

Research Article

On the Performance of Quasiorthogonal STBC with Relay Selection and Phase Rotation Techniques for Decode and Forward Cooperative Communications

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The performance of quasiorthogonal space-time block code with relay-selection and phase-rotation techniques applied to cooperative communications for four communication nodes is investigated. Specifically, by applying relay-selection and phase-rotation techniques, a diversity gain of four can be achieved. In addition, a symbol error rate (SER) performance analysis with closed-form expression and power allocation are investigated and compared with simulation results. The results show that theoretical SER curves are close to the simulation results. In addition, a code rate of the proposed scheme is two times higher than ordinary cooperative communications. The computer simulation results also show a significant probability of error improvement of about 2.8 dB over the conventional decode-and-forward protocol.

1. Introduction

In recent years, the utilization of multiple antennas at transmitters and receivers has gained popularity due to the potential of increasing the system capacity [1]. The increased spectral efficiency of such systems is also important because the bandwidth is a precious commodity, and by using multiple antennas at transmitters and receivers, the spectral efficiency can be drastically increased. Systems with multiple transmit and multiple receive antennas, more commonly known as multiple-input multiple-output (MIMO) systems, can provide a spatial diversity gain. This gain is obtained by transmitting or receiving copies of a signal through different antennas. This is an effective approach to combat fading in wireless channels and to improve the performance of the communication system.

Recently, a generalized MIMO system, called a cooperative communication, has been proposed for realizing the advantages of the conventional MIMO system, for example, the diversity gain [2]. By means of the cooperation of the active users equipped with a single antenna in the wireless

networks, the generalized MIMO system can be established in a distributed fashion. In addition, the coverage range of such communication is also expanded, which results in lower power consumption for a particular user communicating with far-away destinations, and in turn prolongs the battery life.

Another approach for increasing the transmission rate is to employ a transmit diversity based on a space-time block code (STBC) technique [3]. However, the complex-valued STBC which provides a full code rate, and a full diversity gain does not exist for more than two transmit antennas [3]. In fact, orthogonal-STBC designed for more than two antennas can achieve full diversity gain, but its code rate is less than unity. On the other hand, quasi-orthogonal STBC (QO-STBC) [4], proposed for four transmit antennas, achieves the full code rate, but it suffers from a loss in diversity order due to a coupling effect between the symbols in the codeword.

Given the advantages of QO-STBC, it can be applied to cooperative communications for performance enhancement with relay-selection or phase-rotation techniques. The contributions of this paper are as follows.

- (i) It can be shown that the proposed QO-STBC decode-and-forward cooperative communication (QO-DF) system can enhance the performance of the system such that the diversity of four is achieved. In addition, the code rate of the proposed scheme is two times higher than the ordinary cooperative communications.
- (ii) The optimum and suboptimum power allocations are investigated. In addition, the system performance can be enhanced by adopting the optimum power allocation scheme.
- (iii) We analyze the theoretical symbol error rate and compare the theoretical results with the simulation results. It turns out that the theoretical and simulation results are close to each other, which could confirm the validity of the theoretical result.

The rest of this paper is organized as follows. In Section 2, we present a conventional decode-and-forward (DF) protocol for four-node cooperative communications. In Section 3, we describe the proposed QO-DF cooperative communications with relay-selection and phase-rotation techniques. The maximum ratio combining (MRC) and the signal-to-noise ratio (SNR) of the proposed system are described in Section 4. The theoretical SER analysis and optimum power allocation of the proposed system are described in Section 5. The simulation results compared with the theoretical results are shown in Section 6. Finally, we conclude this paper in Section 7.

2. System Model and Conventional Decode-and-Forward Protocol for Wireless Ad hoc Networks

In cooperative wireless communications, for example, wireless ad hoc networks, wireless users can cooperate with neighbouring users to form a generalized MIMO system with a coding scheme, for example, STBC, for enhancing the system performance, for example, a probability of error. The conventional DF cooperative communication system model with a single relay is described in [5]. However, for the sake of exposition, we consider cooperative communications in the case of a wireless network with two phases and four communication nodes (i.e., one user acts as a source node and the other four users act as relay nodes), and one destination node as shown in Figure 1.

In phase I, node 1 transmits a modulated signal to its destination, while nodes 2, 3, and 4 receive this transmitted signal due to the broadcast nature of wireless channels. In phase II, nodes 2, 3, and 4 will retransmit the received signal to node 1's destination in a DF fashion. Likewise, in the next communication periods, node 2, 3, or 4 will act as the source node, and the other users will act as the relay nodes, respectively. In both phases, all nodes transmit the signal through orthogonal channels using time-division multiplexing (TDMA). In this paper, we employ an M-PSK modulation scheme.

