Hindawi Publishing Corporation
International Journal of Distributed Sensor Networks
Volume 2012, Article ID 135054, 9 pages
doi:10.1155/2012/135054

Research Article

Holes Detection in Anisotropic Sensornets: Topological Methods

Wei Wei,[1] Xiao-Lin Yang,[2] Pei-Yi Shen,[3] and Bin Zhou[4]

[1] *School of Computer Science and Engineering, Xi'an University of Technology, Xi'an 710048, China*
[2] *College of Management Science, Chengdu University of Technology, Chengdu 610059, China*
[3] *National School of Software, Xidian University, Xi'an 710071, China*
[4] *College of Science, Xi'an University of Science and Technology, Xi'an 710054, China*

Correspondence should be addressed to Xiao-Lin Yang, yangxlcdut@gmail.com

Received 11 May 2012; Revised 2 September 2012; Accepted 12 September 2012

Academic Editor: Chuan Foh

Wireless sensor networks (WSNs) are tightly linked with the practical environment in which the sensors are deployed. Sensor positioning is a pivotal part of main location-dependent applications that utilize sensornets. The global topology of the network is important to both sensor network applications and the implementation of networking functionalities. This paper studies the topology discovery with an emphasis on boundary recognition in a sensor network. A large mass of sensor nodes are supposed to scatter in a geometric region, with nearby nodes communicating with each other directly. This paper is thus designed to detect the holes in the topological architecture of sensornets only by connectivity information. Existent edges determination methods hold the high costs as assumptions. Without the help of a large amount of uniformly deployed seed nodes, those schemes fail in anisotropic WSNs with possible holes. To address this issue, we propose a solution, named PPA based on Poincare-Perelman Theorem, to judge whether there are holes in WSNs-monitored areas. Our solution can properly detect holes on the topological surfaces and connect them into meaningful boundary cycles. The judging method has also been rigorously proved to be appropriate for continuous geometric domains as well as discrete domains. Extensive simulations have been shown that the algorithm even enables networks with low density to produce good results.

1. Introduction and Motivation

Sensornets are appearing as promising techniques for pervasive data exchange and information sharing. Sensornets are tightly linked with the geometric environment in which they are deployed. Detecting topological holes is a very important task in wireless sensor networks [1]. In many crucial safe-related scenarios, such as earthwork construction and mine exploitation cases, we need to determine whether topological holes in space topological structure exist, thus, we can send the urgent warning for users so as to prevent the disasters that happen suddenly and have enough time to deal with the accidents in time. Many existing countermeasures usually do strong assumptions. As we know, all of mathematical theorems have their own used field. That is to say, before we use these mathematical methods to solve the practical problems, we need to proof at least explain that these mathmatic ways can be used in the specialized domain. Simultaneously, those current methods either enquire customerized hardware devices or have strong assumptions on the network environment, leading to low efficiency and applicability. In this work, we fundamentally analyze the detecting mind of space holes issue by topology methodology and by observing the inevitable topology deviations introduced by holes. We generalize the definition of space holes in practical scenarios and propose a topological approach. Mathmatical proof and simulation results show that our approach can detect and locate various holes and relies solely on topological information of the network. To the best of our knowledge, we try our best to make the first attempt towards a purely topological approach to detect holes distributedly without any rigorous requirements and assumptions. At the same time, we also solve the applied domain problem of mathmaitical theorem by removing the theoretical barriers to finish it. Our approach achieves superior performance and applicability with the least limitations.

On one hand, sensor network applications for example environment monitoring and data collection demand wealthy coverage over the region of interest. On the other hand, the global topology of a WSN has a great influence on the design of basic networking functionalities, for example, point-to-point routing and data collecting mechanisms. In this paper we study the problem of discovering the global geometry of the sensornets field, especially, inspecting sensor nodes on the boundaries (both inner and outer boundaries). The standpoint we take is to regard the sensornet as a discrete sampling of the underlying geometric environment. This is inspired by the fact that sensornets are to offer dense monitoring of the potential space. Therefore, the shape of the sensor field, that is, the boundaries, indicates significant characters of the underlying environment. These boundaries usually have physical correspondences, such as a building floor plan, a map of a transit network, topography changes, and barriers (skyscrapers, subsidence areas, etc). Holes can also map to events that are being monitored by the sensornet. If we consider the sensors with readings above a threshold to be "invalid", then the hole borders are essentially isocontours of the landscape of the property of interest.

Cases include the identification of regions with overheated sensors or abnormal chemical contamination. Holes are also important indicators of the universal health of a sensornet, for example insufficient coverage and connectivity. The detection of holes divulges groups of destroyed sensors because of physical destruction or power consumption, where additional sensor deployment is demanded. Besides the real scenario mentioned above, understanding the global geometry and topology of the sensor field is of great importance in the design of basic networking operations. For example, in the sensor deployment problem, if we are desirous to spread some mobile sensors in an unknown region formed by static sensor nodes, knowing the border of the region permits us to guarantee that newly added sensors are deployed only in the expected region.

A number of networking protocols also exploit geometric intuitions for simplicity and scalability, for instance geographical greedy forwarding [2, 3]. Such algorithms based on local greedy advances may fail at local minima if the sensor networks have nontrivial topology. Backup methods, for instance face, routing on a explanate subgraph, can assist packets avoid local minima, but build high traffic on hole boundaries, and eventually destroy the network lifetime [2, 3]. This artificial product is not amazing because any algorithm with a strong geometrical application, for example geographical forwarding, ought to stick to the genuine shape of the sensor field. Currently, there are lots of routing schemes that address explicitly the importance of topological properties and propose routing with virtual coordinates that are adaptive to the inner geometric features [4, 5]. The construction of these virtual coordinate systems needs the identification of topological features. We focus on developing a judgment method that detects hole boundaries based on the Poincare Conjecture theory.

