

Research Article

Distributed Control of Mobile Sensor Networks under RF Connectivity Constraints

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This paper addresses the problem of coordinating the motion of the nodes in a mobile sensor network for area coverage applications under RF communication limitations. During network evolution, the area sensed by the network increases until it reaches optimum configuration, while information for decision making is acquired distributively among the nodes via a prespecified number of hops. Unlike previous works, radio range is not demanded to be at least twice the sensing range, imposing an extra constraint in the overall problem setup. The proposed control scheme guarantees end-to-end RF connectivity of the network, while attaining optimum area coverage. Results are further verified via simulation studies.

1. Introduction

Distributed coordination of robotic swarms has been studied widely in the last years due to its direct application in missions where human interference may be risky or even prohibited. Mobile platforms with sensing, computational, and communication capabilities are in most cases spread in areas of interest in order to investigate various physical quantities and/or even take responsibility of surveying the area assuming intruder detection, patrolling, or exploration scenarios.

Although the agents in the group share a common objective, usually optimizing an *a priori* aggregate objective function [1–3], the way they collaborate is performed in a spatially distributed manner, rather than a global-coordinating one, due to the physical restriction in the acquired information via the antennas' range. Hence, connectivity preservation during the deployment stage is an issue of major importance, since this ensures information flow among the mobile nodes in order to cooperate for achieving their common goal.

Assuming both limited-range sensing and communication abilities of the platforms, it is evident that demand for connectivity preservation and area coverage optimality

cannot be achieved simultaneously, and thus there is trade-off to be balanced [4, 5]. Distributed coordination of mobile networks via intuitive nearest neighbour rules has been proposed by the authors in [6]. Connectivity control of networks has been examined in [7, 8], while more generalized coordinate-free theoretical approaches have been developed in [9, 10].

Coverage of a region by a set of nodes has been examined in previous works neglecting sensing range and thus is approached via a more networked pointed of view. Recent works have utilized topology control techniques in order to reduce the number of redundant links in congested wireless sensor networks [11–14], while antagonistic approaches have also been proposed [15, 16]. Distributed connectivity preservation during the deployment stage has been examined in [17] via estimation of the eigenvalues of the network's Laplacian [18].

In most of the works in the existing literature on the field of distributed sensors deployment, the communication range of the nodes' antennas is assumed either variable but unbounded (in terms of no upper limit) [19, 20] or bounded but greater than twice the sensing range [1, 21, 22]. This dependence of the radio range on the sensing one, although not met in practice, remarkably lets us surpass any network

connectivity issues and concentrate on optimization of the covered area.

In this paper, though, the nodes' radio range is assumed to be fixed and can be less than the aforementioned bound (i.e., twice the sensing range). This restriction imposition, although met most often in practical scenarios (where the sensors' and antennas' ranges are uncorrelated), leads in inability to apply already presented area coverage-oriented coordination schemes [2, 6, 22, 23]. Despite the aforementioned radio range constraint, the algorithm proposed in this paper guarantees network connectivity in a finite predefined number of hops, while leading the nodes in an optimal state, considering coverage terms.

The rest of the paper is organized as follows. In Section 2 the problem of surveillance of a region by a mobile sensor network is introduced, along with the main preliminaries on Voronoi partitioning. In Section 3 the background concerning network connectivity is presented, while connectivity in a finite number of hops among two nodes is analysed from a graph perspective. The proposed coordination scheme that takes into account the network's coverage performance along with communication constraints imposed due to radio restrictions is presented in Section 4. Simulation results in Section 5 further confirm efficiency of the proposed scheme, while concluding remarks are provided in the last section.

2. Coverage Problem Formulation

Consider n in number mobile robotic agents responsible for the sensing coverage of an area of interest D , defined as a convex compact set in \mathbb{R}^2 . Let $I_n = \{1, 2, \dots, n\}$ be the set of unique identifiers of the nodes, while their positions on the Euclidean plane are denoted by x_i , $i \in I_n$.

