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Research Article

Joint Relay Ordering and Linear Finite Field Network Coding for Multiple-Source Multiple-Relay Wireless Sensor Networks

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Network coding (NC) is significant for the spectral efficiency promotion of the multiple-source multiple-relay wireless sensor networks (WSNs). However, the existing scheme ignored further exploitation of the communications between relays. To address this issue, a relay ordering algorithm based on finite field NC (FFNC) is proposed in this paper. In the scheme, the relays who initially fail to decode from sources are kept listening and searching for the opportunity to decode the signals from other relays, so as to recover the failure links. The outage probability of the scheme and the corresponding lower bound are theoretically deduced under two-source two-relay assumption. Moreover, the scheme is proved to own the merit of diffusion effect, which makes the diversity improvement more efficient by simply increasing the relay number in the network. Simulation results correspond well with the analysis and demonstrate that the proposed scheme always outperforms FFNC in outage behavior and attains more opportunities to supply full diversity for the network. Moreover, it shows that the diffusion effect enables this scheme to be very suitable for the multiple-relay WSN.

1. Introduction

The concept of cooperative communication [1] is investigated quite intensively in recent years, because it can achieve the diversity gain of multiple-antenna with low complexity. In a large network such as WSN, the neighbouring users can potentially behave as the distributed antennas for a source node by relaying signals to the destination, thus enhancing the transmission reliability. However, this leads to the loss of spectral efficiency, since the relays have to consume time to receive and forward the signals from the sources. To overcome such shortcoming, network coding (NC) [2, 3] has been actively extended to the cooperation systems [4, 5]. In NC cooperation, instead of forwarding data separately, the relay nodes combine all the received messages to the whole one, so as to save the channel use.

Researches regarding NC cooperation have been carried out focusing on two typical scenarios, one is in the context of cooperative multiple access channels [6–9], while the other one is in that of two-way (TW) relay channels [10–12]. Since the NC operation introduces correlation between coded signals, the error propagation caused by relay will seriously degenerate the diversity performance. To account for this issue, [6] proposes adaptive NC, in which only "good" packets are coded. In [7], the authors proposed a power-efficient scaling scheme to reduce the impacts of error bits. Alternatively, [8] employs LDPC to supply noiseless bit pipes in order to achieve full diversity gain. In [10], Nguyen et al. make a tradeoff between spectral efficiency and outage behaviour, by exploiting log likelihood ratio threshold to identify the quality of signals.

In all these literatures, the source number $M = 2$ and NC are operated in binary fields. To further enhance the total spectral efficiency, [9] enables multiple users to share a common relay equipped with multiple antenna. However, in single antenna set, when $M > 2$, the scheme suffers diversity loss, because the binary field cannot supply enough codeword for all the sources. To overcome this drawback, in [13], the authors propose the linear finite field network coding (FFNC), which can achieve full diversity with arbitrary

M ($M \geq 2$) sources. More generally, [14] extends this scheme to the multiple-source multiple-relay scenario, that is, for arbitrary M sources and N relays; each one of the sources can achieve the diversity order of $N + 1$.

However, [13, 14] only considered the so-called two-hop cooperation, where the signals sent by a relay were only received by the destination. In fact, by the broadcast nature of wireless channels, the signals transmitted from a relay may be received by the other relays as well as the destination. Reference [15] has found that the order in which the multiple relays forward the signals significantly affects the end-to-end performance and proposed the relay ordering (RO) algorithms to improve the outage behaviour in amplify-and-forward (AF) cooperation systems, while [16] proposes a similar one for the decode-and-forward (DF) mode. To the best of our knowledge, however, such RO has not been studied for multiple-source multiple-relay network, and it motivates the work of this paper.

In this paper, we introduce RO into FFNC for the WSN with arbitrary M ($M \geq 2$) sources and N ($N \geq 2$) relays. The main contribution of the new scheme is to exploit the signals received by other relays when a relay transmits, so as to improve the full diversity probability, which denote the probability when each one of M sources achieves a diversity of $N+1$. It is generally known that through NC, a relay is able to supply the diversity only when it can successfully decode all the signals from the sources. In [14], if the relay fails to decode all the received signals, it will keep silent, thus supplying no diversity. Different from that, in the proposed scheme, the transmission order is scheduled according to the outage link number, and those who have decode successfully transmit firstly, while those who fail to decode keep listening to recover and update the outage link instantaneously. Through this scheduling, even when partial relays fail to decode initially, the whole system can still access to the $N + 1$ diversity at some extent. We deduce the outage probability and the corresponding lower bound for the proposed scheme in the network when $M = 2$, $N = 2$. Further, the scheme is extended to the scenario when $M \geq 2$ and $N \geq 2$. Simulation results verify the derived theoretical outage expressions and demonstrate that the proposed scheme always outperforms FFNC and offers more opportunities to supply the full diversity.

The remainder of this paper is organized as follows: Section 2 briefly reviews the related work. Section 3 describes the system model and the proposed scheme. Outage analysis and discussion are presented in Section 4. Simulation results are provided in Section 5. Finally, Section 6 concludes the paper.

2. Related Work

Multiple-source multiple-relay networks have recently attracted substantial research efforts with the goal of improving performance in terms of diversity order or throughput. In [17], Zhang et al. addressed the relay assignment problem in the network of M source pairs and N relays, and the proposed optimal and suboptimal schemes can, respectively, achieve the diversity of $N+1$ and N. However, these schemes are only efficient when $N \geq M$. In [18], Jeon et al. proposed the user pair selection to increase the throughput of the multiple-pair TW system. Through the channel norm-based and minimum distance-based criterions, multiple-pair TW transmission can adapt to the state of wireless channel well. But for each transmission, only the selected pair other than all the sources can access the diversity gain. Zhang et al. in [19] proposed a relay assignment for the multiple-pair multiple-relay network. The scheme is based on the max-min criterion and aims at attaining full diversity for all the pairs. This solution also owns the constraint that $N \geq M$. Besides, $2M$ channel uses are consumed for the whole transmission.