The rest of this paper is organized as follows. We first give a brief overview of this scheme in Section 2. And then, we present the PPA design principle in a continuous domain and offer the solid and complete theoretical proof to describe how the traditional and continuous topological theory (Poincare-Perelman Theorem) can be suitably (appropriately) applied to discrete and practical scenarios. As a result, we utilize the Poincare-Perelman Theorem to judge (determine) whether there are existing holes in real topological spaces. Namely, the constructing topological structure of continuous deployment of sensors over the Euclidean plane can also be used to justify whether holes in practical applications exist. We can efficiently detect holes danger and therefore send alert notice in real and safe field applications. In Section 3, we perform the problem formulation and holes detection in discrete environments. Section 4 extends the discussion into the practical discrete context. Section 5 evaluates the proposed scheme through comprehensive simulations and compares it with state of the art-area-based approaches localization schemes. We conclude the work in Section 5.

2. Prior Works

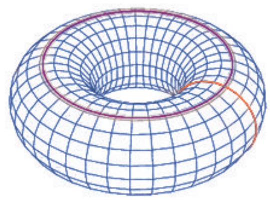
A lot of methods have been presented to judge sensor locations in WSNs. A universal overview of the state-of-the-art localization schemes is available in [6].

Existing researches on edges recognition can be separated into three classifications: geometric, statistical, and topological methods. Geometric methods that were proposed by Fang et al. [1] for boundary detection use geographical location information. This method assumes that the sensor nodes can sense their geographical locations and that the communication graph follows the UDG (Unit Disk Graph) assumption, when two nodes are connected by an edge if and only if their interval is at most 1. The description of holes in [1] is closely interrelated with geographical forwarding so that a packet can only get stuck at a node of hole edges. Fang et al. also presented a simple algorithm that greedily sweeps along hole boundaries and eventually discovers boundary cycles. Statistical methods for boundary detection usually make assumptions about the probability distribution of the sensor deployment. Fekete et al. [7] proposed a border detection algorithm for sensors (uniformly) randomly deployed inside a geometric region. The primary idea is that boundaries nodes have much lower average degrees than nodes in the "interior" of the network. Statistical arguments cause an appropriate degree threshold to differentiate border nodes. An statistical way is to calculate the "restricted stress centrality" of a vertex v, which measures the quantity of shortest paths going through v with a bounded length [7]. Nodes in the interior tend to have a higher centrality than nodes on the boundary. With a sufficient nodes density, the centrality of the nodes holds dual features so that it can be used to detect boundaries. The dominating weak points of these two algorithms are the idealized request on sensor deployment and density: the mean density needs to be 100 at least. In real scenario, the sensors are not as dense and they are unnecessarily arranged uniformly and randomly. There are also topological methods to prime deficient sensor coverage and holes. Ghrist and Muhammad [8] presented an algorithm that detects holes via homology with no knowledge of sensor locations; on the contrary, the algorithm is centralized, with assumptions that both the sensing range and communication range are disks with radii carefully tuned. Kröller et al. [9] presented an algorithm by probing for combinatorial structures called flowers and augmented cycles. They make less restrictive assumptions on the problem setup, modeling the communication graph by a quasi UDG, with nodes p and q demonstrably linked by an edge if $d(p,q) \leq \sqrt{2}/2$ and not connected if $d(p,q) > 1$. The success of this algorithm critically depends on the identification of at least one flower structure, which might not often be the case specially in a sparse network. For a real scenario, Funke [10] developed a simple heuristic with only connectivity information. The essential idea is to build iso contours with hop count from a root node and identify where the contours are broken. Under the unit-disk graph assumption and adequate sensor density, the algorithm outputs nodes marked as border with certain guarantees. Definitely, for each node of the geometry boundary, the algorithm enables to mark a corresponding sensor node within distance 4.8, and each node marked as boundary is within distance 2.8 from the actual geometry boundary [11]. The simplicity of the algorithm is appealing; however, the algorithm only identifies nodes that are near the boundaries but does not show how they are connected in a meaningful way. The density requirement of the algorithm is also rather high; so as to obtain good results, the average degree generally needs to be at least 16.

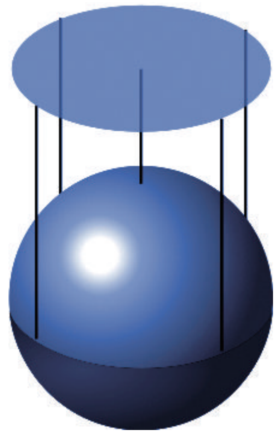
From mathematics aspect, the Poincare Conjecture [12] is a theorem about the specification of the three-dimensional sphere among three-dimensional manifolds. Original conjectured is proposed by Henri Poincare, the claim considers a space that locally resembles ordinary three-dimensional space but is connected, finite in size, and lacks any boundary (a closed three-dimensional manifold). The Poincare Conjecture states that if each loop in such a space can be continuously tightened to a point, then it must be a three-dimensional sphere. An similar result has been proved in higher dimensions. (Some related content is partially referred to the Wikipedia information).

(a) A loop can be contracted to a point without leaving the sphere surface

(b) Gradual Construction of a torus based on a rectangle



(c) Any chromatic loops cannot be contracted to a point without leaving the torus surface



(d) Hemispheres mapped to a Whole Sphere

Figure 1: Diagram of manifold.

2.1. Definitions of Manifold. A manifold is a space made by conglutinating together pieces of Euclidean space, which is called charts. For example you could take two-dimensional disks and bend them around two hemispheres and then stick them together to form a two-dimensional sphere. (See also in Figure 1(d)).

