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Research Article

A Minimax Unbiased Estimation Fusion in Distributed Multisensor Localization and Tracking

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A minimax estimation fusion in distributed multisensor systems is proposed, which aims to minimize the worst-case squared estimation error when the cross-covariances between local sensors are unknown and the normalized estimation errors of local sensors are norm bounded. The proposed estimation fusion is called as the Chebyshev fusion estimation (CFE) because its geometrical interpretation is in coincidence with the Chebyshev center, which is a nonlinear combination of local estimates. Theoretically, the CFE is better than any local estimator in the sense of the worst-case squared estimation error and is robust to the choice of the supporting bound. The simulation results illustrate that the proposed CFE is a robust fusion in localization and tracking and more accurate than the previous covariance intersection (CI) method.

1. Introduction

Multi-sensor networks have received an increasing attention in recent years, due to their huge potential in applications, such as communication, signal process, routing and sensor management, and many other areas. In this paper, we focus on a specific and simple estimation fusion model in a distributed multi-sensor system, which is in fact a two-level optimization in the estimation fusion. Every sensor first optimally estimates the state of target based on its own measurements and then transmits its estimate to the fusion center. The problem of estimation fusion is to find an optimal state estimator based on all the received local estimates. Although the centralized fusion which directly makes use of all measurements from the local sensors in time is theoretically the best fusion strategy, sometimes communication or reliability constraints make it impossible to transmit all the sensor measurements to a fusion center. In contrast, the distributed fusion which only needs to fuse all received local estimates has the advantages of lower communication requirements, improved robustness, and so forth.

However, the fusion algorithms in distributed system have to deal with troubles that do not exist in centralized fusion. One of the difficulties is that the errors of local estimates to be fused are generally correlated, and as a result the distributed fusion cannot be achieved by a standard centralized algorithm such as the Kalman filter. The reasons of this correlation may be a common process noise in target when the state estimates are not fused at each sampling instant, or common prior information in the estimates from previous communication.

Over the last two decades, much research has been performed on distributed fusion [1–6]. Some approaches are looking for the "optimal" linear combination of local estimates in some criteria, such as weighted least squares or minimum variance [1, 2]. In [7], the authors proposed a new multi-sensor optimal information fusion criterion which is weighted by matrices in the linear minimum variance sense. An optimal Kalman filtering fusion with cross-correlated sensor noises is proposed in [8], which assumes that the correlation of sensor noises is accurately known. A unified model for estimation fusion based on the best linear unbiased estimation (BLUE) is proposed in [9]. However, all of the aforementioned methods rely on two assumptions: one is that the local estimates are unbiased and the other is that the error covariance matrix of all local estimates is known.

There are other approaches attempting to reconstruct the optimal centralized estimate from the local estimates. A random weighting estimation method for fusion of multidimensional position data is proposed in [10]. The method in [5, 6, 11] deduces to a linear combination of local estimates, but is not particularly effective in handling the correlation in measurement noises. In the seminal papers [4, 6, 12, 13], the covariance intersection (CI) algorithm was proposed to deal with this problem. It fuses without assuming any knowledge on the correlation between the local estimation errors. A robust estimation fusion is proposed in [14], which assumes that the correlation between the local estimation errors is not accurately known but belongs to an uncertain set. However, it is also a linear combination of local estimates as the other aforementioned methods. Theoretically, the linear combination may not be an accurate formation of the distributed fusion. Recently, a nonlinear estimation fusion is proposed in [15], where it minimizes the estimation error covariance only for the most favorable realizations of the random matrix and models it as an optimization problem with a chance constraint. Such optimization problem is also nonconvex and with appropriate relaxation it can be simplified to a convex problem. Similar with all the other aforementioned methods, it considers the optimal fusion in the sense of statistics, which do not necessarily lead to a small estimation error. There may be the case that the estimation error is very large even though the optimal criteria considered is small. So far, the robustness of the fusion estimation is still a challenge.

In this paper, we are looking forward to establishing a robust distributed fusion strategy under some basic assumptions. This robust fusion is aimed at minimizing the worst-case fusion error, which is achieved through a minimax problem. Although it is non-convex, we can relax it to a semidefinite program (SDP) following [16]. The resulted SDP problem can be solved quite efficiently in polynomial time by an interior point method; in particular, by the homogeneous self-dual method [17] or toolbox CVX in Matlab. Then the resulted fusion estimate is a form of a nonlinear combination of local estimates. Since the geometrical interpretation of our fusion method is in coincidence with the Chebyshev center, we call it the ion (CFE). The basic assumption of this paper is that the local estimation errors are bounded. Although it is not satisfied theoretically if the estimation error is a Gaussian distributed variable, it can be guaranteed in a nearly 100% probability if the bound is large enough and in practical applications it can always be satisfied. We call this bound the supporting bound, which is directly related to the resulted Chebyshev fusion estimate. So we further investigate the sensitive analysis of the relationship between the Chebyshev fusion estimate and the supporting bound. The result shows that the performance of the proposed Chebyshev fusion estimation is robust to the choice of the supporting bound. Moreover, numerical simulations are used to corroborate the theoretical results which demonstrate the good performance of the proposed CFE method.

The remainder is organized as follows. We briefly introduce the distributed estimation fusion problem in Section 2 and propose the robust CFE method in Section 3. The sensitive analysis about the choice of parameter R in CFE method is provided in Section 4, and some numerical simulations are carried out in Section 5. Section 6 gives conclusions.