The robots are considered to evolve in the interior of D in discrete time via the control inputs u_i , as

$$x_i^{k+1} = x_i^k + u_i^k, \quad u_i \in \mathbb{R}^2, \quad x_i \in D, \quad i \in I_n, \quad (1)$$

where the superscript k denotes the current time step, $k = 0, 1, \dots$. In this paper, at each time step it is assumed that only one node can move; thus, at the first step, $k = 1$, node $i = 1$ will move, while afterwards the node to move is determined in a random manner.

Assuming surveillance purposes, sensors are embedded on the robotic platforms that sense the area in range of r around the nodes, denoted by B_i , that is,

$$B_i = \{x \in \mathbb{R}^2 : \|x - x_i\| \leq r\}, \quad i \in I_n. \quad (2)$$

In an area coverage application, the aggregate objective function under optimization can be expressed as the area of the union of the nodes' sensing regions over the D domain, that is,

$$\mathcal{H} = \int_{D \cap \bigcup_{i \in I_n} B_i} dS, \quad (3)$$

where dS is the elementary surface for integration purposes.

A quite common method to deal with such kind of problems in swarm robotics is to tessellate the space into

subsets via a distance-based metric and assign them among the nodes. Voronoi diagram [24], $\mathcal{V} = \{V_i, i \in I_n\}$, is the most common partitioning among n distinct points x_i , $i \in I_n$, defined as

$$V_i = \{x \in D : \|x - x_i\| \leq \|x - x_j\|, j \in I_n\}, \quad i \in I_n. \quad (4)$$

In other words, V_i is the set of the points of D that are closer to x_i than any other points in $\{x_j, j \in I_n\}$. We refer to V_i as the Voronoi cell of node i .

Let \mathcal{G}_D be the Delaunay graph associated with the corresponding Voronoi partitioning. We assume that the reader is familiar with the main preliminaries on graph theory [18]. Two nodes that share an edge of their Voronoi cells are considered as neighbours in \mathcal{G}_D . The Delaunay neighbours \mathcal{N}_i of an arbitrary node i are then defined as

$$\mathcal{N}_i = \{j \in I_n : V_i \cap V_j \neq \emptyset, j \neq i\}, \quad i \in I_n. \quad (5)$$

Apparently, if $j \in \mathcal{N}_i$, then $i \in \mathcal{N}_j$.

Utilizing the sets V_i and considering \mathcal{H} , one can define in an equivalent manner the r -limited Voronoi cell of an arbitrary node i , V_i^r , as the parts of the corresponding Voronoi cell that are simultaneously sensed by that node, that is,

$$V_i^r = V_i \cap B_i, \quad i \in I_n. \quad (6)$$

It is easily proven that via this definition, the total area sensed by the network, \mathcal{H} , can be expressed as the summation of the r -limited Voronoi cells of the nodes, that is,

$$\mathcal{H} = \sum_{i \in I_n} \int_{V_i^r} dS. \quad (7)$$

Equivalently to the Delaunay graph \mathcal{G}_D , one can define the $2r$ -limited Delaunay one, denoted by \mathcal{G}_D^{2r} , where the neighbours of a node i in this graph are the nodes whose r -limited Voronoi cells share an edge with V_i^r , that is,

$$\mathcal{N}_i^{2r} = \{j \in I_n : V_i^r \cap V_j^r \neq \emptyset, j \neq i\}, \quad i \in I_n. \quad (8)$$

Figure 1 shows graphically the neighbouring relationships among the nodes in the Delaunay, \mathcal{G}_D , and $2r$ -limited Delaunay graphs, \mathcal{G}_D^{2r} .

3. Radio Connectivity Issues

An issue of major importance in coordination of mobile sensor networks is the distributed nature of the designed control schemes. In other words, the nodes should organize their action without global knowledge of the network's state, but via local information from neighbouring nodes, instead.