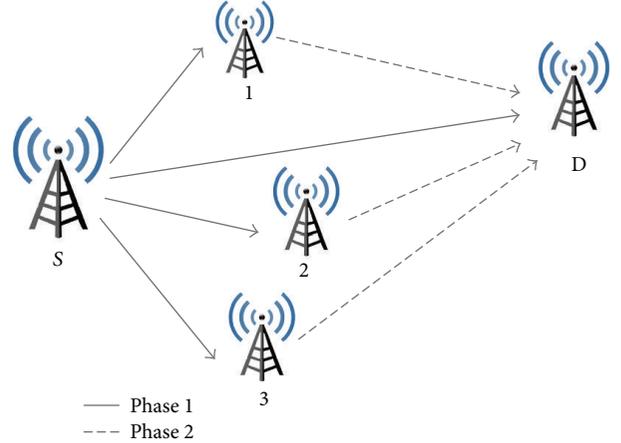


FIGURE 1: A system model of four-node cooperative communications.

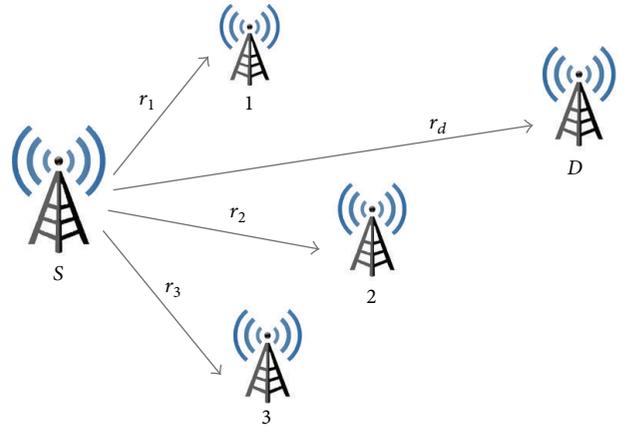


FIGURE 2: A source node broadcasts the transmit signal to relay nodes in phase I.

In phase I, the source node broadcasts its transmit signal to the destination and the relay nodes in the first time t_1 , as shown in Figure 2. The received signal expressions for phase I can be expressed as follows:

$$\begin{aligned}
 r_1 &= \sqrt{P_d} h_{s1} x + n, \\
 r_2 &= \sqrt{P_d} h_{s2} x + n, \\
 r_3 &= \sqrt{P_d} h_{s3} x + n, \\
 r_d &= \sqrt{P_d} h_{sd} x + n,
 \end{aligned} \tag{1}$$

where P_d is the transmit power of the source node, x is the transmitted signal from the source, r_1, r_2 , and r_3 are the received signal at relays 1, 2, and 3, respectively, and r_d is the received signal at the destination. h_{ij} is the channel impulse response from node i to j , and n is the additive white Gaussian noise (AWGN).

After relays 1, 2, and 3 received broadcasting signals from the source node, and decoded these signals using a Maximum-Likelihood (ML) receiver, the decoded symbols

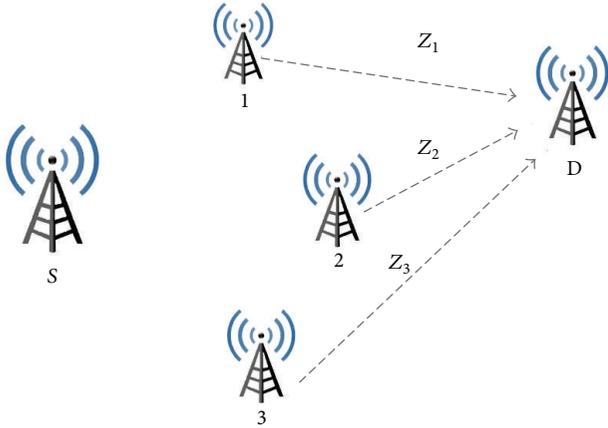


FIGURE 3: Each relay node sends the decoded signals to the destination node in phase II.

can be written as \tilde{x}_1 , \tilde{x}_2 , and \tilde{x}_3 , respectively. The relay nodes will modulate the decoded symbols and retransmit them to the destination node in phase II in the sequential time intervals t_2 , t_3 , and t_4 , respectively, as shown in Figure 3.

The expressions are as follows:

$$\begin{aligned} z_1 &= \sqrt{\frac{P_q}{3}} h_{1d} \tilde{x}_1 + n, \\ z_2 &= \sqrt{\frac{P_q}{3}} h_{2d} \tilde{x}_2 + n, \\ z_3 &= \sqrt{\frac{P_q}{3}} h_{3d} \tilde{x}_3 + n, \end{aligned} \quad (2)$$

where P_q is the transmit power of the relay nodes, z_1 , z_2 , and z_3 are the received signals at the destination node in phase II, which are sent by the relays 1, 2, and 3, respectively. At the destination node, the MRC is performed as follows [6]:

$$y = \sqrt{P_d} \frac{h_{sd}^*}{N_0} r_d + \sqrt{\frac{P_q}{3}} \frac{h_{1d}^*}{N_0} z_1 + \sqrt{\frac{P_q}{3}} \frac{h_{2d}^*}{N_0} z_2 + \sqrt{\frac{P_q}{3}} \frac{h_{3d}^*}{N_0} z_3, \quad (3)$$

where y is the combined received signal at the destination node and N_0 is the variance of noise. In phases I and II, we can observe that the DF cooperative communication uses four time slots to send one symbol so that the code rate is equal to 1/4. Some improvement could be made by properly using a space-time coding scheme in phase II.