In all the mentioned literatures, the full diversity is achieved through specific selection and assignment schemes, which require complex synchronization and signalling. Moreover, since NCs are all operated in binary field, one relay can only support two sources simultaneously, which limit the spectral efficiency promotion.

To overcome the constraint of coding field size, in [13], the authors proposed a multiple-source NC ($M \geq 2$) by coding in the linear finite field. Since the field size is larger than $M + N - 1$, each relay can serve for all sources, thus reducing the channel use. Furthermore, in [14], Xiao et al. extended this scheme into the network with arbitrary M sources and N relays. The scheme is based on maximum distance separable (MDS) codes and is able to attain $N + 1$ diversity order. Nevertheless, this solution ignores the communications between relays, which have been proved to be helpful for system performance in [15, 16]. Motivated by these as presented above, this paper focuses on the performance promotion of FFNC through specific RO scheduling.

3. System Model and Scheme Description

In this section, we first introduce the system model of multiple-source multiple-relay NC. Based on it, the proposed scheme is described in detail.

3.1. System Model. Consider a wireless network where arbitrary sources Si, $i = 1, \ldots, M$, communicate to a common base station (BS), with the aid of arbitrary relays rj, $j = 1, \ldots, N$, as shown in Figure 1. It is assumed that from each Si, there exists a direct path to BS, while there are in addition N paths via the relays. The whole transmission consists of multiple access (MA) and broadcast (BC) phases, as is depicted, respectively, in Figures 1(a) and 1(b). During MA phase, each Si transmits information $\mathbf{I}_i$ at rate R on half-duplex time-division orthogonal channel, while all the relays keep listening. It is assumed that the transmitted signals suffer the effects of path loss, independent quasistatic block fading and additive white Gaussian noise (AWGN), so the received signals at ri and BS can be expressed as

$$\begin{aligned} \mathbf{Y}_{Si,rj} &= \sqrt{E_s} h_{Si,rj} \mathbf{X}_{Si} + \mathbf{n}_{rj}, \\ \mathbf{Y}_{Si,D} &= \sqrt{E_s} h_{Si,D} \mathbf{X}_{Si} + \mathbf{n}_{D}, \end{aligned} \quad (1)$$

where E_s is the transmitting power and $\mathbf{Y}$ and $\mathbf{X}$ are the received and transmitted channel codeword respectively.

$h_{Si,rj}$ and $h_{Si,D}$ capture the effects of frequency nonselective block fading from Si to rj and BS, respectively, which are modelled as zero-mean, independent, circular-symmetric complex Gaussian random variables with variances $\sigma^2_{Si,rj}$ and $\sigma^2_{Si,D}$. $\mathbf{n}_{rj}$ and $\mathbf{n}_D$ capture the effect of AWGN and are modelled as zero-mean complex Gaussian random variables with variance N_0. Suppose the channel state information is available only at the receivers for all the links. Similar to [14], the perfect channel coding is assumed and FFNC is implemented on top of it. In BC phase, if rj successfully extracts $\mathbf{I}_i$ from $\mathbf{Y}_{Si,rj}$, it will code them as $\mathbf{C}_j$ and convert it into $\mathbf{X}_{rj}$ for the transmission. Thus there are in total $M+N$ time slots (TSs) consumed by all sources and relays. Due to the broadcast property of the wireless medium, when rj transmits, all the other relays also receive the corresponding codeword, so the received signals at BS and ri can be written as

$$\mathbf{Y}_{rj,D} = \sqrt{E_s} h_{rj,D} \mathbf{X}_{rj} + \mathbf{n}_D$$
$$\mathbf{Y}_{rj,i} = \sqrt{E_s} h_{rj,i} \mathbf{X}_{rj} + \mathbf{n}_D, \quad i \neq j. \tag{2}$$

The core of FFNC in [14] is the transfer matrix $\mathbf{K}$, which is expressed as

$$\mathbf{K} = (\mathbf{U}, \mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_N) = \begin{pmatrix} 1 & 0 & \cdots & 0 & \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,N} \\ 0 & 1 & \cdots & 0 & \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,N} \\ \vdots & & & \cdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 & \gamma_{M,1} & \gamma_{M,2} & \cdots & \gamma_{M,N} \end{pmatrix} \tag{3}$$

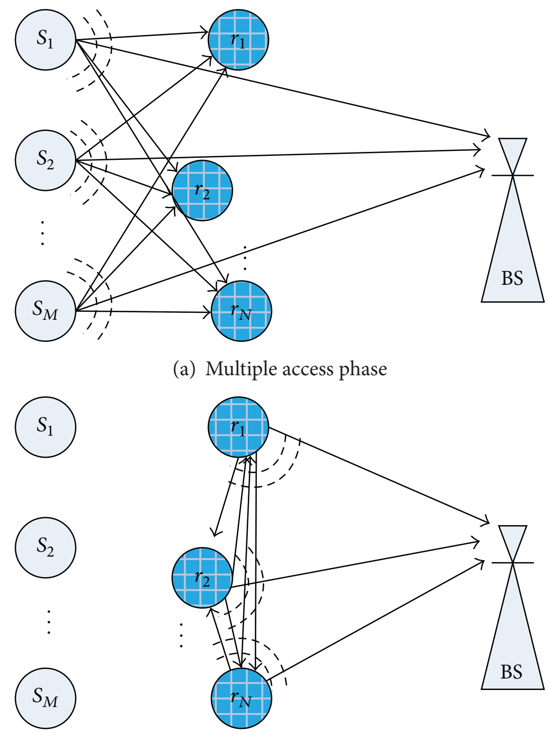
$\mathbf{U}$ is the global encoding kernels (GEKs) of direct transmission corresponding to $S1, S2, \ldots, SM$. $\mathbf{G}_i = [\gamma_{1,i}, \gamma_{2,i}, \ldots, \gamma_{M,i}]^T$ is the GEK of ri, which reflects the linear relation between the outgoing codeword and input information by $\mathbf{C}_i = \mathbf{I} \cdot \mathbf{G}_i$, where $\mathbf{I} = [\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_M]$ represents the source messages originating at $S1, S2, \ldots, SM$. Note that $\gamma_{i,j}$ is normally in a finite field (Galois field) GF($|A|$), where $|A| \geq M+N+1$ is the alphabet size. Thus, BS will receive $M+N$ signals from M sources and N relays in total. By exploiting the generator matrix of MDS codes as $\mathbf{K}$, any M out of $M+N$ columns is nonsingular [14], so $\mathbf{I}$ can be recovered by any M simultaneous equations.