Each node is assumed to be equipped with radio transceivers in order to be able to exchange spatial information with other neighbouring nodes in range. The antennas are assumed to transmit omnidirectionally around x_i up to a radius R . Unlike the majority of previous works

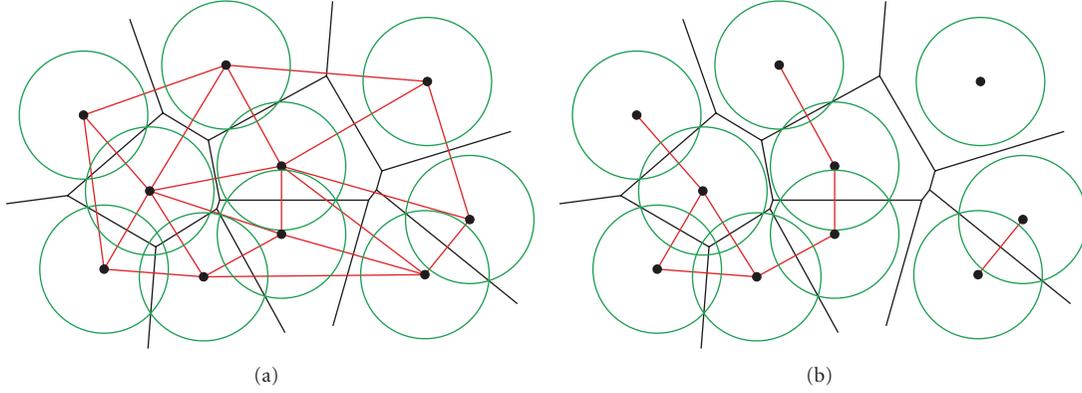


FIGURE 1: Graphical representation of the Delaunay (a) and $2r$ -limited Delaunay (b) neighbours in a sensor network.

in the existing literature, where the antennas' radii are considered variable, in this paper the latter is assumed fixed, same for all nodes, and not demanded to be at least twice the sensing range. In fact, the case $R \geq 2r$ is examined in detail in the literature, since network connectivity is trivially guaranteed that way, in the network's area-optimal state.

Apparently, a bidirectional communication link exists among any two nodes i and j if and only if $\|x_i - x_j\| \leq R$, for spatial information exchange purposes. Graphically, the neighbouring relationships among the nodes from a communication aspect (i.e., ignoring the nodes' sensing abilities) can be represented in the communication graph of the network, denoted by \mathcal{G}_c , where an edge exists among two nodes if and only if one is in radio-range of the other, and vice versa. It should be noted that although the Delaunay and $2r$ -limited Delaunay graphs are somehow correlated, the communication graph is totally independent, since it does not rely on the nodes' sensory domains, B_i , but is determined only by the nodes' positions and the common communication radius R . As far as the neighbouring relationships in the graph \mathcal{G}_c are considered, let us state the following definitions.

Definition 1. Two nodes i and j in the communication graph \mathcal{G}_c associated with a wireless sensor network are called (directly) connected, while denoted by $i \sim j$, if and only if $\|x_i - x_j\| \leq R$, where R is the common radio range.

Definition 2. Given the communication graph \mathcal{G}_c of a sensor network, a routing path of length ℓ among two nodes i and j is a sequence of $\ell + 1$ nodes $i, k_1, k_2, \dots, k_{\ell-1}, j$ such that $i \sim k_1, k_1 \sim k_2, \dots, k_{\ell-1} \sim j$.

Definition 3. Two nodes i and j in the communication graph \mathcal{G}_c are called N -hops connected, while denoted by $i \sim_N j$, if and only if the minimum-length path among them (if there exists a path) has length N . When two nodes are 1-hop connected, we will simply refer to them as connected, that is, $i \sim j$.

Given a sensor network and the communication graph \mathcal{G}_c , one can conclude (global) connectivity of the latter if there exists a routing path from any node of the network to any other, a.k.a. end-to-end connectivity. This can also

be expressed in algebraic terms via examining positiveness of the second smallest eigenvalue of the Laplacian matrix that corresponds to the aforementioned graph. For more information on algebraic graph theory the reader is encouraged to refer to [18]. The main issue, however, in controlling network connectivity via the graph's Laplacian is the fact that it is a centralized approach and thus inapplicable in cases of communication range constraints imposed by the nodes' antennas physical characteristics.

The motivation for introducing N -hop connectivity term among the nodes (Definition 3) is the fact that motion is performed in discrete time. Thus, from a practical point of view, one can assume that between two consequent motion time steps k and $k + 1$, the nodes perform a transmit/receive action N times. That way, each node is informed at each time step k for the positions of the nodes that it is N -hops connected with, or equivalently, its N -hop neighbours in the \mathcal{G}_c graph. Hence, distributed approaches can be developed that are based on the aforementioned set for the node to move at each step, without requiring global knowledge of the state of the network.