A torus (the surface of a donut) can be established utilizing a rectangular diagram as shown in this image. The colored parallelograms explain how a pattern on the associated surface would arise in case the edges were once again disconnected. (See also in Figures 1(b) and 1(c)). A pair of solid balls can made a three-dimensional sphere. It should be required to discern every point of the first ball boundary with the corresponding point of the second one. Other kinds of manifolds can be established by the mimetic ways.

2.2. Explanations of Homeomorphic. Generally, two shapes are homeomorphic if one of these shapes can be transformed into the other without pause or discontinuity. *A* homeomorphism is a function of continuous domain that maps points of one object into another.

Two spaces are regarded as homeomorphic if a homeomorphism between them exists. Such as, a two-dimensional sphere is homeomorphic to the surface of a cube; homoplastically, a three-dimensional sphere is homeomorphic to the three-dimensional boundary of a four-dimensional hypercube. In the above-mentioned primary concepts, they can aid to comprehend the specification of the Poincare-Perelman Theorem: the Poincare-Perelman Theorem says that a three-dimensional manifold which is compact, has no boundary and is simply connected must be homeomorphic to a three dimensional-sphere. The original phrasing was as follows: consider a compact three dimensional manifold V without boundary. Is it possible that the fundamental group of V could be trivial, even though V is not homeomorphic to the three-dimensional sphere? This centurial challenging problem is proofed by Grigori Perelman [12] in 2006.

Here is the standard form of the conjecture: every simply connected, closed tridimensional manifold is homeomorphic to the triaxial sphere. Therefore, a centralized method of collecting all of the information to a central server is not feasible for large sensor networks.

2.3. Preliminaries. Materials for topology theory. The following definitions in topology theory can be found in [13].

Topology. Let X be a set. $\mathcal{T} \subseteq 2^X$ is called a topology on X if (1) $\emptyset, X \in \mathcal{T}$; (2) if $A, B \in \mathcal{I}$, then $A \cap B \in \mathcal{T}$; (3) if $\{A_i \mid i \in I\} \subseteq \mathcal{T}$, then $\bigcup_i A_i \in \mathcal{T}$. The pair $(X, \mathcal{T})$ is called a topological space. The members of $\mathcal{T}$ are called open sets. If Y is a subset of X, then $\mathcal{T} \mid Y = \{U \cap Y \mid U \in \mathcal{T}\}$ is a topology on Y and called the induced topology of $(X, \mathcal{T})$. A bijection $f : (X_1, \mathcal{T}_1) \to (X_2, \mathcal{T}_2)$ between two topological spaces is called a homeomorphism if $B \in \mathcal{T}_2$ iff $f^{-1}(B) \in \mathcal{T}_1$ for any $B \subseteq X_2$. In this case, $(X_1, \mathcal{T}_1)$ and $(X_2, \mathcal{T}_2)$ are said to be homeomorphic to each other.

Dense Set. Let $(X, \mathcal{T})$ be a topological space and $C \subseteq X$. A point $x \in X$ is called a cluster of C if $U \cap C \neq \emptyset$ any $U \in \mathcal{T}$ with $x \in U$. Denote C^- as the set of all cluster of C, called the closure of C. The set C is called a closed set if $C = C^-$. A set is called a clopen set if it is simultaneously open and closed. A set C is called dense of $(X, \mathcal{T})$ if $C^- = X$. Dense set is an important and useful concept in topology. For example, every continuous map from a dense set of a topological space to another topological space can be extended onto the whole topological space. Thus dense sets in a topological space may share some same topological properties as the whole topological space, for example the connectedness as Theorem 1 shows.

Partition. The specification of partition £ for a set X, a family of subsets $\{X_i \mid i \in I\}$ is called a partition of X, if $\bigcup_i X_i = X$ and $X_i \cap X_j = \emptyset$ for all $i, j \in I$ with $i \neq j$.

2.4. Our Contributions. We develop a practical and efficient determination solution for boundary detection in sensor networks, using only the communication graph and not making unrealistic assumptions. We do not assume any location information, angular information, or distance information. More importantly, we do not request that the communication graph obeys the unit disk graph model or the quasi-unit disk graph model. Actual communication ranges are not circular disks and are often quite irregularly shaped [14]. Algorithms that depend on the unit disk graph model fail in practice (e.g., the extraction of a planar subgraph by the relative neighborhood graph or Gabriel graph [15].

Our PPA method also readily provides other topological and geometric information, such as the number of holes (genus), the nearest hole to any given sensor, and the sensor field's medial axis (the collection of nodes with at least two closest boundary nodes), which is useful for virtual coordinate systems for load-balanced routing [4]. Simulation results show that our algorithm correctly determines useful borders for sensor networks with rational node density (average degree 10 and above) and distribution (e.g., uniform). The algorithm also works well for nonuniform distributions. The algorithm is efficient. The entire procedure involves only three network flooding procedures and greedy shrinkage of paths or cycles. Further, as a theoretical ensure, we prove that for a continuous geometric space bounded by polygonal obstacles, the case in which node density approaches infinity, the algorithm correctly discovers all of the boundaries. More definitely, we investigate the fact that a legitimate multihop sensor network deployed on the surface of a geometric terrain, (even possibly including irregular boundaries, inner obstacles, or even on a non-2D plain) PPA solution is able to accurately estimate the node-to-node distances and calculate node locations with only 3 seeds, thus increasing system scalability and usage as well as lowering hardware costs. In addition, PPA does not presume the superior communication capability of seeds, that is, with much larger radio range than those of the ordinary nodes [16].