3. The Proposed Quasi-Orthogonal STBC Decode-and-Forward Cooperative Communications

Now, we consider four cooperative communication nodes as a multiple-input single-output (MISO) communication system, as shown in Figure 4. We also consider the source and three relays in the cooperative communications as four transmit antennas in the MISO communication, and apply QO-STBC [4], as shown in Figure 5.

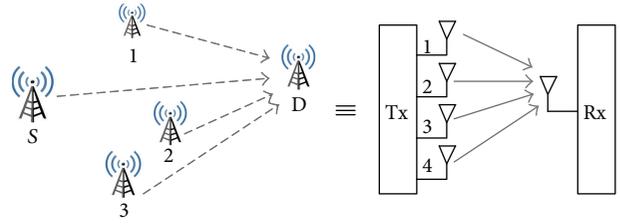


FIGURE 4: An equivalent system diagram of cooperative communications in phase II and a MISO system.

In phase I, a source node encodes four consecutive symbols to a QO-STBC codeword and broadcasts to the relay nodes and destination node in four time slots. Then the relay nodes will decode the received signals and send them individually to the destination node. We regard the cooperation in phase II of three relays and the source node as four transmit antennas for the QO-STBC scheme, in which relay 1, relay 2, relay 3, and the source act as the first antenna, second antenna, third antenna, and fourth antenna, respectively. Therefore, four data blocks are transmitted over four consecutive block intervals through four antennas using the following 4×4 QO-STBC code matrix [7],

$$C = \begin{array}{c} \text{Space} \\ \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{array} \\ \text{Time} \end{array} \quad (4)$$

where C is a QO-STBC code matrix. In addition, the channels can be modelled as a matrix of 4×1 , whose coefficients are the same as the frequency response of the channels h_{ij} . In the first block interval, the blocks s_1 , s_2 , s_3 , and s_4 are transmitted by transmitting a power of $P_q/4$ simultaneously from the first, second, third, and fourth antennas, respectively. The received signal corresponding to these blocks is expressed by r_1 . In a similar way, the blocks of $-s_2^*$, s_1^* , $-s_4^*$, and s_3^* are transmitted during the second block interval simultaneously over four antennas, and the corresponding received block is expressed by r_2 , and so on for the third and the fourth block intervals. The received signal blocks y_1 , y_2 , y_3 , and y_4 in the first, second, third, and fourth block intervals, respectively, can be written as

$$\begin{aligned} y_1 &= \sqrt{\frac{P_q}{4}} (h_{1d}s_1 + h_{2d}s_2 + h_{3d}s_3 + h_{sd}s_4) + n_1, \\ y_2 &= \sqrt{\frac{P_q}{4}} (-h_{1d}s_2^* + h_{2d}s_1^* - h_{3d}s_4^* + h_{sd}s_3^*) + n_2, \\ y_3 &= \sqrt{\frac{P_q}{4}} (-h_{1d}s_3^* - h_{2d}s_4^* + h_{3d}s_1^* + h_{sd}s_2^*) + n_3, \\ y_4 &= \sqrt{\frac{P_q}{4}} (h_{1d}s_4 - h_{2d}s_3 - h_{3d}s_2 + h_{sd}s_1) + n_4. \end{aligned} \quad (5)$$

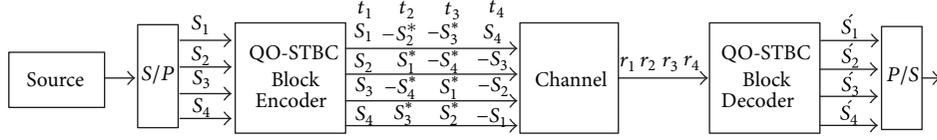


FIGURE 5: An equivalent block diagram of the proposed QO-STBC.

For the sake of simplicity, we replace h_{1d}, h_{2d}, h_{3d} , and h_{sd} by h_1, h_2, h_3 , and h_4 , respectively. Hence, we have

$$\begin{bmatrix} y_1 \\ y_2^* \\ y_3^* \\ y_4 \end{bmatrix} = \sqrt{\frac{P_q}{4}} \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix}. \quad (6)$$

We can derive (6) in a vector form as follows,

$$\mathbf{y}_k = \mathbf{h}_k \mathbf{s}_k + \mathbf{n}_k, \quad k = 1, 2, 3, 4. \quad (7)$$

At the receiver, the matched filtering is performed as follows [8]:

$$\bar{\mathbf{y}}_k = \mathbf{h}_k^H \mathbf{y}_k = \mathbf{h}_k^H \mathbf{h}_k \mathbf{s}_k + \mathbf{h}_k^H \mathbf{n}_k, \quad (8)$$

where \mathbf{h}_k^H is a Hermitian matrix of \mathbf{h}_k .