Assumption 1. Initially, each node is ℓ -hops connected with all its $2r$ -limited Delaunay neighbours, where $\ell \in I_N$ for any *a priori* given $N \in I_{n-1}$, that is,

$$i \sim_{\ell} j, \quad j \in \mathcal{N}_i^{2r}, \quad \ell \leq N \leq n - 1, \quad i \in I_n. \quad (9)$$

The aforementioned assumption becomes clearer via Figure 2, presenting a sensor network of 7 nodes with ratio R/r equal to 1.6. The network's $2r$ -limited Delaunay graph along with the sensed domain is presented in Figure 2(a), while the communication graph along with the radii R is depicted in Figure 2(b). The two parts, sensing and communication ones, have been separated in order to avoid confusion, while they refer to the same network. Examining nodes 6 and 7, one can verify via \mathcal{G}_D^{2r} that $6 \in \mathcal{N}_7^{2r}$ and vice versa. However, as concluded by the communication graph \mathcal{G}_c of the network, these nodes are not directly connected but are 5-hops connected, instead, via the path $\{7, 2, 3, 4, 5, 6\}$, that is, $6 \sim_5 7$.

In fact, it is apparent that if $R \geq 2r$, then each node is directly connected (from a communication aspect) with all

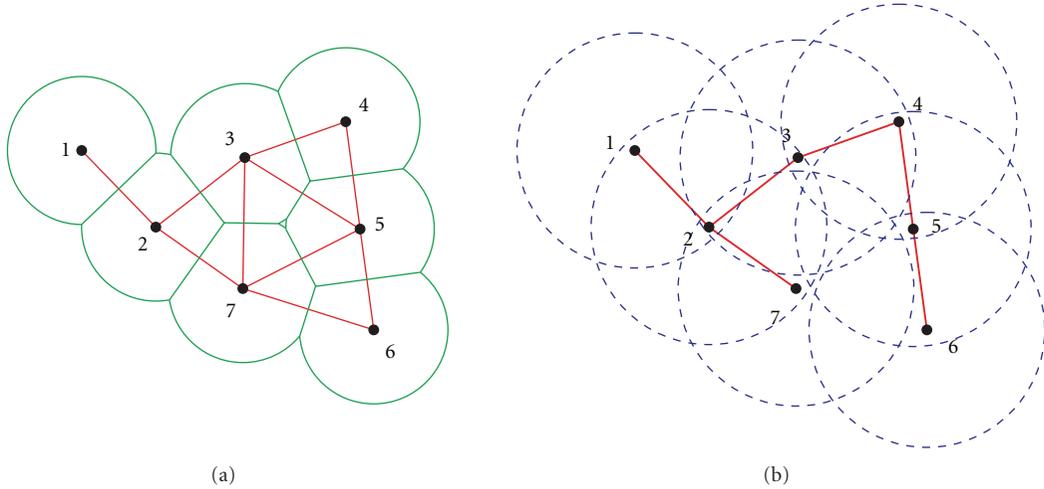


FIGURE 2: Graphical example in order to associate \mathcal{G}_D^{2r} (a) and \mathcal{G}_c (b) graphs.

of its $2r$ -limited Delaunay neighbours, that is, $j \in \mathcal{N}_i^{2r} \Rightarrow j \sim i$, in general. The scope of the following section is the development of a distributed control law, such that the network is led in an area-optimal configuration, given communication range constraint R , and at most N -hop connectivity demand over the $2r$ -limited Delaunay sets.

4. Connectivity-Aware Coordination Scheme

Recalling \mathcal{H} as defined in (7), the primary objective of the nodes in the network should be to self-deploy themselves in a way that the area of the sensed part of D (expressed as the summation of the areas of the corresponding r -limited Voronoi cells) is as high as possible, assuming bounded sensing domains. Furthermore, coordination at this stage should be performed in a distributed manner via local information, as exchanged among the nodes considering their communicational capabilities. As far as the coverage-part of the objective is concerned, let us first state the following lemma which will form the basis of the coordination stage.