3.2. Scheme Description. We name the proposed scheme as RO-FFNC. Denote L_i as the number of outage links at ri. Define the index sets of the available and unavailable relays as Φ_a and Φ_u, respectively, where $\Phi_a = \{i \mid L_i = 0\}$, $\Phi_u = \overline{\Phi}_a$. The scheme consists of the following steps.

(1) In MA phase, Si broadcasts the codeword $\mathbf{X}_i$ at the ith TS, $i = 1, \ldots, M$.

(2) Each relay decodes the received $\mathbf{Y}_i$ and counts L_j, $j = 1, \ldots, N$. If $L_j = 0$, then j is included in Φ_a, otherwise, it is included into Φ_u.

(3) For each $j \in \Phi_a$, rj codes converts $\mathbf{C}_j$ into $\mathbf{X}_j$ for transmission. After that, j is deleted from Φ_a.



(a) Multiple access phase

(b) Broadcast phase

Figure 1: System model of multiple-source multiple-relay NC network.

(4) When rj is transmitting, for each $i \in \Phi_u$, ri keeps listening and trying to decode $\mathbf{Y}_{rj,i}$; if $\mathbf{C}_j$ is successfully extracted, then $L_i = L_i - 1$.

(5) If $L_i = 0$, then i is removed from Φ_u to Φ_a. ri can recover $\mathbf{I}$ by the following M simultaneous equations:

$$\begin{aligned} &\mathbf{I}_b = \mathbf{I}_b \\ &\vdots \\ &\gamma_{1,j}\mathbf{I}_1 + \gamma_{2,j}\mathbf{I}_2 + \cdots + \gamma_{M,j}\mathbf{I}_M = \mathbf{C}_j, \quad j \in \Phi_a, \end{aligned} \tag{4}$$

where b is the index number of the sources whose information is successfully received by ri at MA phase.

(6) If $\Phi_a \neq \phi$, go to Step (2). Otherwise, for each $i \in \Phi_u$, ri keeps silent in BC phase and the algorithm terminates.

4. Performance Analysis

The outage probability of the link between Si and rj can be expressed as

$$P_{Si,rj} = \Pr\left(I_{Si,rj} < R\right) = 1 - \exp\left(-\lambda_{Si,rj}\beta\right), \tag{5}$$

where $I_{Si,rj} = \log(1 + |h_{Si,rj}|^2 \rho)/2(M+N)$ is the instantaneous mutual information between $\mathbf{Y}_{Si,rj}$ and $\mathbf{X}_{Si,rj}$ and $\rho = E_s/N_0$

is the signal-to-noise ratio (SNR). The factor $1/(M+N)$ is caused by the fact that the total transmission time is shared by $M+N$ TS. $\beta = (2^{2(M+N)R}-1)/\rho$, $\lambda_{Si,rj} = 1/\sigma^2_{Si,rj}$ is the parameter of $|h_{Si,rj}|^2$, which is exponentially distributed. For simplicity, we examine the outage performance of the proposed RO-FFNC in the symmetric network of $M = N = 2$. The closed form of outage probability as well as the full diversity probability is derived and compared with the ones of FFNC.

The outage probability of RO-FFNC can be expressed as

$$P_o^{\text{RO-FFNC}} = \sum_{i=0}^{2} P_{\text{o}|w} \Pr(w=i), \tag{6}$$

where w represents the number of the relays who transmit signals and $P_{o|w}$ denotes the conditional outage probability of the system. $w = 0$ means no relay transmission, so the probability is

$$\begin{aligned}\Pr(w=0) &= \prod_{i=1}^{2} \Pr(L_i \neq 0) \\ &= \prod_{i=1}^{2} \left(1 - \Pr(I_{S1,ri} > R)\Pr(I_{S2,ri} > R)\right) \\ &= \prod_{i=1}^{2} \left(1 - \exp\left(-\sum_{j=1}^{2} \lambda_{Sj,ri}\beta\right)\right).\end{aligned} \tag{7}$$

For $w = 1$, let $i, j \in \{1,2\}$, $i \neq j$, so the set can be expressed as $\{w=1\} = \{i \in \Phi_a,\ L_j = 2\} \cup \{i \in \Phi_a,\ L_j = 1,\ I_{ri,j} < R\}$. $\{i \in \Phi_a,\ L_j = 2\}$ denotes the events when ri is available for transmission, but it is not sufficient to recover rj, where both two source-relay links are in outage. $\{i \in \Phi_a,\ L_j = 1,\ I_{ri,j} < R\}$ denotes the event when rj failed to be recovered because of the link outage between it and ri. These two probabilities can be expressed as

$$\begin{aligned}&\Pr\left(i \in \Phi_a,\ L_j = 2\right) \\ &\quad = \Pr(I_{S1,ri} > R)\Pr(I_{S2,ri} > R) \\ &\qquad \times \Pr\left(I_{S1,rj} < R\right)\Pr\left(I_{S2,rj} < R\right) \\ &\quad = \exp\left(-\sum_{k=1}^{2} \lambda_{Sk,ri}\beta\right) \\ &\qquad \times \prod_{k=1}^{2}\left(1 - \exp\left(-\sum_{k=1}^{2} \lambda_{Sk,rj}\beta\right)\right)\end{aligned} \tag{8}$$