2. Distributed Estimation Fusion Problem

Consider the following l-sensor distributed dynamic system:

$$\begin{aligned} \mathbf{x}_{t+1} &= \Phi \mathbf{x}_t + \mathbf{v}_t, \quad (t = 1, \dots, T), \\ \mathbf{y}_t^i &= \mathbf{H}^i \mathbf{x}_t + \mathbf{w}_t^i, \quad (i = 1, \dots, l), \end{aligned} \tag{1}$$

where $\mathbf{x}_t \in \mathbf{R}^n$ is the state vector, $\Phi \in \mathbf{R}^{n\times n}$ is the transition matrix, $\mathbf{y}_t^i \in \mathbf{R}^{m_i}$ and $\mathbf{H}^i \in \mathbf{R}^{m_i \times n}$, $i = 1, \dots, l$, are the observations and measurement matrices of l local sensors respectively, and $\mathbf{v}_t \in \mathbf{R}^n$ and $\mathbf{w}_t^i \in \mathbf{R}^{m_i}$ are the process noise, and the measurement noise respectively, which are norm-bounded zero mean random processes with covariance matrices $E(\mathbf{v}_t \mathbf{v}_t') = \mathbf{V}, E(\mathbf{w}_t \mathbf{w}_t') = \mathbf{W}$ and independent across sensors and time t.

Kalman's filtering is the best known recursive least mean square (LMS) algorithm to optimally estimate the unknown state of a dynamic system for a single sensor. Thus, the unbiased estimates $\widehat{\mathbf{x}}_t^i$ and corresponding error covariances $\mathbf{P}_t^i = E[(\mathbf{x}_t - \widehat{\mathbf{x}}_t^i)(\mathbf{x}_t - \widehat{\mathbf{x}}_t^i)']$ $(i = 1, \dots, l)$ are available by the Kalman filter. The distributed fusion problem is to generate an "optimal" estimate $\widehat{\mathbf{x}}_t$ from $\widehat{\mathbf{x}}_t^i$ for $i = 1, \dots, l$.

There are three possible architectures in distributed fusion depending on the sources of $\widehat{\mathbf{x}}_t^i$ [6]. In this paper, we consider the "Arbitrary distributed fusion," that is, $\widehat{\mathbf{x}}_t^i$ $(i = 1, \dots, l)$ are l arbitrary estimates to be fused, and no prior information or memory is available. The main problem is caused by correlated estimation errors, because in general $\mathbf{P}_t^{ij} = E[(\mathbf{x}_t - \widehat{\mathbf{x}}_t^i)(\mathbf{x}_t - \widehat{\mathbf{x}}_t^j)'] \neq \mathbf{0}$ for $i \neq j$ and their values may not be known.

In order to simplify the derivations, we start by reformulating the local estimate $\widehat{\mathbf{x}}_t^i$ in terms of a mixture of uncorrelated components $\mathbf{e}_t^i$. More specifically, let us define $\mathbf{e}_t^i \in \mathbf{R}^n$ to be the normalized random vector $\mathbf{e}_t^i = \mathbf{P}_t^{i^{-1/2}}(\mathbf{x}_t - \widehat{\mathbf{x}}_t^i)$ such that $E[\mathbf{e}_t^i] = \mathbf{0}$ and $E[\mathbf{e}_t^i \mathbf{e}_t^{i'}] = \mathbf{I}$. Moreover, because the noises of the dynamic system are norm bounded, we make the following assumption.

Assumption 1. There exists a ball of radius R_t that contains the entire support of the unknown distribution of $\mathbf{e}_t^i$ for all $i = 1, \dots, l$. More specifically, there exists $R_t \geq 0$ such that

$$P\left(\left(\mathbf{x}_t - \widehat{\mathbf{x}}_t^i\right)' \mathbf{P}_t^{i^{-1}} \left(\mathbf{x}_t - \widehat{\mathbf{x}}_t^i\right) \leq R_t^2\right) = 1. \tag{2}$$

We believe that Assumption 1 is reasonable, because in practice the estimation error of the local sensor is impossible to be infinitely large, and we can always find a bound on it. In practical applications, even when we have no additional information about $\mathbf{x}_t$ and $\mathbf{e}_t^i$, we believe that an educated and conservative guess about the magnitude of R_t is available. We will also revisit this issue in Section 4 where we discuss the sensitivity of the resulting fusion estimation with respect to the choice of R_t. In the rest part of this paper, a robust fusion estimation strategy will be derived based on Assumption 1.

3. The Robust Chebyshev Fusion Estimation Strategy

3.1. The Minimax Fusion Strategy. The most widely used fusion strategy is calculating the "best" linear combination of local estimates to minimize some criteria in statistics, such as minimum variance or weighted least squares. However, there may be some nonlinear formations to fuse the local information that performs better, which is at least as good as the linear combination because the linear combination is a special case of non-linear formation.

Moreover, the optimal fusion strategy in statistical meaning is not necessarily to get a good estimate with respect to the estimation error $\|(\mathbf{x}_t - \hat{\mathbf{x}}_t)\|^2$. Especially for the methods which depend on the unknown correlated estimation errors $\mathbf{P}_t^{ij}$, the performance of the fusion result may be considerably poor when the estimated $\hat{\mathbf{P}}_t^{ij}$ are not accurate enough. Because of these uncertainties in the distributed fusion, we propose the following robust mini-max fusion estimation.

