimensional fuzzy estimate $\overline{\wp_{i+1}}$ including the fuzzy target locations and fuzzy velocity at time t_i is

$$\begin{aligned} &\mu\left((x_{i+1}, y_{i+1}, v_{i+1}) \mid \overline{\wp_{i+1}}\right) \\ &\quad = \sup\{\alpha : (x_{i+1}, y_{i+1}, v_{i+1}) \in \Omega_{i+1}(\alpha)\}. \end{aligned} \tag{20}$$

3.2. Fuzzy Geometry Theory for the Case of Fuzzy Linear Observer Trajectory in R^2 Space. Figure 3 shows the fuzzy

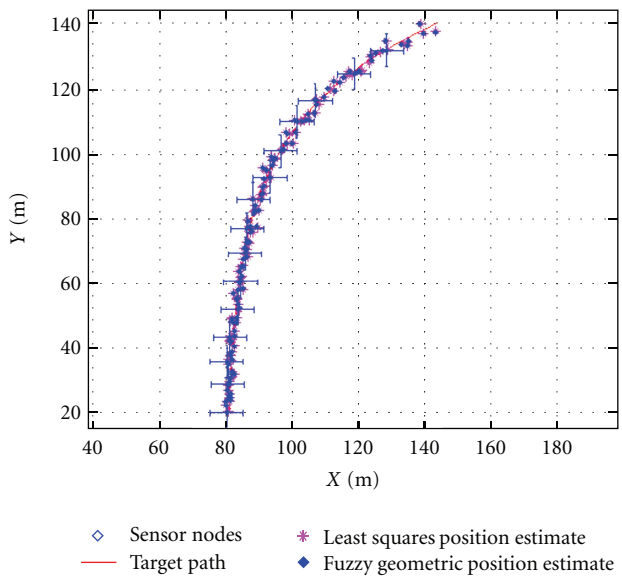

Figure 4: illustration of the moving target trajectory, defuzzified position estimate and some interval values, and least squares position estimate.

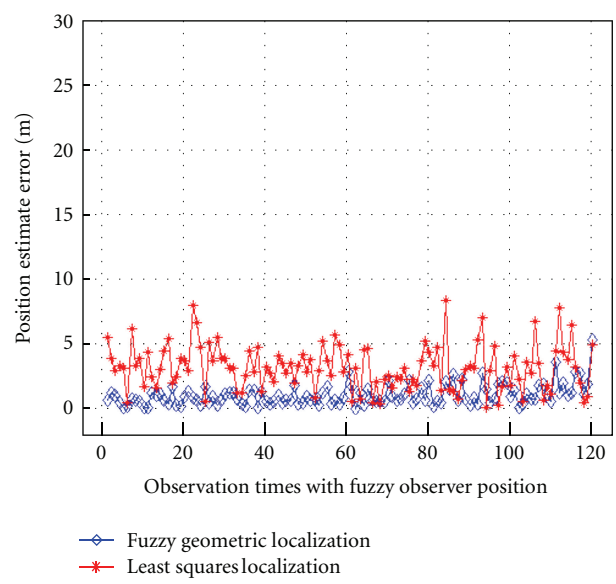

Figure 5: Position estimate error comparison with fuzzy observer positions.

geometry of target localization for the case of fuzzy linear observer trajectory. Let observer location $\overline{P_1}$ be the origin of the fuzzy plane, denoted as $\overline{P_1}(0,0)$.