$$\begin{aligned}&\Pr\left(i \in \Phi_a,\ L_j = 1,\ I_{ri,j} < R\right) \\ &\quad = C_2^1 \Pr(I_{S1,ri} > R)\Pr(I_{S2,ri} > R) \\ &\qquad \cdot \Pr\left(I_{S1,rj} < R\right)\Pr\left(I_{S2,rj} > R\right)\Pr\left(I_{ri,j} < R\right) \\ &\quad = 2\exp\left(-\left(\sum_{k=1}^{2} \lambda_{Sk,ri} + \lambda_{S2,rj}\right)\beta\right) \\ &\qquad \times \left(1 - \exp\left(-\lambda_{S1,rj}\beta\right)\right)\left(1 - \exp\left(-\lambda_{ri,j}\beta\right)\right).\end{aligned} \tag{9}$$

The factor C_2^1 is due to the reciprocity of $I_{S1,rj}$ and $I_{S2,rj}$. With (8)-(9), $\Pr(w=1)$ can be written as

$$\begin{aligned}\Pr(w=1) = C_2^1 \Big(&\Pr\left(i \in \Phi_a,\ L_j = 2\right) \\ &+ \Pr\left(i \in \Phi_a,\ L_j = 1,\ I_{ri,j} < R\right)\Big),\end{aligned} \tag{10}$$

where C_2^1 is due to the reciprocity of i and j. Similarly, for $w = 2$, there is $\{w=2\} = \{i, j \in \Phi_a\} \cup \{i \in \Phi_a,\ L_j = 1,\ I_{ri,j} > R\}$. $\{i, j \in \Phi_a\}$ denotes the events when there is no source-relay link in outage at both ri and rj, and $\{i \in \Phi_a,\ L_j = 1,\ I_{ri,j} > R\}$ denotes the events when one link experiences outage at rj but is recovered by exploiting the transmission of ri. The probabilities can be expressed, respectively, as

$$\begin{aligned}\Pr\left(i, j \in \Phi_a\right) &= \prod_{i=1}^{2}\prod_{j=1}^{2} \Pr\left(I_{Si,rj} > R\right) \\ &= \exp\left(-\sum_{i=1}^{2}\sum_{j=1}^{2} \lambda_{Si,rj}\beta\right),\end{aligned} \tag{11}$$

$$\begin{aligned}&\Pr\left(i \in \Phi_a,\ L_j = 1,\ I_{ri,j} > R\right) \\ &\quad = C_2^1 \Pr(I_{S1,ri} > R) \\ &\qquad \times \Pr(I_{S2,ri} > R) \cdot \Pr\left(I_{S1,rj} < R\right) \\ &\qquad \times \Pr\left(I_{S2,rj} > R\right)\Pr\left(I_{ri,j} > R\right) \\ &\quad = 2\exp\left(-\left(\sum_{k=1}^{2} \lambda_{Sk,ri} + \lambda_{S2,rj} + \lambda_{ri,j}\right)\beta\right) \\ &\qquad \cdot \left(1 - \exp\left(-\lambda_{S1,rj}\beta\right)\right),\end{aligned} \tag{12}$$

$$\begin{aligned}\Pr(w=2) = C_2^1 &\Pr\left(i \in \Phi_a,\ L_j = 1,\ I_{ri,j} > R\right) \\ &+ \Pr\left(i, j \in \Phi_a\right).\end{aligned} \tag{13}$$

Substitute (11)-(12) into (13) and $\Pr(w=2)$ can be calculated.

At BS, there are $2+w$ received signals in total if at least two of them are successfully decoded, $\mathbf{I}$ can be recovered; otherwise, the system experiences outage. With different values of w analyzed above, the corresponding conditional outage probabilities are given as

$$\begin{aligned}P_{o|w=0} &= 1 - \prod_{i=1}^{2} \Pr(I_{Si,D} > R) \\ &= 1 - \exp\left(-\sum_{i=1}^{2} \lambda_{Si,D}\beta\right) \overset{\rho\to\infty}{\approx} \rho^{-1}\sum_{i=1}^{2} \lambda_{Si,D}\left(2^{2R}-1\right),\end{aligned} \tag{14}$$

$$\begin{aligned}P_{o|w=1} = &\prod_{i=1}^{2} \Pr(I_{Si,D} < R) + \Pr(I_{S1,D} < R) \\ &\times \Pr(I_{rk,D} < R) + \Pr(I_{S2,D} < R)\Pr(I_{rk,D} < R) \\ &- 2\prod_{i=1}^{2} \Pr(I_{Si,D} < R)\Pr(I_{rk,D} < R)\end{aligned}$$

$$\begin{aligned}
&= \prod_{i=1}^{2} (1-\exp(-\lambda_{Si,D}\beta)) + (1-\exp(-\lambda_{S1,D}\beta)) \\
&\quad \times (1-\exp(-\lambda_{rk,D}\beta)) + (1-\exp(-\lambda_{S2,D}\beta)) \\
&\quad \times (1-\exp(-\lambda_{rk,D}\beta)) \\
&\quad - 2\prod_{i=1}^{2} (1-\exp(-\lambda_{Si,D}\beta)) \cdot (1-\exp(-\lambda_{S1,D}\beta)) \\
&\quad \times (1-\exp(-\lambda_{rk,D}\beta)) (1-\exp(-\lambda_{S2,D}\beta)) \\
&\overset{\rho\to\infty}{\approx} \rho^{-2} 2\left(\sum_{i=1}^{2}\lambda_{Si,D} + \lambda_{rk,D}\right)\left(2^{2R}-1\right),
\end{aligned} \tag{15}$$