Based on Assumption 1, we have observed that the state $\mathbf{x}_t$ must lie in the ellipsoid $E_i = \{\mathbf{x} : (\mathbf{x} - \hat{\mathbf{x}}_t^i)' \mathbf{P}_t^{i^{-1}} (\mathbf{x} - \hat{\mathbf{x}}_t^i) \le R_t^2\}$, so the intersection of the l quadratic ellipsoids is nonempty, which is defined as

$$Q = \{\mathbf{x} : \ f_i(\mathbf{x}) = \mathbf{x}' \mathbf{A}_i \mathbf{x} + 2\mathbf{b}_i' \mathbf{x} + c_i \le 0, 1 \le i \le l\}, \tag{3}$$

where $\mathbf{A}_i = \mathbf{P}_t^{i^{-1}}$, $\mathbf{b}_i = -\mathbf{P}_t^{i^{-1}} \hat{\mathbf{x}}_t^i$, and $c_i = (\hat{\mathbf{x}}_t^i)' \mathbf{P}_t^{i^{-1}} \hat{\mathbf{x}}_t^i - R_t^2$. Therefore, we have $P(\mathbf{x}_t \in Q) = 1$. In order to get a robust fusion estimation without the information on correlated local estimation errors, we directly treat the estimation error and suggest minimizing the worst-case error over Q, which is equivalent to finding the Chebyshev center of Q:

$$\min_{\hat{\mathbf{x}} \in \mathbf{R}^n} \max_{\mathbf{x} \in Q} \|\hat{\mathbf{x}} - \mathbf{x}\|^2. \tag{4}$$

The geometrical interpretation of the Chebyshev center is the center of the minimum radius ball enclosing Q. Thus, problem (4) can be equivalently written as

$$\min_{\hat{\mathbf{x}}, r} \left\{ r : \|\hat{\mathbf{x}} - \mathbf{x}\|^2 \le r, \ \forall \mathbf{x} \in Q \right\}. \tag{5}$$

However, computing the Chebyshev center (4) is a difficult optimization problem in general, because the inner maximization is nonconvex quadratic problem. Recent research in the context of quadratic optimization [3] shows that the Chebyshev center can be calculated efficiently when Q is the intersection of two ellipsoids in the complex domain, despite the nonconvexity. While in the real domain and when there are more than two constraints, a relaxed Chebyshev center (RCC) is proposed in [16].

3.2. The Relaxed Chebyshev Center Fusion Estimation. The RCC of Q, which is denoted as $\hat{\mathbf{x}}_{\mathrm{RCC}}$, is obtained by replacing the non-convex inner maximization in (4) by its semidefinite relaxation and then solving the resulting convex-concave min-max problem, and for more details, one can refer to [16]. Therefore, an explicit representation of $\hat{\mathbf{x}}_{\mathrm{RCC}}$ can be achieved by the following theorem.

Theorem 2. *The RCC of Q is given by*

$$\hat{\mathbf{x}}_{RCC} = -\left(\sum_{i=1}^{l} \alpha_i \mathbf{A}_i\right)^{-1} \left(\sum_{i=1}^{l} \alpha_i \mathbf{b}_i\right), \tag{6}$$

where $\{\alpha_1, \alpha_2, \ldots, \alpha_l\}$ is an optimal solution of the following convex optimization problem in l variables:

$$\min_{\alpha_i} \left\{ \left(\sum_{i=1}^{l} \alpha_i \mathbf{b}_i\right)' \left(\sum_{i=1}^{l} \alpha_i \mathbf{A}_i\right)^{-1} \left(\sum_{i=1}^{l} \alpha_i \mathbf{b}_i\right) - \sum_{i=1}^{l} \alpha_i c_i \right\} \tag{7}$$

$$\text{s.t.} \quad \sum_{i=1}^{l} \alpha_i \mathbf{A}_i^{-1} \ge \mathbf{I}, \quad \alpha_i \ge 0, \ i = 1, 2, \ldots, l. \tag{8}$$

It is not difficult to cast the optimization problem (7) as the following SDP:

$$\min_{\alpha_i} \left\{ t - \sum_{i=1}^{l} \alpha_i c_i \right\} \tag{9}$$

$$\text{s.t.} \begin{pmatrix} \sum_{i=1}^{l} \alpha_i \mathbf{A}_i & \sum_{i=1}^{l} \alpha_i \mathbf{b}_i \\ \sum_{i=1}^{l} \alpha_i \mathbf{b}_i' & t \end{pmatrix} \ge 0 \tag{10}$$

$$\sum_{i=1}^{l} \alpha_i \mathbf{A}_i \ge \mathbf{I}, \quad \alpha_i \ge 0, \ i = 1, 2, \ldots, l. \tag{11}$$

We see that the fusion estimate $\hat{\mathbf{x}}_{\mathrm{RCC}}$ is completely a nonlinear combination with all the available local information, including the estimates $\hat{\mathbf{x}}_t^i$ and error covariances $\mathbf{P}_t^i$, and the coefficients α_i are solved by an SDP (9), which can be calculated with high efficiency. The local estimates $\hat{\mathbf{x}}_t^i$ are just the fusion estimate $\hat{\mathbf{x}}_{\mathrm{CFE}}$ when $\alpha_j = \delta_{ij}$, where $\delta_{ij} = 1$ when $i = j$, and $\delta_{ij} = 0$ when $i \neq j$. From Proposition IV.2 in [16], $\hat{\mathbf{x}}_{\mathrm{CFE}}$ is unique and feasible. So the worst-case estimation error of $\hat{\mathbf{x}}_{\mathrm{CFE}}$ is smaller than or at least as small as that of local estimators in the relaxed sense.

Remark 3. Note that from the definition of Q given in (3) and Theorem 2, the optimal fusion coefficients α_i are actually relative to the local estimates $\hat{\mathbf{x}}_t^i$. Therefore, the optimal fusion coefficients α_i are time varying and need to be solved at every sampling time t. Fortunately, the optimization problem (9)–(11) is an SDP, which is a class of convex optimization problems and can be solved in polynomial time using efficient algorithms, such as the software package SeDuMi or CVX toolbox in MATLAB. Therefore, this could satisfy real-time processing when the number of sensors l is not too large.