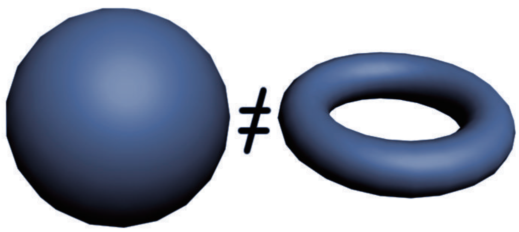
Due to all mentioned above assumptions based on UDG graph model and its basis on the symptom of packing number, it is thus inaccurate under non-UDG graphs. Indeed, there are still no perfect symptoms found to establish an all-round method in the resource-limited sensornets. Our design is originated from the perspective of topological observation and is based on the theory of Poincare Conjecture, our solution is orthogonal to existing approaches and takes a step towards relaxing these assumptions and expanding the applicability of methods.

3. Problem Formulation and Holes Detection

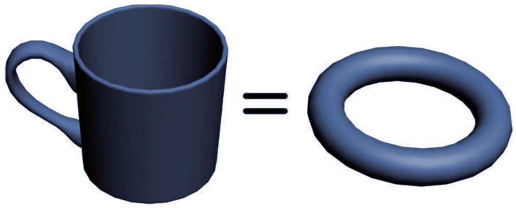
The definition has been given under the constraints of the UDG communication graph model, which has been proven far from practical in many analytical and experimental works. Second, the distance-based definition in Euclidean space naturally binds the hole features with external geometric environments and thus neglects the inherent topological impacts introduced by holes. We hereby present a more general and fundamental definition of the hole based only on network topologies and aim to present the inherent characteristics of holes. According to the Poincare-Perelman Theorem, in the three dimensions space, the donut topology is homeomorphous to the coffee cup topology (see also Figure 2(b)). As shown in Figure 2(a), since these two topologies are not equivalent (namely, not homeomorphism), we can determine that the holes in the monitored areas based on the Poincare-Perelman Theorem exist (see also Figures 2(a) and 2(b)). Since these two topologies are not equivalent (namely, not homeomorphism), we can determine that the holes in the monitored areas based on the Poincare-Perelman Theorem exist.

In real scenario, we will treat the multihole condition. But in this proposed solution, we currently do not differentiate the numbers of holes. In future work, we will discuss and deal with this condition.

Owing to constructing the network topological structure of monitored areas, in given the surface S, we first select an arbitrary point in S as the root and run a continuous Dijkstra shortest path algorithm [17, 18] to construct the topology structure (manifold) of monitored areas. As shown in Figure 3(a) and Figure 3. Consequently, we can determine whether any closed and simply connected manifold is



(a) Topology inequivalent



(b) Topology inequivalent

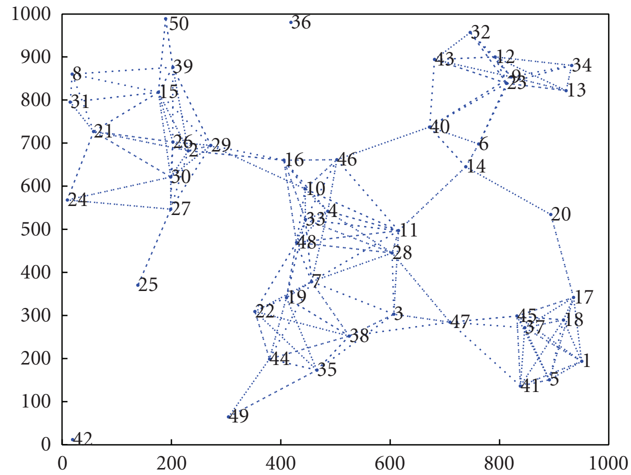
Figure 2: Diagram of manifold.

homeomorphic to the three dimensions sphere. If it is not homeomorphic to the three dimensions sphere. It refers to there is/are hole(s) in the monitored areas. (See also in Figure 3(b)).

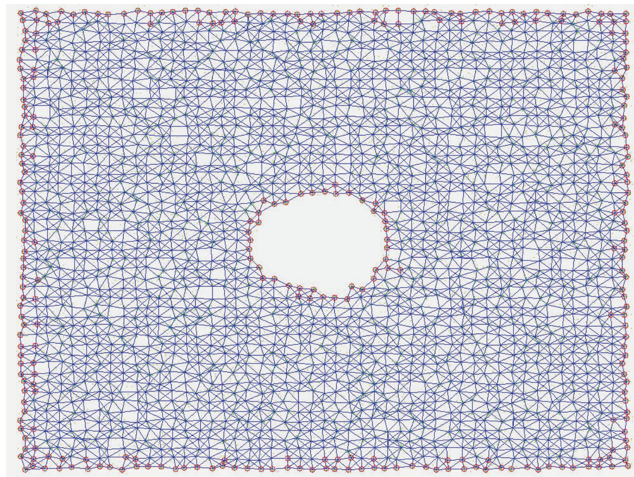
3.1. Holes Detection in Discrete Environments. We have characterized the impact of holes and described the principles of holes detection under continuous settings in the previous section. In a real multihop network, however, nodes are deployed discretely on the field. In this section, we focus on solving the mapping question from the discrete domain to continuous geometric domain. Since the theory of Poincare-Perelman Theorem belongs to the judgement of continuous geometric domain, we need to proof the correctness and applicability of this topological judgement. By means of the following solid proof, we can transfer the discrete topological space to the continuous geometric domains. Namely, we utilize the partial discrete topology structure to substitute the whole continuous geometric topology. As a result, we can apply the theory of Poincare-Perelman Theorem to judge the existence of holes in the monitored areas by WSNs.