Definition 4. A set of n distinct points in D is called an r -limited centroidal Voronoi configuration if and only if each point lies at the centroid of its own r -limited Voronoi cell.

Lemma 5 (see [23]). *The area covered by a set of nodes in an r -limited centroidal Voronoi configuration is locally maximum.*

In this paper, since coordination of the network is assumed to be performed in discrete time-steps, the nodes are selected to move towards the centroid of their corresponding r -limited Voronoi cells via fixed step sizes, until they reach optimum configuration where the total sensed area is maximum. More specifically, considering (1), the control law u_i for the corresponding node to move at each step can be selected as

$$u_i = \sigma \frac{\text{centr}(V_i^r) - x_i}{\|\text{centr}(V_i^r) - x_i\|}, \quad (10)$$

where the sample instance superscripts have been omitted to avoid notation complexity. In the previous expression, $\text{centr}(\cdot)$ stands for the centroid of the compact-set argument, while $\sigma = \min(\varepsilon, \|\text{centr}(V_i^r) - x_i\|)$, for any arbitrarily small $\varepsilon > 0$.

It should be noted that monotonicity of \mathcal{H} is not guaranteed during the transition of the network towards the r -limited centroidal Voronoi configuration; however, since one node moves at each time step (as stated in Section 2), the network will reach asymptotically area optimal configuration.

However, in order to characterize the control scheme (10) as decentralized, in the sense that the node to move does not require global network knowledge to apply it, the aforementioned node should be able to evaluate its r -limited Voronoi cell via information from the nodes in range. In most of previous works in the literature, this communication issue is surpassed by allowing the nodes' communication range to be at least twice the sensing one, since only the nodes in range of $2r$ are needed for distributed evaluation of V_i^r [1].

In this paper, however, an extra constraint is considered via limiting the communication range R to *any* arbitrary value, which in practice is imposed by the antennas' radio characteristics. The proposed control action is based mainly on preserving at most N -hop radio connectivity among the $2r$ -limited Delaunay neighbours, where N is a fixed number of hops *a priori* defined and independent of the nodes' sensing radius, so that (10) is distributively evaluated.

Let us denote by i the node to move at an arbitrary time step k . The latter decides on making a move towards the centroid of its r -limited Voronoi cell, in order to increase network's coverage performance. However, node i is assumed to have spatial information acquired from the nodes that it is N -hops connected with. It is clear that if after the motion a new node joins the set of the nodes that i is connected with, then no issue arises, since this just provides an extra link in the \mathcal{G}_c graph.

In the case, however, where a link is about to break if motion is performed, then the node to move checks if that

- (1) [time-step k ; node i is to move]
- (2) identify N -hop neighbouring nodes
- (3) evaluate V_i^r
- (4) evaluate control law u_i
- (5) estimate locally altered communication graph (with i at new position, but before performing the motion)
- (6) **if** Assumption 1 does not hold (at new position) **and** bisection-max-depth not reached, **then**
- (7) $\sigma \leftarrow \sigma/2$
- (8) **go to** 4
- (9) **end if**

ALGORITHM 1: Connectivity-aware control scheme.

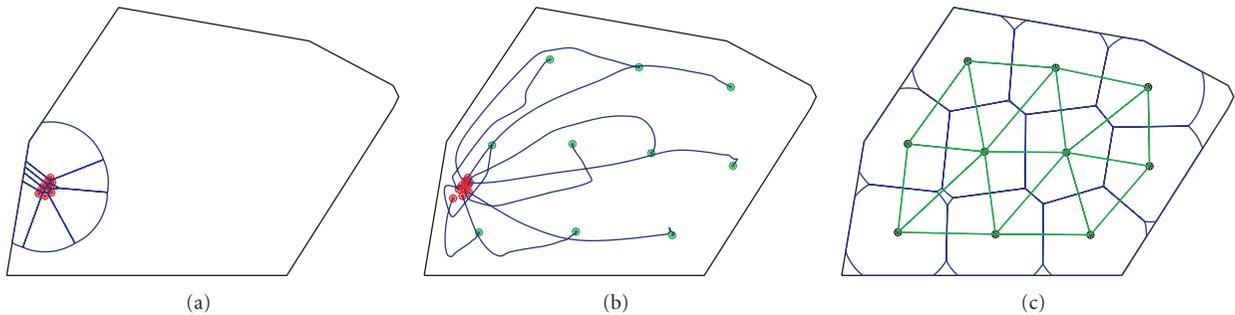


FIGURE 3: Case Study I: coordination results derived via the proposed control scheme. (a) Initial network configuration. (b) Network evolution. The green (red) circles represent the nodes' final (initial) positions. (c) Final network state. Communication graph indicates 1-hop connectivity among the $2r$ -limited Delaunay neighbours.