Among the variables, $\mathbf{A}_i$ and $\mathbf{b}_i$, except c_i, are independent of R_t, that is, the bound of the support of $\mathbf{e}_t^i$. So in Section 4, we focus on the choice of R_t. In what follows, we shall drop the argument t without confusion for notational simplicity.

4. Choosing the Support Bound R

From the expression of $\hat{\mathbf{x}}_{\text{RCC}}$ in (7), the fusion estimate is determined by the parameters α_i, which is the solution of the SDP problem (9). Because R appears only in the optimal object, the choice of R does not infect the feasible set of (9). First of all, we discuss the sensitivity of the choice of R in CFE of distributed fusion estimation.

4.1. The Sensitivity of the Choice of R. Let us write the SDP problem (9) in the standard literature on linear semidefinite programs by

$$\begin{aligned}(\mathrm{P})\quad & \max \quad \mathbf{g}'\mathbf{y} \\ & \text{s.t.} \quad \mathcal{A}^*(\mathbf{y}) + \mathbf{S} = \mathbf{C} \quad \mathbf{S} \succeq \mathbf{0},\end{aligned} \tag{12}$$

where $\mathcal{A}^*(\mathbf{y}) := \sum_{i=1}^{l+1} \mathbf{y}_i \mathbf{F}_i$, $\mathbf{g} = [c_1, \ldots, c_l, -1]'$, $\mathbf{y} = [\alpha_1, \ldots, \alpha_l, t]'$, for $i = 1, \ldots, l$, $\mathbf{E}_i = \operatorname{diag}(\mathbf{e}_i)$, $\mathbf{e}_i(j) = 1$ if $i = j$, else $\mathbf{e}_i(j) = 0$, and

$$\begin{aligned}\mathbf{F}_i &= \begin{pmatrix} \mathbf{A}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_i & \mathbf{b}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{b}_i' & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{E}_i \end{pmatrix}_{(2n+l+1)\times(2n+l+1)}, \\ \mathbf{F}_{l+1} &= \begin{pmatrix} \mathbf{0}_{(2n\times 2n)} & 0 & \mathbf{0} \\ \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & 0 & \mathbf{0} \end{pmatrix}_{(2n+l+1)\times(2n+l+1)}, \\ \mathbf{C} &= \begin{pmatrix} \mathbf{I}_{n\times n} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}_{(2n+l+1)\times(2n+l+1)}.\end{aligned} \tag{13}$$

The dual of the primal program is

$$\begin{aligned}(\mathrm{D})\quad & \min \quad \mathbf{C} \bullet \mathbf{X} \\ & \text{s.t.} \quad \mathcal{A}(\mathbf{X}) = \mathbf{g} \qquad X \succeq \mathbf{0},\end{aligned} \tag{14}$$

where $\mathbf{C} \bullet \mathbf{X} := \operatorname{trace}(\mathbf{C}'\mathbf{X})$ and $\mathcal{A}(\mathbf{X}) := [\mathbf{F}_1 \bullet \mathbf{X}, \ldots, \mathbf{F}_{l+1} \bullet \mathbf{X})]$. The discussion of the sensitivity of the choice of R is based on the following assumption.

Assumption 4. The programs (P) and (D) are strictly feasible and there exist $\overline{\mathbf{y}}$, $\overline{\mathbf{S}}$, and $\overline{\mathbf{X}}$ which are unique and strictly complementary solutions of (P) and (D), that is,

$$\begin{gathered}\mathcal{A}(\overline{\mathbf{X}}) = \mathbf{g}, \quad \mathcal{A}^*(\overline{\mathbf{y}}) + \overline{\mathbf{S}} = \mathbf{C}, \quad \overline{\mathbf{X}}\,\overline{\mathbf{S}} = \mathbf{0}, \\ \overline{\mathbf{S}} \succeq \mathbf{0}, \quad \overline{\mathbf{X}} \succeq \mathbf{0}, \quad \overline{\mathbf{X}} + \overline{\mathbf{S}} \succ \mathbf{0}.\end{gathered} \tag{15}$$

Based on the above assumption, we consider the solutions of the programs (P) and (D) when there is a perturbation $\delta\mathbf{g}$ on $\mathbf{g}$ with the following theorem.

Theorem 5. *If the programs (P) and (D) satisfy Assumption 4 and the data $\mathbf{g}$ is changed by sufficiently small perturbation $\delta\mathbf{g}$, then the optimal solutions of the perturbed semidefinite programs are differentiable functions of perturbation $\delta\mathbf{g}$. Moreover, the derivatives $\dot{\mathbf{y}} := D\overline{\mathbf{y}}(\delta\mathbf{g})$, $\dot{\mathbf{S}} := D\overline{\mathbf{S}}(\delta\mathbf{g})$ and $\dot{\mathbf{X}} := D\overline{\mathbf{X}}(\delta\mathbf{g})$ at $\overline{\mathbf{y}}$, $\overline{\mathbf{S}}$, $\overline{\mathbf{X}}$ satisfy*

$$\begin{gathered}\mathcal{A}^*(\dot{\mathbf{y}}) + \dot{\mathbf{S}} = \mathbf{0}, \\ \mathcal{A}(\dot{\mathbf{X}}) = \delta\mathbf{g}, \\ \dot{\mathbf{X}}\overline{\mathbf{S}} + \overline{\mathbf{X}}\dot{\mathbf{S}} = \mathbf{0}.\end{gathered} \tag{16}$$

Remark 6. The perturbation $\delta\mathbf{g}$ does not infect the feasible set of (P), and so does Slater's condition of (P). By continuity, Slater's condition of (D) is also satisfied for all sufficiently small perturbation $\delta\mathbf{g}$. The result in this theorem is based on the fact that Assumption 4 is still satisfied when perturbed $\mathbf{g}$ by $\delta\mathbf{g}$.