Let $\mathcal{T}$ be a topology on a set X and $Y \subseteq X$. Then $\mathcal{T} \mid Y = \{U \cap Y \mid U \in \mathcal{T}\}$ is a topology on Y, called the induced topology on Y and in this case $(Y, \mathcal{T} \mid Y)$ is always called a subspace of $(X, \mathcal{T})$ and $\mathcal{T}$ an extension of $\mathcal{T} \mid Y$ from Y to X. We confirm that these two topological spaces $(X, \mathcal{T})$ and $(Y, \mathcal{T} \mid Y)$ have the same topological properties. In a topological space X, a subset U is called dense if $U^- = X$, where U^- is the closure of U in $(X, \mathcal{T})$. A topological space $(X, \mathcal{T})$ is called connected if there exists no clopen (simultaneously closed and open) subset except empty and whole set X.

Theorem 1. *Suppose that $(Y, \mathcal{T}_1)$ is a dense subspace $(X, \mathcal{T}_2)$, then $(Y, \mathcal{T}_1)$ is connected if and only if $(X, \mathcal{T}_2)$ is connected.*



(a) Dijkstra process



(b) Hole detection process

Figure 3: Diagram of hole boundary construction.

Proof. Suppose that $(Y, \mathcal{T}_1)$ is connected. If $(X, \mathcal{T}_2)$ is not connected, then there exists a clopen set U in $(X, \mathcal{T}_2)$ and $U \notin \{X, \varnothing\}$. Put $V = U \cap Y$, we have that V is a clopen set in $(Y, \mathcal{T}_1)$, which implies that $V = Y$ and $Y \subseteq U$ (obviously, $V \neq \varnothing$). Since Y is dense in $(X, \mathcal{T}_1)$, we have $X = Y^- \subseteq U^- = U$, which contradicts to $U \neq X$. Conversely, suppose that $(X, \mathcal{T}_2)$ is connected. If $(Y, \mathcal{T}_1)$ is not connected, then it contains a clopen set V which is neither Y nor $\varnothing$. For this V, there exist a clopen set U in $(X, \mathcal{T}_2)$ such that $V = U \cap Y$. In order to induce a contradiction, we only need to show that $U \neq X$. If $U = X$, then $V = U \cap Y = Y$, which is another contradiction to $V \neq Y$. The proof is complete. □

Remark 2. If a topological space $(X, \mathcal{T})$ is n-connected topological space $(n \geq 2)$, then $(X, \mathcal{T})$ can exactly be separated into $n-1$ connected subspace $\{(X_i, \mathcal{T} \mid X_i) \mid i = 1, 2, \ldots, n-1\}$ such that $\{X_i \mid i = 1, 2, \ldots, n-1\}$ is a partition of X. By Theorem 1, if $(Y, \mathcal{T}_1)$ is a dense subspace $(X, \mathcal{T}_2)$, then $(Y, \mathcal{T}_1)$ is n-connected if and only if $(X, \mathcal{T}_2)$ is n-connected. As mentioned in the above Remark, the definition of concept partition for a set X, a family of subsets $\{X_i \mid i \in I\}$ is called a partition of X, if $\bigcup_i X_i = X$ and $X_i \cap X_j = \emptyset$ for all $i, j \in I$ with $i \neq j$.

The following verdict is usually held for any set that satisfies the requirement of the theory. Whatever the set is finite or infinite. We assume that a certain area deployed the WSNs. This area can be considered as a smooth curve equipped with the traditional Euclidean topology, the set of all sensors equipped with their own topology can be considered as a subspace of the former one. Furthermore, we assume that the set of all sensors is dense in this area.

We assume that the whole sensor nodes set, which completely cover the monitored area, constructs a dense set. A dense set is the monitored area which is abundantly and completely covered by the large quantities of sensor nodes. Therefore, the network topology can be continuously expanded to the monitored area. Specially, some part of sensornets can be destroyed by some accidents so that it will lead to form a hole in the architecture of topology. As a result, there exists a hole in the corresponding practical area. The sensornets corresponding geometric structure is a universal Euclid topology, particularly, if a hole in this monitoring area exists. If and only if the topology of sensornets is sub dense space of area topology space. Furthermore, If and only if the geometric topology of monitored area is connected completely, consequently, the constructing topology of sensornets is interconnected. Simultaneously, if and only if there exists holes in the geometric topology of monitored area, as a result, there exists holes in the constructing topology of sensornets.

Thus detecting whether there are holes existing in sensornets topology is equivalent to detecting whether there are holes in the monitored area.

Steps. Symbolic Interpretation. Area S, $\mathcal{T}$ is the Euclidean topology of S. The set C denotes the sensornets while C_1 denotes the efficient sensornets. Precondition: set C is dense in $(S, \mathcal{T})$.

(1) Let S_1 be the closure of C_1 on $(S, \mathcal{T})$. If it exists holes, then $S1 \subseteq S$. (2) Obtaining $\mathcal{T}_1$ while the topology $\mathcal{T}$ of S is constrained in the S_1. Therefore, $(S_1, \mathcal{T}_1)$ is a subspace of $(S, \mathcal{T})$. (3) In the above mentioned, S_1 is continuous set. Consequently, we can depend on the Poincare Conjecture theory to determine whether there are holes that existed in monitoring area. If there are holes in the topology structure of $(S_1, \mathcal{T}_1)$, then there are holes in the topology structure of C. The above-mentioned theory can guarantee this determination.