Theorem 2. *Suppose that the neighboring observers are separated by the fuzzy distance q, then*

(1) *any fuzzy geometry of target localization for the case of fuzzy linear observer trajectory can be equivalently represented by Figure 3, which can be realized with the rotation and translation of absolute coordinates of the target location and observer locations,*

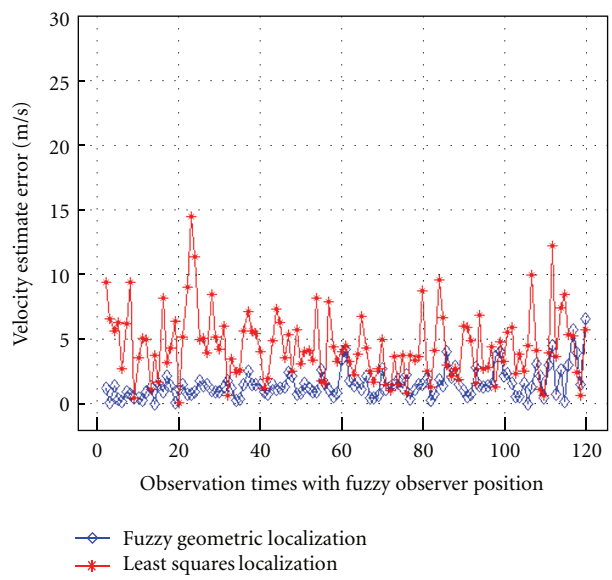

Figure 6: Velocity estimate error comparison with fuzzy observer positions.

(2) *for the fuzzy linear observer trajectory shown in Figure 3, the fuzzy centroid of the observer locations is*

$$c = \left[\frac{1}{2}(N-1)q \quad 0\right]. \tag{21}$$

Proof. Obvious by fuzzy geometry theory. □

4. Experiments Analysis

Three sensor nodes are deployed in R^2 space to implement the 3-dimensional fuzzy estimation including the target position and velocity. Due to the uncertainty in practical sensor networks, sensor observation positions are estimated by $\overline{S_1}(0,0)$, $\overline{S_2}(100,0)$, and $\overline{S_3}(200,0)$ based on fuzzy theory, where the coordinates of sensor $(\overline{X},\overline{Y})$ are a triangle fuzzy number, denoted as $(x-5/x/x+5,\ y-5/y/y+5)$, which determine a fuzzy linear observer trajectory. For the same bearing measurements, let the standard deviation of measurement error be 0.01 rad, time interval $\overline{T}(1) = 1\,\text{s}$. The bearings-only localizations based on fuzzy geometry and least squares methods [10] are analyzed and compared in Figure 4.

The sensor positions are estimated as the arbitrary points which belong to the fuzzy points $\overline{S_1}(0,0)$, $\overline{S_2}(100,0)$, and $\overline{S_3}(200,0)$, respectively. Figure 4 illustrates the moving target trajectory, the defuzzified position estimate of the target using weighted average operator and some interval values, and the position estimate based on least squares method. Figures 5 and 6 compare the position estimate error and velocity estimate error between the two methods. It shows that the precision of fuzzy geometric localization is better than that of least squares localization in uncertain sensor networks. The resulting position estimate outperforms the

traditional least squares approach for bearings-only localization with uncertainty. When the target arrived at the coordinate (112, 120), set $\alpha = 0.6$, then the fuzzy position estimate interval is ([110.8, 114.8], [117.3, 121.3])(0.6). The defuzzified position estimate is (112.8, 119.3), and the defuzzified estimate error is 1.05 m, and the defuzzified estimate velocity is 3.01 m/s. Therefore, the defuzzified 3-dimensional fuzzy estimation is (112.8, 119.3, 3.01). Simulation results validate the rationality and the effectiveness that fuzzy geometry is applied in the bearings-only target localization for two-dimensional sensor networks.

5. Conclusion

A fuzzy geometric localization approach using passive sensor networks in smart grid is proposed based on fuzzy geometry. The fuzzy analytical bias expressions and the constraints are derived considering fuzzy measurements and fuzzy observer positions. The interplay between the target localization geometry and the fuzzy estimation bias is analyzed in detail for the case of fuzzy linear observer trajectory. The experiment results validate that the resulting fuzzy estimate outperforms the traditional least squares approach in a number of respects for localization with uncertainty. Future work will focus on the various kinds of fuzzy observer trajectories and higher-dimensional localization problem in practical sensor networks.

Acknowledgments

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