Remark 7. The result in this theorem is a special case in Theorem 1 in [18], which gives a comprehensive sensitivity result on the perturbation of all data of programs (P) and (D). Thus, our theorem could be a direct corollary from it.

Remark 8. Although the derivatives $\dot{\mathbf{y}}$, $\dot{\mathbf{S}}$, and $\dot{\mathbf{X}}$ are characterized by a system of linear equations (16), it is an overdetermined system of $l + 1 + (2n + l + 1)(3n + 3l/2 + 2)$ linear equations for the $l+1+(2n+l+1)(2n+l+2)$ unknowns.

Theorem 9. *The derivatives $\dot{\mathbf{y}}$, $\dot{\mathbf{S}}$, and $\dot{\mathbf{X}}$ in (16) can be given as the unique solution of the following nonsingular system of $l + 1 + (2n + l + 1)(2n + l + 2)$ linear equations for the $l + 1 + (2n + l + 1)(2n + l + 2)$ unknowns.*

Proof. By the conditions in Assumption 4, $\overline{\mathbf{X}}\,\overline{\mathbf{S}} = \mathbf{0} = \overline{\mathbf{S}}\,\overline{\mathbf{X}}$, and thus the matrices $\overline{\mathbf{X}} \succeq \mathbf{0}$ and $\overline{\mathbf{S}} \succeq \mathbf{0}$ commute. This guarantees that there exists a unitary matrix $\mathbf{U}$ that simultaneously diagonalizes $\overline{\mathbf{S}}$ and $\overline{\mathbf{X}}$. Therefore, by Corollary 1 in [18], the derivatives $\dot{\mathbf{y}}$, $\dot{\mathbf{S}}$, and $\dot{\mathbf{X}}$ can be solved from the following system:

$$\begin{gathered}\mathcal{A}^*(\dot{\mathbf{y}}) + \dot{\mathbf{S}} = \mathbf{0}, \\ \mathcal{A}(\dot{\mathbf{X}}) = \delta\mathbf{g}, \\ \Pi_{\text{up}}\left(\mathbf{U}'\left(\dot{\mathbf{X}}\overline{\mathbf{S}} + \overline{\mathbf{X}}\dot{\mathbf{S}}\right)\mathbf{U}\right) = \mathbf{0},\end{gathered} \tag{17}$$

where $\Pi_{\text{up}}(\mathbf{X})$ denotes the upper triangular of $\mathbf{X}$. □

So far, we have theoretically analyzed the sensitivity of a perturbation $\delta\mathbf{g}$ for SDP (P). The derivatives of the optimal solution to the perturbation could be calculated by a nonsingular system of linear equations. Because the variable R only exists in the object parameter $\mathbf{g}$, the change of R leads to a perturbation $\delta\mathbf{g}$ on the direction $[1, \ldots, 1, 0]'$. If the value of $\dot{\mathbf{y}}$ is sufficiently small, the performance of the proposed CFE is robust due to the choice of R.

4.2. The Geometrical Interpretation of R. From the expression in (3), we see that R in fact determines the size of the l

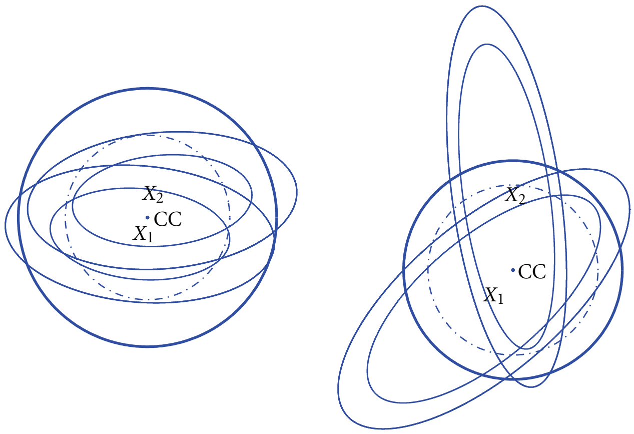

Figure 1: The illustration of the insensitivity on the choice of R.

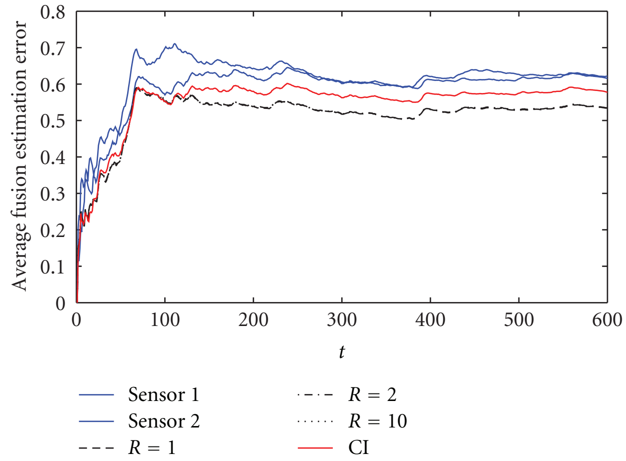

Figure 2: The average estimation error with respect to t for the local sensors, CFE, and CI method for $R_{\mathbf{v}} = [0.05\ \ 0; 0\ \ 0.05]$, $R_{\mathbf{w}}^{(i)} = [1\ \ 0; 0\ \ 2]$, where the CFE is calculated for $R = 1, 2, 10$ separately.

ellipsoids. We illustrate in Figure 1, that the RCC of two interacting ellipsoids is still the same when changing the sizes simultaneously.