3.2. Topological Boundary Recognition. Suppose a large number of sensor nodes are scattered in a geometric region with nearby nodes communicating with each other directly. Our goal is to discover the nodes on the boundary of the sensor field, using only local connectivity information. We propose a solution that identifies boundary cycles for the sensor field. For compact 2-dimensional surfaces without boundary, if every loop can be continuously tightened to a point, then the surface is topologically homeomorphic to a 2 spheres, usually just called a sphere. The Poincare Conjecture asserts that the same is true for 3-dimensional surfaces. (See also in Figure 1(a)). Practically, for obtaining the topology of monitored areas, we firstly use the Dijkstra Shortest Path algorithm [17] to construct the topology (manifold) of monitored areas. Consequently, we can determine whether any closed and simply connected manifold is homeomorphic to the three dimensions sphere. If it is not homeomorphic to the three dimensions sphere, it refers to there is/are hole(s) in the monitored areas (see also Figure 3(b)).

In the following, we first outline the Dijkstra Shortest Path algorithm and then explain each step in detail.

Algorithm allows the node at which we are starting to be called the initial node. Let the distance of node Y be the distance from the initial node to Y. Dijkstra's algorithm that allocates some initial distance values and will try to increase them step-by-step. Assign to every node a distance value. Set it to zero for our initial sensor node and to infinity for all other nodes. Mark all nodes as unvisited. Set initial sensor node as current. For current node, consider all its unvisited neighbors and calculate their distance (from the initial node). For instance, if current node (A) has distance of 6, and an edge connecting it with another node (B) is 2, the distance to B through A will be $6 + 2 = 8$. If this distance is less than the previously recorded distance (infinity in the beginning, zero for the initial node), overwrite the distance. When we are done considering all neighbors of the current node, mark it as visited. A visited node will not be checked ever again; its distance recorded now is final and minimal. If all nodes have been visited, finish. Otherwise, set the unvisited node with the smallest distance (from the initial node) as the next "current node". Suppose you want to find the shortest path between two intersections on a map, a starting point and a destination. To accomplish this, you could highlight the streets (tracing the streets with a marker) in a certain order, until you have a route highlighted from the starting point to the destination. The order is conceptually simple: at each iteration, create a set of intersections consisting of every unmarked intersection that is directly connected to a marked intersection, and this will be your set of considered intersections. From that set of considered intersections, find the closest intersection to the destination (this is the "greedy" part, as described above) and highlight it and mark that street to that intersection, draw an arrow with the direction, then repeat. In each stage mark just one new intersection. As getting to the destination, follow the arrows backwards. There will be only one path back against the arrows, the shortest one. The basic idea is to detect the existence of holes by judging whether if the existing topology is equivalent to sphere in the three-dimension space. Based on the mentioned above, we can construct a topology of monitored areas. Intuitively, it is very hard to determine the existence of holes by the two-topology structure. We assume our method can obtain the whole monitored topology, and then we can compare this obtained topology with sphere topology. Finally, we can determine whether holes in the monitored area exist.

4. Simulations

We performed extensive simulations in various scenarios, with the goal to evaluate the performance of the algorithm with respect to the network topology, node density and distribution, so on. We particularly note that our method works well even in cases of very low average degree, such as less than 10, or even as low as 10 in some models. Its ability is also similar to average degree 20 condition. Degree 6 has been shown to be optimal for mobile networks [19]. For each figure in this part, we assume a root node in the upper left corner and middle to illustrate the communication range of the sensor field.

4.1. Random Distribution of Sensors. In this experiment, we first assume that the network connectivity and link quality are good enough. In terms of a uniform distribution, we randomly deploy 1600 nodes in a square region with one hole. The average degree of the graph is discriminated by regulating the communication radius. As expected, Figures 4(a) and 4(b) show the results of our method. We can efficiently judge the hole existing in the monitored area. Connectivity is necessary for computing the shortest path tree. Practically, this low-degree graph with insufficient connectivity is the major troubling issue for prior boundary detection methods. Since our method only requires the communication graph, we can use several simple policies to raise artificially the average degree. For a disconnected network, we use the largest connected component of the graph to build our shortest path tree. Then we artificially enlarge the communication radius by taking two/three hops neighbors as fake one-hop neighbors. According to this means, the connectivity of the graph will be made better, and the results will be improved correspondingly by this simple strategy. The result using three hops neighbors has fewer incorrectly marked extremal nodes, and the final judgement is in good result except that the boundary cycle is not very tense. This is understandable since we make the communication range artificially larger, so that more nodes could be equivalently to distribute on the boundary now. Therefore, based on our solution, we can efficiently find holes in the supervised area.