For the orthogonal STBC, \mathbf{h}_k would be a unitary matrix, and, hence, $\mathbf{h}_k^H \mathbf{h}_k$ would be a diagonal matrix. In this case, $\bar{\mathbf{y}}_k$ would provide an estimate of \mathbf{s}_k . However, for the case of four antennas as in the QO-STBC scheme, the matrix $\mathbf{h}_k^H \mathbf{h}_k$ is not diagonal, but it is in the following form:

$$\mathbf{h}_k^H \mathbf{h}_k = \begin{bmatrix} \beta & 0 & 0 & \alpha \\ 0 & \beta & -\alpha & 0 \\ 0 & -\alpha & \beta & 0 \\ \alpha & 0 & 0 & \beta \end{bmatrix}, \quad (9)$$

where

$$\beta = \sum_{k=1}^4 |h_k|^2, \quad (10)$$

$$\alpha = 2 \operatorname{Re} \{h_1 h_4^* - h_2 h_3^*\}.$$

It can be seen that the signal-to-noise ratio (SNR) can be maximized if α is forced to zero. For an orthogonal STBC design, α is zero regardless of the channel coefficient values. However, for the case of four antennas, we require some feedbacks from the receiver in order to force α to zero. The techniques to force a coupling term α to zero, and to obtain a diversity order of four are relay-selection and phase-rotation techniques.

3.1. Relay-Selection Technique. Assuming that symbols from each relay are sent and multiplied by real-valued variables θ_k , $k = 1, 2, 3, 4$, where the coefficients θ_k have a binary value $\{0, 1\}$, as explained below. The new coupling term becomes

$$\alpha = 2 \operatorname{Re} \{\theta_1 h_1 \theta_4 h_4^* - \theta_2 h_2 \theta_3 h_3^*\}. \quad (11)$$

The variables θ_k can be chosen such that

$$\begin{aligned} \text{If } |h_1|^2 > |h_4|^2, & \text{ then } \theta_1 = 1, \theta_4 = 0, \\ & \text{else } \theta_1 = 0, \theta_4 = 1, \\ \text{If } |h_2|^2 > |h_3|^2, & \text{ then } \theta_2 = 1, \theta_3 = 0, \\ & \text{else } \theta_2 = 0, \theta_3 = 1. \end{aligned} \quad (12)$$

The new received expressions can be expressed as follows:

$$\begin{aligned} y_1 &= \sqrt{\frac{P_q}{4}} (\theta_1 h_1 s_1 + \theta_2 h_2 s_2 + \theta_3 h_3 s_3 + \theta_4 h_4 s_4) + n_1, \\ y_2 &= \sqrt{\frac{P_q}{4}} (-\theta_1 h_1 s_2^* + \theta_2 h_2 s_1^* - \theta_3 h_3 s_4^* + \theta_4 h_4 s_3^*) + n_2, \\ y_3 &= \sqrt{\frac{P_q}{4}} (-\theta_1 h_1 s_3^* - \theta_2 h_2 s_4^* + \theta_3 h_3 s_1^* + \theta_4 h_4 s_2^*) + n_3, \\ y_4 &= \sqrt{\frac{P_q}{4}} (\theta_1 h_1 s_4 - \theta_2 h_2 s_3 - \theta_3 h_3 s_2 + \theta_4 h_4 s_1) + n_4. \end{aligned} \quad (13)$$

Then, the new matrix \mathbf{h}_k^H becomes

$$\mathbf{h}_k^H = \begin{bmatrix} \theta_1 h_1^* & \theta_2 h_2 & \theta_3 h_3 & \theta_4 h_4^* \\ \theta_2 h_2^* & -\theta_1 h_1 & \theta_4 h_4 & -\theta_3 h_3^* \\ \theta_3 h_3^* & \theta_4 h_4 & -\theta_1 h_1 & -\theta_2 h_2^* \\ \theta_4 h_4^* & -\theta_3 h_3 & -\theta_2 h_2 & \theta_1 h_1^* \end{bmatrix}. \quad (14)$$

This technique would force α to zero, and we could obtain a diversity order of four, while preserving the total transmitted power. Basically, this technique chooses the best two channels in the antenna pairs (1, 4) and (2, 3) according to the channel quality indicated at the receiver. The QO-STBC scheme is then applied to use the best antenna in each pair. Therefore, it provides a diversity gain of four.