A geometrical interpretation about this phenomenon is that the RCC reflects the center point of the intersection of some ellipsoids in some sense. When simultaneously enlarges or reduces the sizes of these ellipsoids, the resulted RCC still represents the center location in the same sense, so it is not strange that the RCC is insensitive to the choice of R. In fact, as in the simulations in Section 5, we illustrate that the influence of the value of R on the fusion estimation is trivial.

However, we should certify that when changing the value of R, these ellipsoids own a common interaction area. Therefore, we suggest making a conservative choice of R. In practice, we can estimate it from the experienced learning or prior information.

Also from Figure 1, we see that the RCC of two ellipsoids may be either the linear combination of the centers of the ellipsoids or not. So the CFE varies a larger space comparing with the other linear fusion methods.

5. Simulation Experiments in Localization and Tracking

In this section, some simulation experiments are designed to show the performance of the proposed CFE method in localization and tracking and compare it with the result of the previous CI method. In addition, we have designed a numerical simulation to test the sensitiveness of the choice of the value R as well.

5.1. Simulation of Dynamic System. We consider the following dynamic system:

$$\begin{aligned} \mathbf{x}_{t+1} &= \Phi \mathbf{x}_t + \mathbf{v}_t, \quad (t = 1, \ldots, T) \\ \mathbf{y}_t^{(i)} &= \mathbf{H}^{(i)} \mathbf{x}_t + \mathbf{w}_t^{(i)} \quad (i = 1, 2). \end{aligned} \tag{18}$$

Case 1. Consider

$$\Phi = \begin{pmatrix} \cos\left(\dfrac{2\pi}{300}\right) & \sin\left(\dfrac{2\pi}{300}\right) \\ -\sin\left(\dfrac{2\pi}{300}\right) & \cos\left(\dfrac{2\pi}{300}\right) \end{pmatrix}, \quad \mathbf{H}_i = \begin{pmatrix} \dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \\ -\dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \end{pmatrix}, \tag{19}$$

and the noises $\mathbf{v}_t$ and $\mathbf{w}_t^{(i)}$ are normally distributed with zero means and covariances $R_{\mathbf{v}} = [0.05\ \ 0; 0\ \ 0.05]$ and $R_{\mathbf{w}}^{(i)} = [1\ \ 0;\ \ 0\ \ 2]$, respectively. $\hat{\mathbf{x}}_t^i$ $(i = 1, 2)$ are 2 local estimators of $\mathbf{x}_t$ with covariance $\mathbf{P}_t^i$, respectively, which are calculated by a standard Kalman filter. The two sensors transmit their local estimates and covariance matrices to the fusion center, so it has the information of $\hat{\mathbf{x}}_t^i$ and $\mathbf{P}_t^i$.

We use the CFE and CI methods to fuse the two local estimates tracking the target for $t = 1, \ldots, 600$, where the CFE is calculated by solving the SDP problem (9) with the software package SeDuMi. The CI fusion is calculated following the method in [6]. The tracking performances are evaluated by the average estimation error, which is defined as

$$\mathrm{ARE}(\mathbf{x}_t) = \frac{\sum_{l=1}^{L} \left\| \overline{\mathbf{x}}_t^l - \mathbf{x}_t \right\|}{L}, \tag{20}$$

where $\overline{\mathbf{x}}_t^l$ denotes the estimation fusion of the state $\mathbf{x}_t$ at ensemble l and $L = 1000$ is the number of ensemble runs. The tracking performances of the local sensors, CFE, and CI method are illustrated in Figure 2, which shows the results of the average estimation error with respect to sampling time t for the local sensors, CFE, and CI method, respectively, where the CFE is calculated for $R = 1, 2, 10$ separately.

From Figure 2, we see that the average estimation error of CFE is consistently smaller than the local sensors as well as the CI method for all the choice of $R = 1, 2, 10$, which

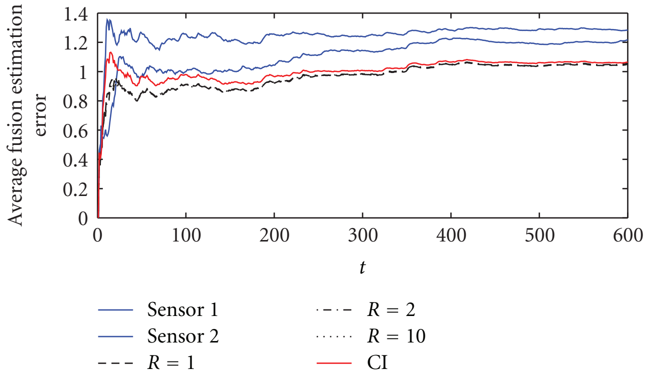

FIGURE 3: The average estimation error with respect to t for the local sensors, CFE, and CI method for $R_{\mathbf{v}} = [0.5\ \ 0; 0\ 0.5]$ and $R_{\mathbf{w}}^{(i)} = [3\ \ 0; 0\ \ 4]$, where the CFE is calculated for $R = 1, 2, 10$ separately.