link was included in the shortest path in \mathcal{G}_c to determine N -hop connectivity among any two nodes of the set of its radio-connected neighbours. If so, then motion is tried to be performed with half the step size σ , and so on, till a predefined search depth. In the case where neither of the examined cases is able to preserve connectivity among the aforementioned set of nodes, then the node stays idle and does not perform any motion. The previous procedure is summarized in Algorithm 1.

Although the control scheme described previously incorporates some kind of conservatism, it is still able to (i) increase network's coverage performance from one step to another, while (ii) preserving N -hop radio connectivity among the $2r$ -limited Delaunay neighbours, as required for distributed evaluation of network's coverage increase. The fact that no motion is performed if a link that is about to break determines connectivity of the communication graph is the reason for the existence of overlapping among the nodes' sensory domains in the optimal state.

5. Simulation Results

Simulations were conducted in order to further verify efficiency of the proposed coordination scheme. The region D to be surveyed is considered as a convex compact planar set of total area $\int_D dS = 6.2015$ units². The latter is identical to the one used in [25]. Two simulation studies are examined. In the first case, the network consists of $n = 10$ nodes

with $r = 0.5$ units. The communication radius R was selected equal to $2r$, while the nodes were demanded to attain 1-hop network connectivity. In the second study, the network consists of $n = 20$ mobile nodes with sensing radii equal to $r = 0.2$ units. In order to emphasize in connectivity issues, the communication radius of the nodes' antennas was selected equal to r , $R = 0.2$ units, imposing a quite hard constraint in connectivity preservation during the coordination stage, while the nodes are *a priori* demanded to retain at most 3-hop connectivity.

The nodes are initially deployed randomly in D , so that Assumption 1 holds, for both cases. Given the set D , the maximum possible coverage ratio achieved that is evaluated as the summation of n circles (best case scenario) is equal to 100% and 40.5% of D , respectively, for each case. The network's initial configuration, evolution through time, and the final network's state, for the first case study are shown in Figure 3, in this order.

It is apparent that the network achieves optimum coverage, while the nodes retain radio connectivity with their $2r$ -limited Delaunay neighbours. Figure 5(a) presents the network's coverage during evolution, where it is seen that almost maximum performance is attained.

As far as concerns the second scenario, $R = r$, the network's initial configuration, evolution, and the final network's state are presented in the top part of Figure 4, in this order.

Figure 5 depicts the evolution the normalized network's coverage, that is, \mathcal{H} as a ratio of the area of D when