3.2. Phase-Rotation Technique. Another way to force α to be zero is to use a phase-rotation approach. We consider that the symbols transmitted from the third and the fourth antennas are rotated by a common phasor $e^{j\theta}$. Note that this operation does not change the transmitted power. Since the phase rotation on transmitted symbols is effectively equivalent to rotating the phases of the corresponding channel coefficients, the new coupling term can be written as

$$\alpha = 2 \operatorname{Re} \{(h_1 h_4^* - h_2 h_3^*) e^{-j\theta}\}. \quad (15)$$

Let $\rho = (h_1 h_4^* - h_2 h_3^*)$, in order to force α to zero, the product of ρ and $e^{-j\theta}$ should be a complete imaginary number. This can be achieved when angle $(\rho) - \theta$ is either $-\pi/2$ or $\pi/2$. Therefore, θ is determined by

$$\theta = \frac{\pi}{2} - \text{angle}(\rho) \quad \text{or} \quad \frac{3\pi}{2} - \text{angle}(\rho). \quad (16)$$

Hence, the phase angle is limited to $\theta \in [-\pi/2, \pi/2]$. The new expressions for received signals in (13), respectively, can be expressed as follows:

$$\begin{aligned} y_1 &= \sqrt{\frac{P_q}{4}} (h_1 s_1 + h_2 s_2 + e^{j\theta} h_3 s_3 + e^{j\theta} h_4 s_4) + n_1, \\ y_2 &= \sqrt{\frac{P_q}{4}} (-h_1 s_2^* + h_2 s_1^* - e^{j\theta} h_3 s_4^* + e^{j\theta} h_4 s_3^*) + n_2, \\ y_3 &= \sqrt{\frac{P_q}{4}} (-h_1 s_3^* - h_2 s_4^* + e^{j\theta} h_3 s_1^* + e^{j\theta} h_4 s_2^*) + n_3, \\ y_4 &= \sqrt{\frac{P_q}{4}} (h_1 s_4 - h_2 s_3 - e^{j\theta} h_3 s_2 + e^{j\theta} h_4 s_1) + n_3. \end{aligned} \quad (17)$$

Then, the new matrix \mathbf{h}_k^H becomes

$$\mathbf{h}_k^H = \begin{bmatrix} h_1^* & h_2 & e^{j\theta} h_3 & e^{-j\theta} h_4^* \\ h_2^* & -h_1 & e^{j\theta} h_4 & -e^{-j\theta} h_3^* \\ e^{-j\theta} h_3^* & e^{j\theta} h_4 & -h_1 & -h_2^* \\ e^{-j\theta} h_4^* & -e^{j\theta} h_3 & -h_2 & h_1^* \end{bmatrix}. \quad (18)$$

Furthermore, the relay-selection and phase-rotation techniques can be applied to eight or more nodes, as described in [9].

4. Maximum Ratio Combining and Signal to Noise Ratio Analysis

In DF cooperative communications, a destination jointly combines the signal received from a source in phase I and the signal received from the relays in phase II by using the maximum ratio combining (MRC) method and detects the combined received symbols by using the ML receiver.

4.1. Maximum Ratio Combining. In the proposed system, we can use an MRC combiner at the destination node by combining a direct signal from the source in phase I and retransmitted signals in phase II. The MRC combining expressions are as follows:

$$\begin{aligned} u_1 &= w_d h_{sd}^* r_1 + w_q (y_1 h_1^* + y_2 h_2 + y_3 h_3 + y_4 h_4^*), \\ u_2 &= w_d h_{sd}^* r_2 + w_q (y_1 h_2^* - y_2 h_1 + y_3 h_4 - y_4 h_3^*), \\ u_3 &= w_d h_{sd}^* r_3 + w_q (y_1 h_3^* + y_2 h_4 - y_3 h_1 - y_4 h_2^*), \\ u_4 &= w_d h_{sd}^* r_4 + w_q (y_1 h_4^* - y_2 h_3 - y_3 h_2 + y_4 h_1^*), \end{aligned} \quad (19)$$

where u_1, u_2, u_3 , and u_4 are outputs of the MRC combiner to be used for decoding $\bar{s}_1, \bar{s}_2, \bar{s}_3$, and \bar{s}_4 , respectively, and

$$w_d = \frac{\sqrt{P_d}}{N_0}, \quad w_q = \frac{\sqrt{P_q/4}}{N_0}, \quad (20)$$

where w_d is a weighting coefficient of the direct signal from phase I and w_q is a weighting coefficient of the retransmit signal from phase II. We then employ an ML detector to detect $\bar{s}_1, \bar{s}_2, \bar{s}_3$, and \bar{s}_4 , respectively.

However, it is worth noting that the MRC combining yields the maximum SNR to (19), given that the estimated symbols $\bar{s}_1, \bar{s}_2, \bar{s}_3$, and \bar{s}_4 at the relay nodes are correctly decoded. Specifically, in practical applications, the correctness of $\bar{s}_1, \bar{s}_2, \bar{s}_3$, and \bar{s}_4 depends solely on the quality of the channel links from the source-to-relay link. Hence, the MRC combining cannot guarantee the maximum SNR, as mentioned in [6]. The most useful method for improving the performance of the proposed system is to employ a power allocation scheme, which will be described later.