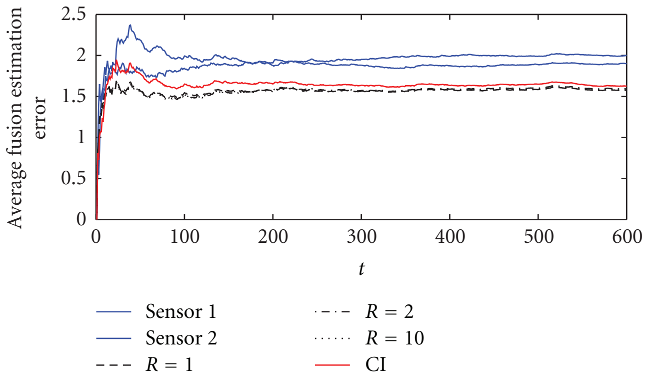

FIGURE 4: The average estimation error with respect to t for the local sensors, CFE, and CI method for $R_{\mathbf{v}} = [5\ \ 0; 0\ 5]$ and $R_{\mathbf{w}}^{(i)} = [3\ \ 0; 0\ \ 4]$, where the CFE is calculated for $R = 1, 2, 10$ separately.

verified that the proposed CFE method is more accurate compared with the CI method. Also, the average estimation errors are almost the same with respect to different values of R, which experimentally illustrate that CFE is insensitive to the value of R.

The next simulation is carried out for the same dynamic system as above, but the covariances of the noises $\mathbf{v}_t$ and $\mathbf{w}_t^{(i)}$ are $R_{\mathbf{v}} = [0.5\ \ 0; 0\ \ 0.5]$ and $R_{\mathbf{w}}^{(i)} = [3\ \ 0; 0\ \ 4]$, respectively. The resulted tracks and average estimation errors are shown in Figure 3. We can achieve the same results from this simulation that the CFE method is more accurate than CI method and the performances of CFE for different values of R are very close to each other.

Figure 4 is the tracks and average estimation errors when $R_{\mathbf{v}} = [5\ \ 0; 0\ \ 5]$ and $R_{\mathbf{w}}^{(i)} = [3\ \ 0; 0\ \ 4]$. The maximal estimation error through the process in the three simulations are listed in Table 1.

Case 2. Consider

$$\mathbf{H}_1 = \begin{pmatrix} \dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \\ -\dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \end{pmatrix}, \quad \mathbf{H}_2 = \begin{pmatrix} 1 & -0.25 \\ 0.25 & 1 \end{pmatrix}. \tag{21}$$

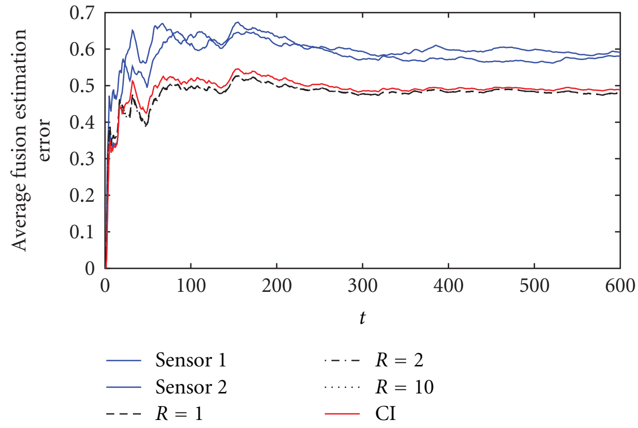

FIGURE 5: The average estimation error with respect to t for the local sensors, CFE, and CI method for $R_{\mathbf{v}} = [0.05\ \ 0; 0\ \ 0.05]$ and $R_{\mathbf{w}}^{(i)} = [1\ \ 0; 0\ \ 2]$, where the CFE is calculated for $R = 1, 2, 10$ separately.

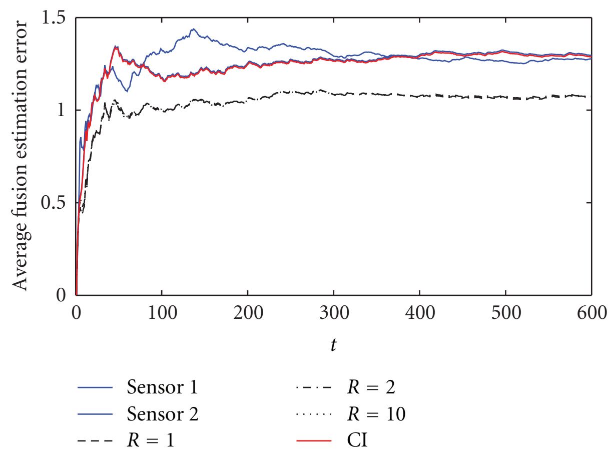

FIGURE 6: The average estimation error with respect to t for the local sensors, CFE, and CI method for $R_{\mathbf{v}} = [0.5\ \ 0; 0\ 0.5]$ and $R_{\mathbf{w}}^{(i)} = [3\ \ 0; 0\ \ 4]$, where the CFE is calculated for $R = 1, 2, 10$ separately.

In this case, also three simulations are carried out for different values of the covariances of the noises $\mathbf{v}_t$, and $R_{\mathbf{w}}^{(i)}$ respectively, and the other conditions are the same as in Case 1.

The tracks in this case are the same with Case 1. The average estimation errors through the process are illustrated in Figures 5–7. The improved performances of CFE are evidently better than CI when fusing the two local estimates, especially when the covariances of the noises are larger as in Figures 6 and 7. In fact, the performance of CI method in these two simulations are almost the same with local sensor 2, which is more accurate than local sensor 1. This comparison shows that CFE is a more stable method for distributed fusion because it always has a significant improvement when fusing the local estimates, while the CI method may just lead to the a local sensor estimate.

The maximal estimation error through the process in the three simulations are listed in Table 2.