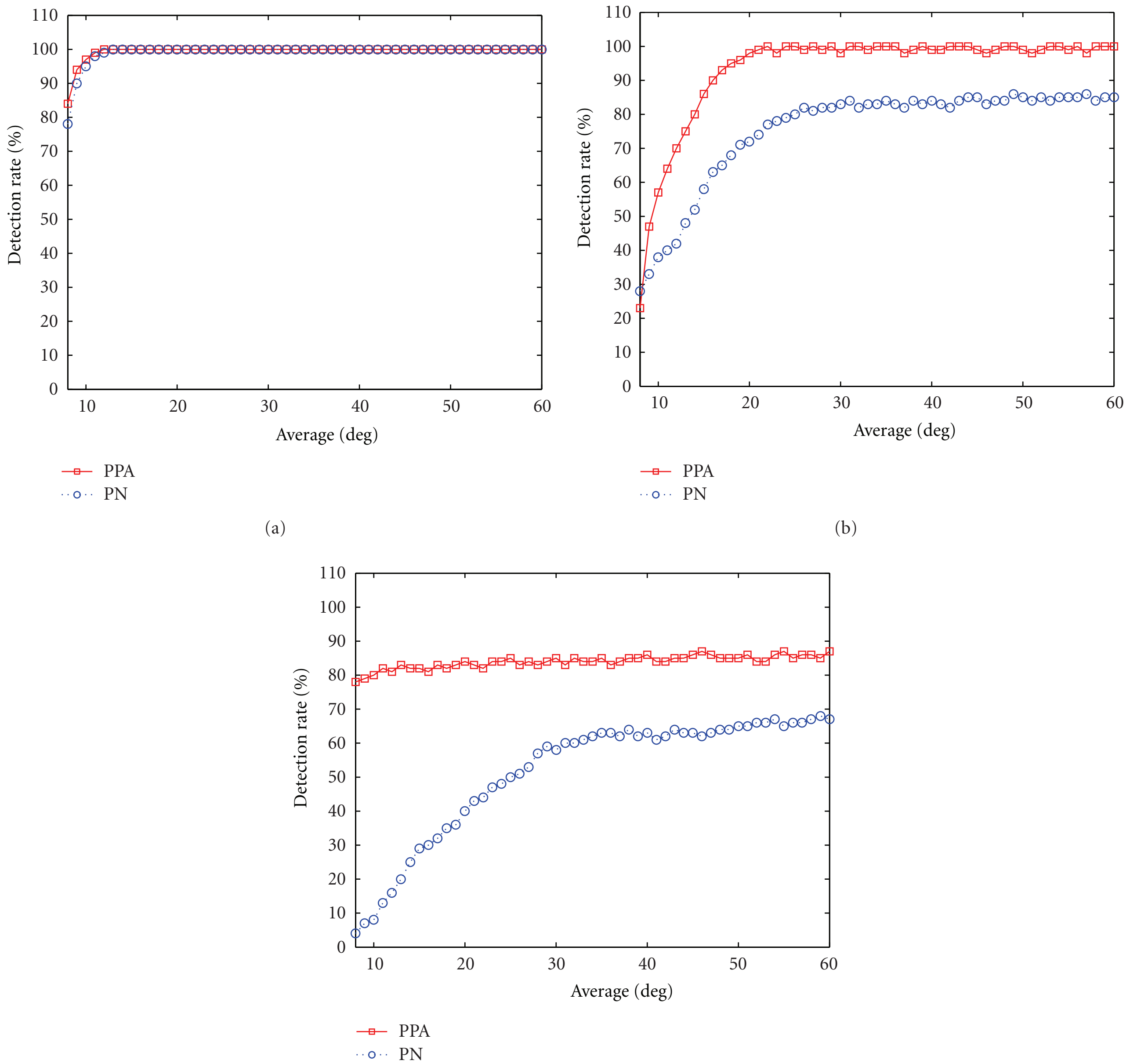
4.2. Grid with Random Perturbation. In this simulation, we put about 1600 nodes on a grid and then perturbed each point by a random shift. Especially, for each original grid node we create two random numbers modulo the length and the width of each block of the grid and use these two small numbers to perturb the positions of the nodes. This distribution may be a good approximation of manual deployments of sensors; it also gives an alternative means of modeling "uniform" distributions, while avoiding clusters and holes that can arise from the usual continuous uniform distribution or Poisson process. As the theoretic verification considering, our method generates very good results, while average degree of graphs is ten or more.

4.3. Low Density, Sparse Graphs. In the experiments, we spread sensor nodes in a square region with one hole. In order to guarantee good connectivity, the nodes are distributed on a randomly perturbed grid. Our experiments show that if we amend the communication radius and decrease the density of nodes, our solution is performed very well, even for low density or sparse condition, as long as the average degree is at about ten or more. See also Figure 4(c).

5. Conclusion and Discussion

We devote our most efforts to explore the application of Poincare Conjecture to resolve the holes detection of safety-monitored areas in WSNs. Based on the theoretic specification, we can judge whether there are holes in the detected area. Because the detected network topology is not homeomorphous to the three dimensions sphere, it can be confirmed to have holes in the detected topology architecture. Therefore, we can accomplish the detecting holes purposes. The proposed new detection solution enables us to find holes in the continuous case, in discrete sensors networks several implementation issues arise. First, even for a given homotopy type, there needs not be a unique shortest path between two nodes. Thus, the boundary topology discovered by our solution, as shown in the simulations, may not tightly surround the real boundaries. Currently, we have two approaches to improve it. One is to make use of the fact that the nodes with lower degree are more likely to be on the boundary; thus, we implemented a preferential scheme for low-degree nodes when computing shortest paths. Another approach is to use an iterative method to find more extremal nodes and then refine the topology; this can also help to address the issue that several extremal points may have the same positions because we use hop counts to approximate true distances. Second, deciding the correct orderings of the extremal nodes requires some care. In the continuous case, extremal nodes project to their nearest node. In the discrete case, since we employ hop counts to approximate the true distance, it is possible that different extremal points are mapped to the same position on the inner boundary, obscuring their ordering. Again, by using an iterative procedure, we delete all the extremal nodes with duplicate positions except one and then iteratively find more extremal points and refine the boundary gradually. In real scenarios, the sensor nodes often know some partial location information or relative angular information. Such positional information can help to improve the performance of our holes detection solution, for example, when we utilize the shortest path algorithm to construct the topology of monitored areas. If the nodes have knowledge of a general arctic direction, it is easier to distinguish the extremal nodes in the interior and exterior of rough boundary. Also, if we have estimated distance or other rough localization information, other than pure hop count, the procedure to find shortest paths will become more reliable. Finally, our method discussed until now assumes a sensor field with holes. We remark that the case with no holes can be solved as well.

Finally, our method discussed until now assumes a sensor field with holes. We remark that the case with no holes can be solved as well. If a network topology is equivalent to



(a)

(b)

(c)

Figure 4: Percent Detected against various node density.