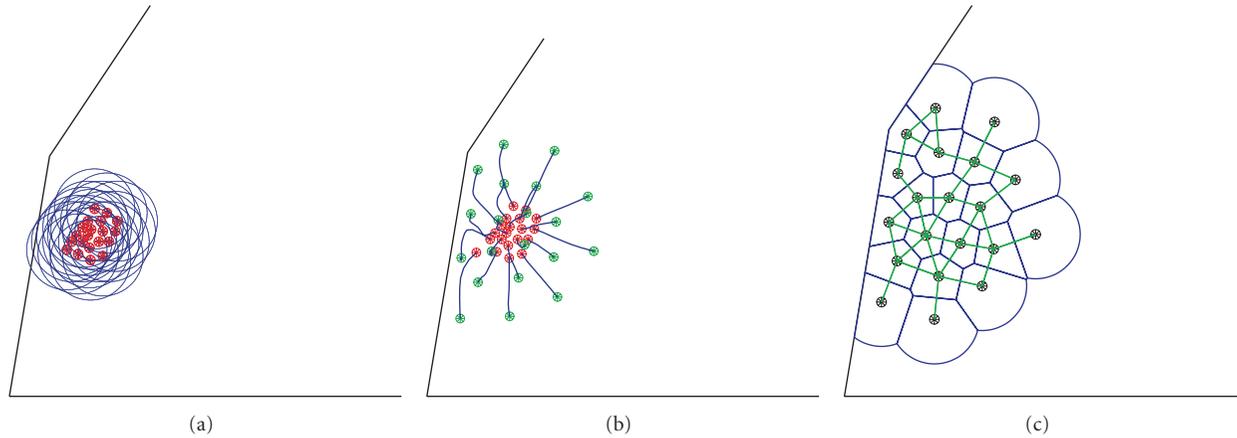


FIGURE 4: Case Study II: coordination results derived via the proposed control scheme. (a) Initial network configuration. (b) Network evolution. The green (red) circles represent the nodes' final (initial) positions. (c) Final network state. Communication graph indicates 3-hop connectivity among the $2r$ -limited Delaunay neighbours.

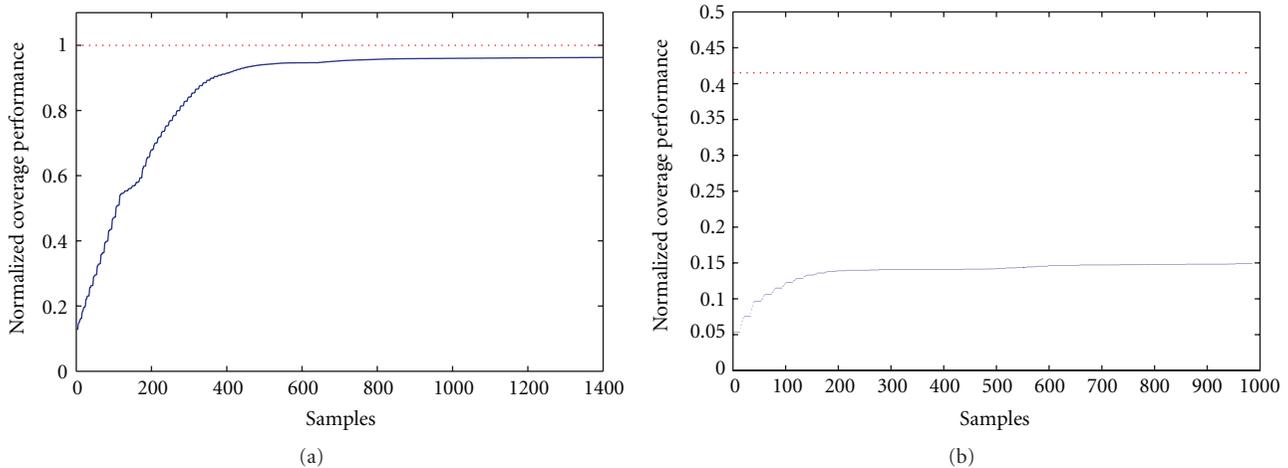


FIGURE 5: Percentage of sensed area during network evolution for Case Studies I (a) and II (b), respectively. The red upper-limit represents the maximum possible coverage ratio, in each case.

coordination scheme of Section 4 is applied (blue line). The red line represents the maximum possible coverage ratio, that is, 40.5%. More specifically, the sensed area percentage, starting from an initial value of 5% (dependent on the initial network configuration), increases as network evolves, until it converges to 14.8%, which is quite satisfactory considering communication constraints imposed.

One can see that the communication constraint imposition retains the network from achieving optimum area coverage, as evaluated via the best case scenario. However, the coverage performance attained is in fact quite satisfactory, taking into account that all nodes preserve 3-hop radio connectivity with all their $2r$ -limited Delaunay neighbours.

6. Conclusions

In this paper, a spatially distributed approach was proposed for leading the mobile nodes of a sensor network towards an area optimal configuration, while simultaneously preserving

radio connectivity among the nodes. The scheme was developed for the general case, where the radio range of the nodes' antennas can be less than twice the sensing range. Each node that moves is ensured to attain N -hop connectivity with the subset of nodes needed for decision making about the spot to move, so that the area coverage of the network is increased. Simulation studies were conducted verifying the efficiency of the proposed scheme.

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