$$\begin{aligned}
P_{o|w=2} &= \prod_{i=1}^{2} \Pr(I_{Si,D} < R) \prod_{j=1}^{2} \Pr\left(I_{rj,D} < R\right) \\
&\quad + \Pr(I_{S1,D} > R) \cdot \Pr(I_{S2,D} < R) \prod_{j=1}^{2} \Pr\left(I_{rj,D} < R\right) \\
&\quad + \Pr(I_{S1,D} < R) \Pr(I_{S2,D} > R) \cdot \prod_{j=1}^{2} \Pr\left(I_{rj,D} < R\right) \\
&\quad + \prod_{i=1}^{2} \Pr(I_{Si,D} < R) \\
&\quad \times \Pr(I_{r1,D} > R) \cdot \Pr(I_{r2,D} < R) \\
&\quad + \prod_{i=1}^{2} \Pr(I_{Si,D} < R) \Pr(I_{r1,D} < R) \Pr(I_{r2,D} > R) \\
&= \prod_{i=1}^{2} (1-\exp(-\lambda_{Si,D}\beta)) \prod_{j=1}^{2} \left(1-\exp\left(-\lambda_{rj,D}\beta\right)\right) \\
&\quad + \exp(-\lambda_{S1,D}\beta) \cdot (1-\exp(-\lambda_{S2,D}\beta)) \\
&\quad \times \prod_{j=1}^{2} \left(1-\exp\left(-\lambda_{rj,D}\beta\right)\right) \\
&\quad + \exp(-\lambda_{S2,D}\beta) \cdot (1-\exp(-\lambda_{S1,D}\beta)) \\
&\quad \times \prod_{j=1}^{2} \left(1-\exp\left(-\lambda_{rj,D}\beta\right)\right) \\
&\quad + \exp(-\lambda_{r1,D}\beta) \cdot (1-\exp(-\lambda_{r2,D}\beta)) \\
&\quad \times \prod_{i=1}^{2} (1-\exp(-\lambda_{si,D}\beta)) \\
&\quad + \exp(-\lambda_{r2,D}\beta) \cdot (1-\exp(-\lambda_{r1,D}\beta)) \\
&\quad \times \prod_{i=1}^{2} (1-\exp(-\lambda_{si,D}\beta)) \\
&\overset{\rho\to\infty}{\approx} \rho^{-3}\left(\sum_{i=1}^{2}\lambda_{Si,D}\prod_{j=1}^{2}\lambda_{rj,D}\right. \\
&\qquad \left. + \sum_{j=1}^{2}\lambda_{rj,D}\prod_{j=1}^{2}\lambda_{Si,D}\right)\left(2^{2R}-1\right).
\end{aligned} \tag{16}$$

Substituting (7), (10), and (13)–(16) into (6), $P_o^{\text{RO-FFNC}}$ can be calculated. According to (14)–(16), it is clear that in high SNR region, $i+1$ diversity order is achieved when $w = i$, $i = 0, 1, 2$. The full diversity probability in [14] can be expressed as

$$P_{\text{full}} = \prod_{i=1}^{N} \Pr(L_i = 0) = \prod_{i=1}^{N}\prod_{j=1}^{M}\left(1 - P_{Si,rj}\right). \tag{17}$$

Following this definition, it can be deduced that $P_{\text{full}}^{\text{RO-FFNC}} = \Pr(w = 2)$ because only when $w = 2$ can the system achieve the full diversity. Thus, the lower bound of $P_o^{\text{RO-FFNC}}$ can be expressed as

$$P_L^{\text{RO-FFNC}} = P_{\text{full}}^{\text{RO-FFNC}} P_{o|w=2} \tag{18}$$

This lower bound tends to be tight as $\rho \to \infty$, because in high SNR region $\Pr(w = 0)$ and $\Pr(w = 1)$ approximate to 0 and $P_o^{\text{RO-FFNC}}$ is dominated by $P_{\text{full}}^{\text{RO-FFNC}}$. For comparison, the outage probability of FFNC in [14] is also given as:

$$P_o^{\text{FFNC}} = \sum_{i=0}^{2} P_{o|w'} \Pr\left(w' = i\right), \tag{19}$$

where $P_{o|w'} = P_{o|w}$, $\Pr(w' = 0) = \Pr(w = 0)$,

$$\begin{aligned}
\Pr\left(w' = 1\right) &= \Pr\left(i \in \Phi_a,\ L_j \neq 0\right) \\
&= \exp\left(-\sum_{i=1}^{2}\lambda_{Si,r1}\beta\right)\left(1-\exp\left(-\sum_{i=1}^{2}\lambda_{Si,r2}\beta\right)\right) \\
&\quad + \exp\left(-\sum_{i=1}^{2}\lambda_{Si,r2}\beta\right)\left(1-\exp\left(-\sum_{i=1}^{2}\lambda_{Si,r1}\beta\right)\right)
\end{aligned} \tag{20}$$

$$\begin{aligned}
\Pr\left(w' = 2\right) &= \Pr\left(i, j \in \Phi_a\right) \\
&= \exp\left(-\sum_{i=1}^{2}\sum_{j=1}^{2}\lambda_{Si,rj}\beta\right).
\end{aligned} \tag{21}$$

Substituting these equations into (19), P_o^{FFNC} can be calculated. Similarly, there is $P_{\text{full}}^{\text{FFNC}} = \Pr(w' = 2)$ and the corresponding lower bound can be also written as $P_L^{\text{FFNC}} = P_{\text{full}}^{\text{FFNC}} P_{o|w'=2}$. Comparing P_L^{FFNC} and $P_L^{\text{RO-FFNC}}$, it can be deduced that

$$\begin{aligned}
P_{\text{full}}^{\text{RO-FFNC}} &= P_{\text{full}}^{\text{FFNC}} + C_2^1\left(\Pr\left(i \in \Phi_a,\ L_j = 1,\ I_{ri,j} > R\right)\right. \\
&> P_{\text{full}}^{\text{FFNC}}
\end{aligned} \tag{22}$$

This result indicates that RO-FFNC has more opportunities to access full diversity compared with FFNC. With this inequality, it is easy to conclude that $P_o^{\text{RO-FFNC}} < P_o^{\text{FFNC}}$ by comparing (6) and (19).