(homeomorphous) the three dimensions sphere, then it have no holes on the monitored areas based on the proofed of Poincare Conjecture, vice versa.

6. Conclusion

In this paper, a novel CT reconstruction model is proposed based on the approximate inverse where the kernel of the FDK method is derived and is used to complete the reconstruction. In order to eliminate the imposed ring artifacts, the kernel is truncated with proper radius. Reconstruction results show that the compact support FDK kernel reconstruction model can suppress the ring artifacts. The proposed reconstruction model preserves the simplicity of the FDK reconstruction method and also provides an alternative to realize the approximate inverse method for circular trajectory. And when the kernel of an algorithm is modified, the corresponding reconstruction formula is also modified accordingly. And this give us another way to improve the existing reconstruction methods.

Acknowledgments

The authors would like to thank the anonymous reviewers for their constructive feedback and valuable input. Thanks are due for the supports to our program from the TI, the

XILINX, and the Software School of Xidian University. This program is partially supported by NSFC (Grant no. 61072105, 61007011) and also supported by the Open Projects Program of National Laboratory of Pattern Recognition. The project is also partially supported by Natural Science Basic Research Plan in Shaanxi Province of China (Program no. 2010JM8005) and Scientific Research Program Funded by Shaanxi Provincial Education Department (Program no. 11JK0504).

References

[1] Q. Fang, J. Gao, and L. J. Guibas, "Locating and bypassing holes in sensor networks," *Mobile Networks and Applications*, vol. 11, no. 2, pp. 187–200, 2006.

[2] P. Bose, P. Morin, I. Stojmenović, and J. Urrutia, "Routing with guaranteed delivery in ad hoc wireless networks," *Wireless Networks*, vol. 7, no. 6, pp. 609–616, 2001.

[3] B. Karp and H. T. Kung, "GPSR: greedy Perimeter Stateless Routing for wireless networks," in *Proceedings of the 6th Annual International Conference on Mobile Computing and Networking (MOBICOM '00)*, pp. 243–254, August 2000.

[4] J. Bruck, J. Gao, and A. Jiang, "MAP: medial axis based geometric routing in sensor networks," in *Proceedings of the 11th Annual International Conference on Mobile Computing and Networking (MobiCom '05)*, pp. 88–102, September 2005.

[5] Q. Fang, J. Gao, L. J. Guibas, V. De Silva, and L. Zhang, "GLIDER: Gradient Landmark-Based Distributed Routing for sensor networks," in *Proceedings of the 24th Conference of the IEEE Communication Society (INFOCOM '05)*, pp. 339–350, March 2005.

[6] J. Hightower and G. Borriello, "Location systems for ubiquitous computing," *Computer*, vol. 34, no. 8, pp. 57–66, 2001.

[7] S. P. Fekete, A. Kröller, D. Pfisterer, S. Fischer, and C. Buschmann, "Neighborhood-based topology recognition in sensor networks," *Lecture Notes in Computer Science*, vol. 3121, pp. 123–136, 2004.

[8] R. Ghrist and A. Muhammad, "Coverage and hole-detection in sensor networks via homology," in *Proceedings of the 4th International Symposium on Information Processing in Sensor Networks (IPSN '05)*, pp. 254–260, April 2005.

[9] A. Kröller, S. P. Fekete, D. Pfisterer, and S. Fischer, "Deterministic boundary recognition and topology extraction for large sensor networks," in *Proceedings of the 17th Annual ACM-SIAM Symposium on Discrete Algorithms*, pp. 1000–1009, January 2006.

[10] S. Funke, "Topological hole detection in wireless sensor networks and its applications," in *Proceedings of the Joint Workshop on Foundations of Mobile Computing*, pp. 44–53, 2005.

[11] S. Funke and C. Klein, "Hole detection or: 'how much geometry hides in connectivity?'," in *Proceedings of the 22nd Annual Symposium on Computational Geometry (SCG '06)*, pp. 377–385, June 2006.

[12] Poincare and Jules Henri, *The American Heritage Dictionary of the English Language*, Houghton Mifflin Company, Boston, Mass, USA, 4th edition, 2000.

[13] J. L. Kelly, *General Topology*, D. Van Nostrand, Princeton, NJ, USA, 1955.

[14] D. Ganesan, B. Krishnamachari, A. Woo, D. Culler, D. Estrin, and S. Wicker, "Complex behavior at scale: an experimental study of low-power wireless sensor networks," Tech. Rep. UCLA/CSD-TR 02-0013, UCLA, 2002.

[15] Y. J. Kim, R. Govindan, B. Karp, and S. Shenker, "Geographic routing made practical," in *Proceedings of the 2nd USENIX/ACM Symposium Networked System Design and Implementation*, pp. 217–230, 2005.

[16] T. He, C. Huang, B. M. Blum, J. A. Stankovic, and T. Abdelzaher, "Range-Free Localization Schemes for Large Scale Sensor Networks," in *Proceedings of the 9th Annual International Conference on Mobile Computing and Networking (MobiCom '03)*, pp. 81–95, September 2003.

[17] E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numerische Mathematik*, vol. 1, no. 1, pp. 269–271, 1959.

[18] F. Li, J. Luo, C. Zhang, S. Xin, and Y. He, "UNFOLD: uniform fast on-line boundary detection for dynamic 3D wireless sensor networks," in *Proceedings of the 12th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc'11)*, pp. 141–152, 2011.

[19] E. M. Royer, P. M. Melliar-Smith, and L. E. Moser, "An analysis of the optimum node density for ad hoc mobile networks," in *International Conference on Communications (ICC '01)*, pp. 857–861, June 2000.