4.2. Signal-to-Noise Ratio Analysis. In this section, we derive an expression of the SNR for the proposed QO-DF cooperative communication system. The SNR output of the MRC combiner at the destination node consists of both direct and relay signals. It can be expressed as follows [6]:

$$\gamma_{\text{QO-DF}} = \gamma_d + \gamma_q, \quad (21)$$

where $\gamma_{\text{QO-DF}}$ is the received SNR at the destination node, γ_d is the SNR of the direct signal in phase I, and γ_q is the SNR of the retransmitted signal in phase II. Assuming that the transmitted symbol of the direct signal in phase I and retransmitted signal in phase II have an average energy of 1, we can derive the SNR of the received signal in each phase as follows:

$$\begin{aligned} \gamma_d &= \frac{P_d |h_{sd}|^2}{N_0}, \\ \gamma_q &= \frac{(P_q/4) |h_1|^2 + (P_q/4) |h_2|^2}{N_0} \\ &\quad + \frac{(P_q/4) |h_3|^2 + (P_q/4) |h_4|^2}{N_0}. \end{aligned} \quad (22)$$

Hence, the total received SNR at the destination can be expressed as

$$\begin{aligned} \gamma_{\text{QO-DF}} &= \frac{P_d |h_{sd}|^2 + (P_q/4) |h_1|^2 + (P_q/4) |h_2|^2}{N_0} \\ &\quad + \frac{(P_q/4) |h_3|^2 + (P_q/4) |h_4|^2}{N_0} \end{aligned} \quad (23)$$

5. Symbol Error Rate Analysis and Optimum Power Allocation

In this section, we consider the symbol error rate (SER) performance analysis of the proposed QO-DF communication system with the M-PSK modulation scheme. First, we consider the SER of the M-PSK signal of the relays in phase I. Let Pe_1 , Pe_2 , and Pe_3 be incorrect decoding probabilities per a symbol of source to relay 1, source to relay 2, and source to relay 3, respectively. According to the SNR analysis in the previous section, we can obtain the SER expression of each relay node in phase I as follows [10]:

$$\begin{aligned} Pe_1 &= \Psi(\gamma_{S1}) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} P_d |h_{s1}|^2}{N_0 \sin^2 \theta}\right) d\theta, \\ Pe_2 &= \Psi(\gamma_{S2}) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} P_d |h_{s2}|^2}{N_0 \sin^2 \theta}\right) d\theta, \\ Pe_3 &= \Psi(\gamma_{S3}) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} P_d |h_{s3}|^2}{N_0 \sin^2 \theta}\right) d\theta, \end{aligned} \quad (24)$$

where $b_{\text{PSK}} = \sin^2(\pi/M)$ and $M = 2^k$ with k even.

Over the Rayleigh fading, we average channels h_{s1} , h_{s2} , and h_{s3} with variances δ_{s1}^2 , δ_{s2}^2 , and δ_{s3}^2 , respectively. Since the fading channels h_{s1} , h_{s2} , and h_{s3} are independent of each other, we can express the incorrect decoding probability of each relay as [5]

$$\begin{aligned} Pe_1 &= F_1\left(1 + \frac{b_{\text{PSK}} P_d \delta_{s1}^2}{N_0 \sin^2 \theta}\right), \\ Pe_2 &= F_1\left(1 + \frac{b_{\text{PSK}} P_d \delta_{s2}^2}{N_0 \sin^2 \theta}\right), \\ Pe_3 &= F_1\left(1 + \frac{b_{\text{PSK}} P_d \delta_{s3}^2}{N_0 \sin^2 \theta}\right), \end{aligned} \quad (25)$$

where

$$F_1(x(\theta)) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \frac{1}{x(\theta)} d\theta. \quad (26)$$

5.1. The Proposed Cooperative Strategy. According to the received signals in phase I, in reality, the relay nodes do not always correctly decode the transmitted symbols. For setting the cooperative protocol strategy in phase II, the relay nodes are assumed to be capable of deciding whether or not it has decoded correctly. This could be achieved through cyclic redundancy check (CRC) codes or approaches by setting an SNR threshold at the relay nodes [11]. In addition, we also assume short-term statistics of the channels [6, 12], that is, channel variances, within a certain period of time to be known to the source node.

For the proposed QO-DF protocol, if no relay incorrectly decodes the symbols, all relays forward the decoded symbols to the destination by quasi-orthogonal space-time coding;

TABLE 1: Cooperative strategy.

Cooperation protocol	No. of incorrect relays	Total received SNR (γ_{total})
Direct signal only (noncooperative)	3	$\gamma_{\text{noncooperative}}$
1-relay cooperative DF	2	$\gamma_{1\text{-relay}}$
2-relay cooperative DF	1	$\gamma_{2\text{-relay}}$
3-relay cooperative QO-DF	0	$\gamma_{\text{QO-DF}}$

otherwise, only the relays that correctly decode the symbols forward them to the destination by a conventional DF method. The total received SNR at the destination depends on the number of relays decoded whose symbols are correct. The cooperative protocol strategy is proposed in Table 1.