5. Numerical Results

In this section, the proposed RO-FFNC is compared with FFNC in terms of outage probability by the Monte-Carlo simulations. The results are also compared with the theoretical

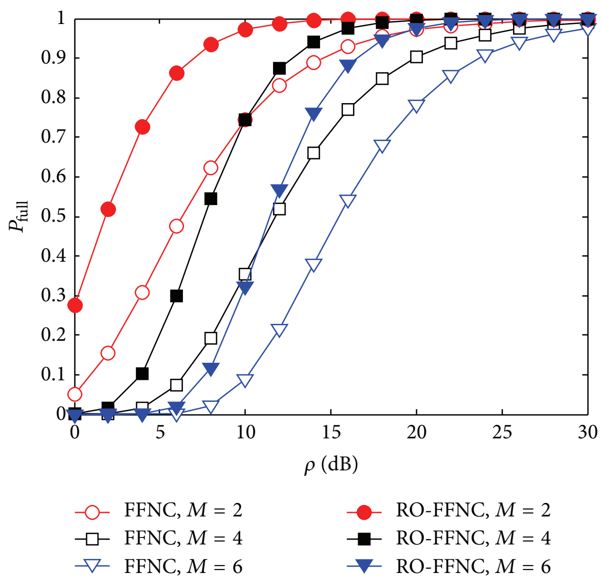


Figure 2: Full diversity probability comparisons when $M = 2, 4, 6$, $N = 2$.

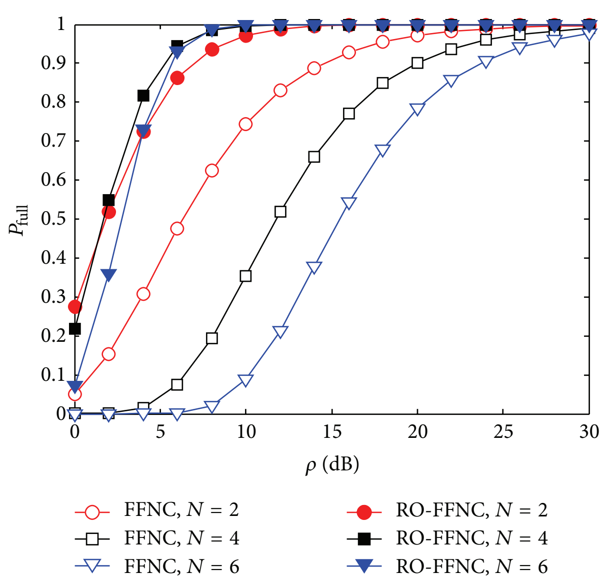


Figure 3: Full diversity probability comparisons when $N = 2, 4, 6$, $M = 2$.

results deduced in Section 4. The simulation parameters are as follows: simulation length is set as 100 million times per point. The channel is nonfrequency-selective Rayleigh block fading model. Assuming the reference distance is $d_{\text{ref}} = 5$ m, $\lambda_{\text{ref}} = 0.25$. Large-scale loss model is established as $\sigma^2/\sigma_{\text{ref}}^2 = (d_{\text{ref}}/d)^{\varphi} = \lambda_{\text{ref}}/\lambda$, where $\varphi = 3.5$ (typical urban) is the path loss factor. All the nodes are located in a symmetric network with $d_{Sj,D} = 30$ m, $d_{Si,rj} = d_{rj,D} = 15$ m, and $d_{ri,j} = 5$ m. Each node transmits at identical rate $R = 0.1$ with the same transmission power and $1/(M + N)$ channel degree of freedom.

Figure 2 depicts the full diversity probability comparisons of RO-FFNC and FFNC when $M = 2, 4, 6$ and $N = 2$.

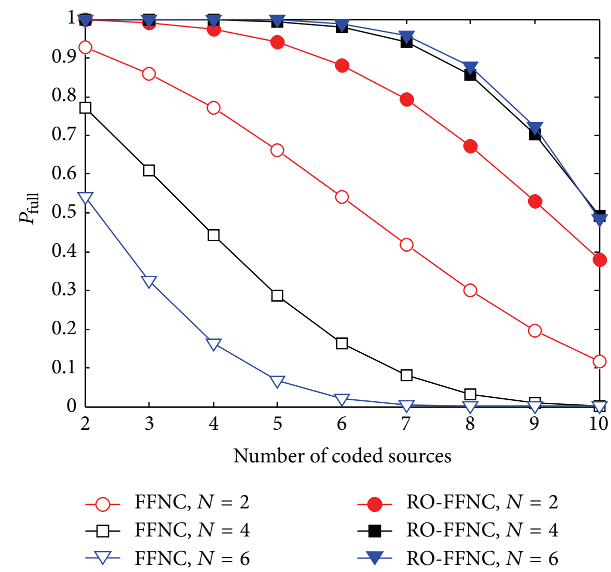


Figure 4: Impact of the source number on the full diversity probability when SNR = 16 dB.

As is shown, for both compared algorithms, P_{full} decreases when the source number M grows. This is reasonable because more signals are required to be successfully decoded simultaneously at relay. Obviously, there are always $P_{\text{full}}^{\text{RO-FFNC}} > P_{\text{full}}^{\text{FFNC}}$ under the same M. Besides, all the P_{full} tend to 1 with the increase of SNR, while $P_{\text{full}}^{\text{RO-FFNC}}$ grows more rapidly than $P_{\text{full}}^{\text{FFNC}}$. This result corresponds well with the analysis in Section 4 and indicates that RO-FFNC outperforms FFNC in the chance to attain full diversity.