Table 1: Comparison of the maximal estimation error through the process in the three simulations.

Simulation	Maximal squared estimation error					
	Sensor 1	Sensor 2	CFE $R = 1$	CFE $R = 2$	CFE $R = 10$	CI
1	3.3750	2.9707	2.7223	2.7601	2.7280	3.2137
2	14.0788	16.3843	9.0723	9.0220	9.1505	10.1641
3	43.6527	30.6748	20.7948	20.8013	21.3269	28.3822

Table 2: Comparison of the maximal estimation error through the process in the three simulations.

Simulation	Maximal squared estimation error					
	Sensor 1	Sensor 2	CFE $R = 1$	CFE $R = 2$	CFE $R = 10$	CI
1	3.2650	2.9283	2.1412	2.1670	2.1692	2.4373
2	15.3545	13.6833	11.8701	11.9099	11.9364	15.3535
3	38.7475	31.1737	27.2534	27.2287	27.3080	30.7442

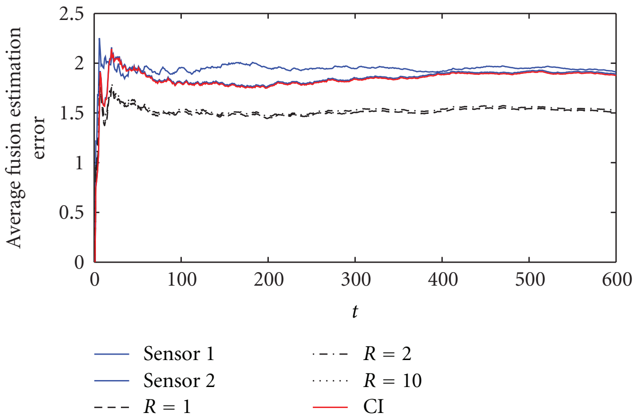

Figure 7: The average estimation error with respect to t for the local sensors, CFE, and CI method for $R_{\mathbf{v}} = [5\ \ 0; 0\ 5]$ and $R_{\mathbf{w}}^{(i)} = [3\ \ 0; 0\ \ 4]$, where the CFE is calculated for $R = 1, 2, 10$ separately.

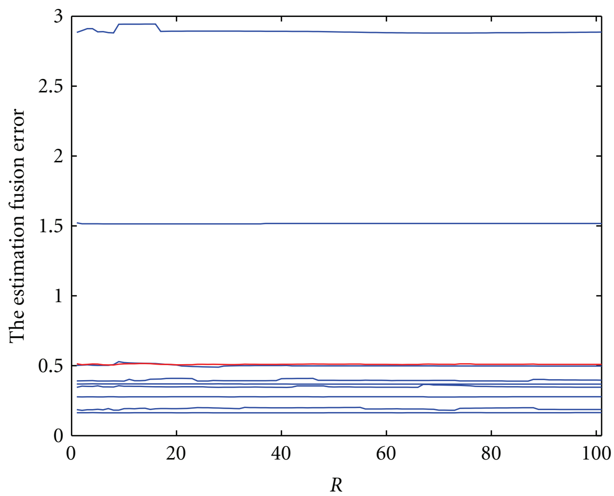

Figure 8: The estimation fusion error with respect to the value of R for 100 runs, where the blue line is the estimation error of the first 10 runs and the red line is the average estimation error for the 100 runs.

From Tables 1 and 2, we can see that the maximal estimation errors of CFE are much smaller than that of CI and the local sensors, which verified that the proposed CFE is a robust fusion estimation. Meanwhile, the performance of CFE is insensitive to the choice of R.

5.2. Sensitivity of the Value of R. In this simulation, we focus on the performance of CFE with respect to different values of R. This experiment explores the average estimation error by Monte-Carlos simulation. Suppose that the true initial state $\mathbf{x}_0$ and the local covariances of estimation error at this moment are known, that is, $\mathbf{x}_0 = [52.3246\ \ 2.2814]$, $\mathbf{P}_0^i = [0.2419\ -0.0456;\ -0.0456\ \ 0.2501]$ $(i = 1, 2)$. The dynamic system is the same as that of Case 1 in last the subsection and $R_{\mathbf{v}} = [1\ \ 0; 0\ \ 1]$ and $R_{\mathbf{w}}^{(i)} = [1\ \ 0; 0\ \ 2]$. We only consider the one step estimation fusion and use the CFE to fuse the one step estimates $\hat{\mathbf{x}}_1^1$ and $\hat{\mathbf{x}}_1^2$ when the value of R varies from 1 to 100.

The fused estimation error with respect to R for 100 runs illustrated in Figure 8, where the blue line is the estimation error of the first 10 runs and the red line is the average estimation error for the 100 runs with respect to R. From Figure 8, we see that the estimation error is nearly unchanged even when the value of R varies from 1 to 100, which verifies that the proposed CFE is not only a robust fusion but also a stable method for the choice of R.

6. Conclusions

In this paper, we propose a method using a mini-max strategy to get a robust fusion estimation in distributed multi-sensor systems for localization and tracking. This method is under the basic assumption that the normalized estimation error of local sensors are norm bounded, thus we can characterize the feasible set of the true state by the intersection of some ellipsoids. Then we proposed the mini-max fusion estimation in order to minimize the worst-case squared error. However, the resulted optimization problem is in fact looking for the Chebyshev center of the interaction of the ellipsoids, which is non-convex in nature. We relax it and get an approximate Chebyshev center by solving a relaxed SDP problem. The resulted estimation fusion is not a linear combination of local estimates. Judging from the simulation results, the proposed CFE method is a robust estimation fusion and more accurate compared with the CI method.

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