This cooperative strategy is expected to achieve a performance diversity order of four. In order to achieve a diversity of order four, all relays have to decode the symbols correctly. The total SER of the proposed QO-DF system can be written as

$$Pe_{\text{total}} = \Psi(\gamma_{\text{total}}) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} \gamma_{\text{total}}}{N_0 \sin^2 \theta}\right) d\theta, \quad (27)$$

in which Pe_{total} greatly depends on the SNR of the cooperative protocol strategy. We can readily express (27) as follows:

$$\begin{aligned} Pe_{\text{total}} &= \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} \gamma_{\text{noncooperative}}}{N_0 \sin^2 \theta}\right) d\theta \\ &\quad \times \text{probability of 3 relays error} \\ &\quad + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} \gamma_{1\text{-relay}}}{N_0 \sin^2 \theta}\right) d\theta \\ &\quad \times \text{probability of 2 relays error} \\ &\quad + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} \gamma_{2\text{-relay}}}{N_0 \sin^2 \theta}\right) d\theta \\ &\quad \times \text{probability of 1 relay error} \\ &\quad + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} \gamma_{\text{QO-DF}}}{N_0 \sin^2 \theta}\right) d\theta \\ &\quad \times \text{probability of none relay error.} \end{aligned} \quad (28)$$

The first component shows the signal received at the destination which is a noncooperative scheme; the second component shows a 1-relay cooperative scheme; the third component shows a 2-relay cooperative scheme; the fourth component shows a 3-relay cooperative with quasi-orthogonal space-time coding.

According to the incorrect decoding probability of each relay in phase I, we can obtain the conditional SER of the proposed QO-DF protocol as follows:

$$\begin{aligned}
Pe_{\text{total}} = & \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} P_d |h_{sd}|^2}{N_0 \sin^2 \theta}\right) d\theta \\
& \times [1 - (1 - Pe_1)^4] [1 - (1 - Pe_2)^4] \\
& \times [1 - (1 - Pe_3)^4] \\
& + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} (P_d |h_{sd}|^2 + P_q |h_1|^2)}{N_0 \sin^2 \theta}\right) d\theta \\
& \times (1 - Pe_1)^4 [1 - (1 - Pe_2)^4] [1 - (1 - Pe_3)^4] \\
& + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} (P_d |h_{sd}|^2 + P_q |h_2|^2)}{N_0 \sin^2 \theta}\right) d\theta \\
& \times (1 - Pe_2)^4 [1 - (1 - Pe_1)^4] [1 - (1 - Pe_3)^4] \\
& + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} (P_d |h_{sd}|^2 + P_q |h_3|^2)}{N_0 \sin^2 \theta}\right) d\theta \\
& \times (1 - Pe_3)^4 [1 - (1 - Pe_1)^4] [1 - (1 - Pe_2)^4] \\
& + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} (P_d |h_{sd}|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/2) |h_1|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/2) |h_2|^2}{N_0 \sin^2 \theta}\right) d\theta \\
& \times (1 - Pe_1)^4 (1 - Pe_2)^4 [1 - (1 - Pe_3)^4] \\
& + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} (P_d |h_{sd}|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/2) |h_1|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/2) |h_3|^2}{N_0 \sin^2 \theta}\right) d\theta \\
& \times (1 - Pe_1)^4 (1 - Pe_3)^4 [1 - (1 - Pe_2)^4] \\
& + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} (P_d |h_{sd}|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/2) |h_2|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/2) |h_3|^2}{N_0 \sin^2 \theta}\right) d\theta
\end{aligned}$$

$$\begin{aligned}
& \times (1 - Pe_2)^4 (1 - Pe_3)^4 [1 - (1 - Pe_1)^4] \\
& + \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b_{\text{PSK}} (P_d |h_{sd}|^2 + (P_q/4) |h_1|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/4) |h_2|^2 (P_q/4) |h_3|^2}{N_0 \sin^2 \theta}\right. \\
& \quad \left. + \frac{(P_q/4) |h_4|^2}{N_0 \sin^2 \theta}\right) d\theta \\
& \times (1 - Pe_1)^4 (1 - Pe_2)^4 (1 - Pe_3)^4, \tag{29}
\end{aligned}$$

where $(1 - Pe_i)^4$ is a correctly decoding probability of QO-STBC codeword at relay i th. In a similar way, $(1 - Pe_i)^4$ is a chance of incorrectly decoding. Furthermore, at higher SNR, we can approximate $1 - (1 - Pe_i)^4 \approx 4Pe_i$.