Figure 3 shows the full diversity probability comparisons when $N = 2, 4, 6$ and $M = 2$ is alternately fixed. For FFNC, the growth of relay number N leads to the decrease of $P_{\text{full}}^{\text{FFNC}}$, because more relays are required to decode successfully. However, for RO-FFNC, it is interesting that $P_{\text{full}}^{\text{RO-FFNC}}$ do not drop similarly; on the contrary, when SNR > 4 dB, it shows that $P_{\text{full}}^{\text{RO-FFNC}}$ increases with the growth of relay number N, which is caused by the diffusion effect, because the more relays exist, the more interrelay links can be exploited to recover the outage ones. This merit indicates that for the network with RO-FFNC, it is an efficient method to improve the diversity order by simply increasing the relay number, while for FFNC, this will lead to the decrease of $P_{\text{full}}^{\text{FFNC}}$, so the efficiency suffers loss.

Figure 4 is the impact of the source number on the full diversity probability when SNR = 16 dB. For both compared algorithms, although the increase of source number leads to the drop of P_{full}, it is obvious that $P_{\text{full}}^{\text{FFNC}}$ suffers more loss. For example, when $P_{\text{full}} = 0.5$ and $N = 2$, FFNC only support 6 sources, while for RO-FFNC, 9 sources can be served. Another observation is that when N increases from 2 to 4 and 6, $P_{\text{full}}^{\text{FFNC}}$ decreases greatly, while on the contrary, $P_{\text{full}}^{\text{RO-FFNC}}$ achieves some improvement. This result is also due to the diffusion effect, so it also verifies the superior of RO-FFNC in the multiple-source multiple-relay network.

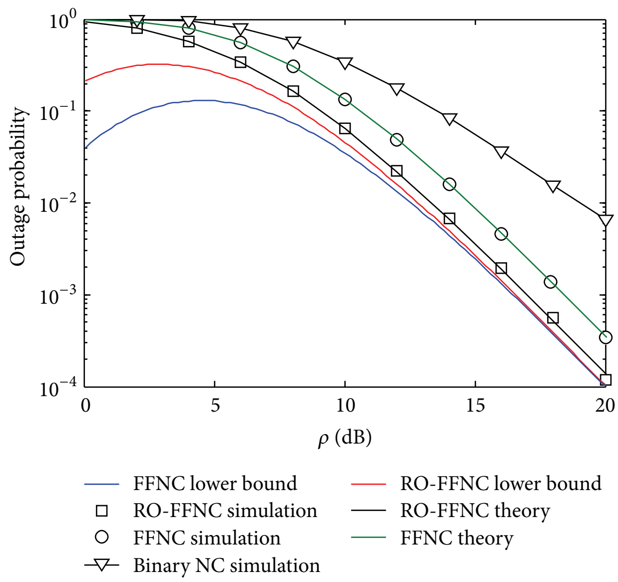


Figure 5: Outage probability comparisons for $M = N = 2$.

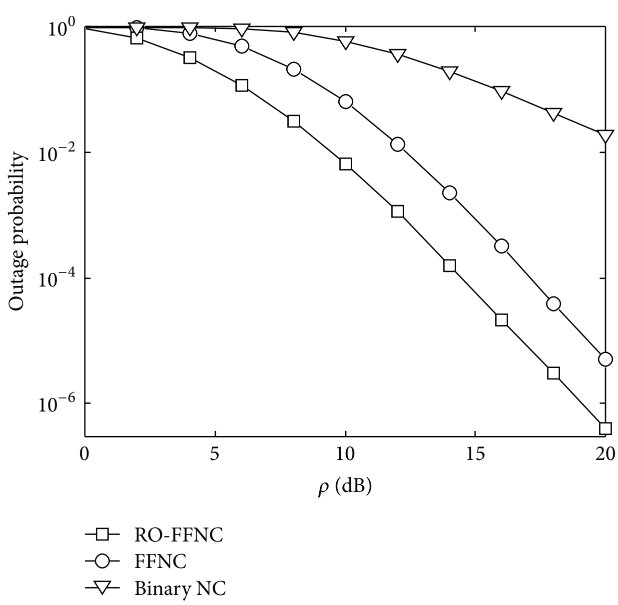


Figure 6: Outage probability comparisons for $M = 2$, $N = 4$.

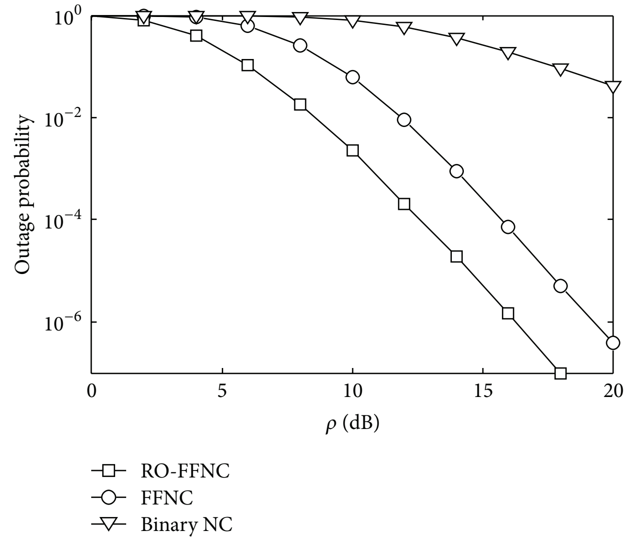


Figure 7: Outage probability comparisons for $M = 2$, $N = 6$.

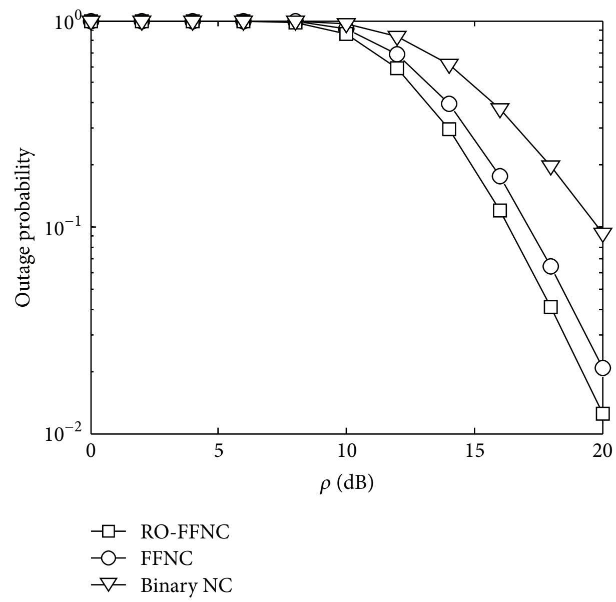


Figure 8: Outage probability comparisons for $M = 4$, $N = 2$.

Figures 5–7 depict the outage probability comparisons for $M = 2$, $N = 2$, 4, and 6, respectively. The outage of conventional binary NC is also shown as the baseline. In Figure 5, the closed-form outage probabilities of RO-FFNC and FFNC are also presented, as well as the corresponding lower bounds. It demonstrates that the theoretical curves essentially overlap the simulation results, and RO-FFNC outperforms FFNC by about 1.5 dB. Besides, the lower bound of RO-FFNC tends to be tight as SNR increases, and is always beyond the one of FFNC, which verifies that $P_{\text{full}}^{\text{RO-FFNC}} > P_{\text{full}}^{\text{FFNC}}$.

Figures 5–7 also show the achieved diversity order by the compared schemes. It can be observed from the curve slope that binary NC always achieves a constant diversity order of 2 when $N > 1$, which had been proved in [14]. Compared with it, RO-FFNC and FFNC both attain the full diversity order of $N + 1$. Besides, the relative gain of RO-FFNC over FFNC grows to 2.5 dB and 3.5 dB, respectively, in Figures 6 and 7. This improvement is achieved simply by the growth of N, and corresponds well with the analysis in Section 4 and the results in Figures 2–4.

Figure 8, Figure 9, and Figure 10 show the simulations of outage probability comparisons for $M = 4$, $N = 2$, 4, and 6, respectively. The situations are similar to the ones

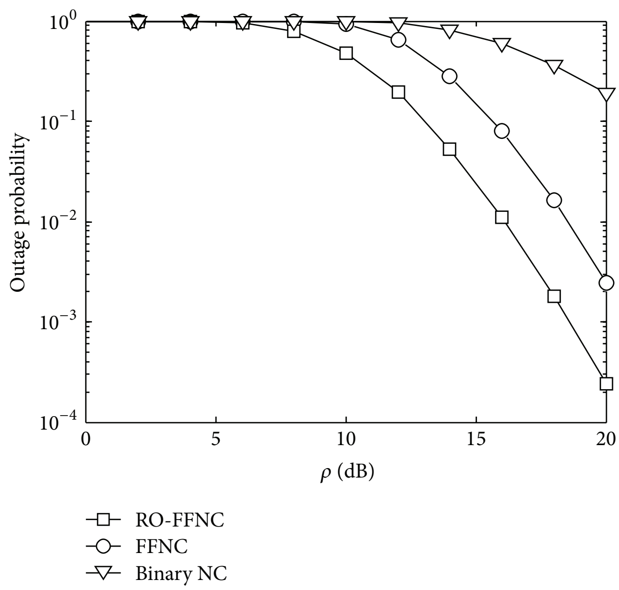


Figure 9: Outage probability comparisons for $M = 4$, $N = 4$.

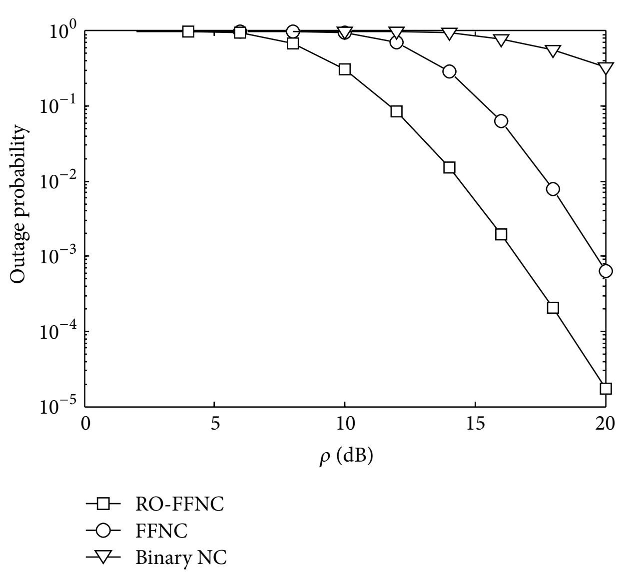


Figure 10: Outage probability comparisons for $M = 4$, $N = 6$.

in Figures 5–7. Although the growth of M deteriorates the outage performance, RO-FFNC is still able to supply full diversity. Moreover, the relative gain over FFNC becomes more considerable.

6. Conclusion

In this paper, we have proposed an RO algorithm based on the FFNC to improve the system outage performance for the multiple-source multiple-relay WSN. We have demonstrated that the proposed scheme can supply more opportunities for all the sources to access full diversity order. Besides, the outage probability and the corresponding full diversity probability of the proposed scheme have been deduced and compared with the ones of FFNC when $M = N = 2$. Simulations corresponded well with the theoretical result, and demonstrated that the proposed scheme is superior to FFNC with a variety of numbers of sources and relays. Besides, we also have verified that the diffusion effect of the RO-FFNC indeed improves the full diversity probability by simply increasing the relay number. This advantage makes the scheme very suitable for the multiple-relay WSN.

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