In the following analysis, we average the channels in phase II over the Rayleigh fading as in phase I. We set up h_{sd}, h_1, h_2, h_3 , and h_4 having variance of $\delta_{sd}^2, \delta_1^2, \delta_2^2, \delta_3^2$, and δ_4^2 , respectively. We are able to obtain the approximate SER equation of the proposed QO-DF cooperative communications with M-PSK modulation in a full closed-form expression as follows:

$$\begin{aligned}
Pe_{\text{total}} = & 64F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{sd}^2}{N_0 \sin^2 \theta}\right) F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s1}^2}{N_0 \sin^2 \theta}\right) \\
& \times F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s2}^2}{N_0 \sin^2 \theta}\right) F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s3}^2}{N_0 \sin^2 \theta}\right) \\
& + 16F_1 \left[\left(1 + \frac{b_{\text{PSK}} P_d \delta_{sd}^2}{N_0 \sin^2 \theta}\right) \left(1 + \frac{b_{\text{PSK}} P_q \delta_1^2}{N_0 \sin^2 \theta}\right)\right] \\
& \times F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s2}^2}{N_0 \sin^2 \theta}\right) F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s3}^2}{N_0 \sin^2 \theta}\right) \\
& \times \left[1 - 4F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s1}^2}{N_0 \sin^2 \theta}\right)\right] \\
& + 16F_1 \left[\left(1 + \frac{b_{\text{PSK}} P_d \delta_{sd}^2}{N_0 \sin^2 \theta}\right) \left(1 + \frac{b_{\text{PSK}} P_q \delta_2^2}{N_0 \sin^2 \theta}\right)\right] \\
& \times F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s1}^2}{N_0 \sin^2 \theta}\right) F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s3}^2}{N_0 \sin^2 \theta}\right) \\
& \times \left[1 - 4F_1 \left(1 + \frac{b_{\text{PSK}} P_d \delta_{s2}^2}{N_0 \sin^2 \theta}\right)\right] \\
& + 16F_1 \left[\left(1 + \frac{b_{\text{PSK}} P_d \delta_{sd}^2}{N_0 \sin^2 \theta}\right) \left(1 + \frac{b_{\text{PSK}} P_q \delta_3^2}{N_0 \sin^2 \theta}\right)\right]
\end{aligned}$$

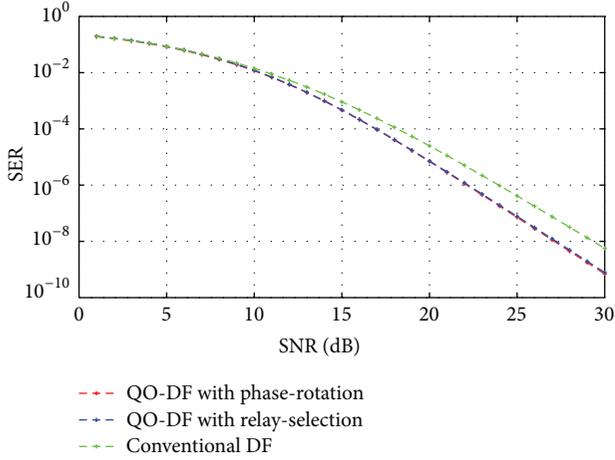


FIGURE 6: SER performance comparison between the proposed QO-DF system and the conventional DF system.

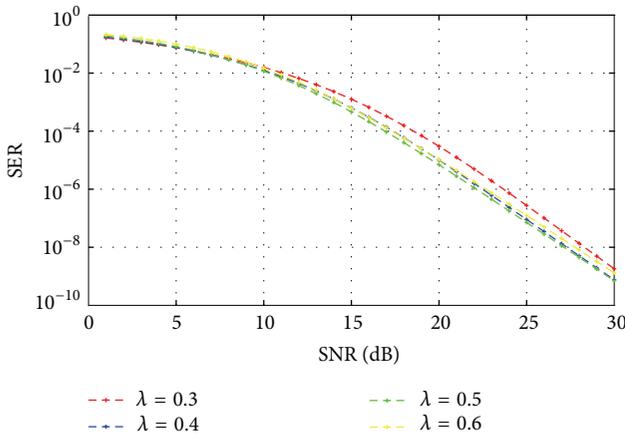


FIGURE 7: SER performance comparison of the proposed QO-DF systems with various values of λ ($\delta_{sr}^2 = \delta_{rd}^2 = 1$).

Specifically, from the SER approximation in (31), we observe that the link between source and destination contributes diversity order of one in the system performance. The cooperation strategy in the second phase also contributes diversity order of four in the system performance. In addition, it depends on the balance of the four channel links from the source to the relays and from the relays to the destination. Therefore, the proposed QO-DF cooperation systems show an overall performance of diversity order of four.

5.2. Optimum Power Allocation. It is common in cooperative communications that the channel variances between source to destination link, source to relay link, and relay to destination link are independent of each other. The MRC expressions in (19) cannot guarantee the maximum of total SNR at the destination. Hence, the power allocation objective is to minimize the approximated SER with respect to users' power, subject to a fixed total power constraint. The concept

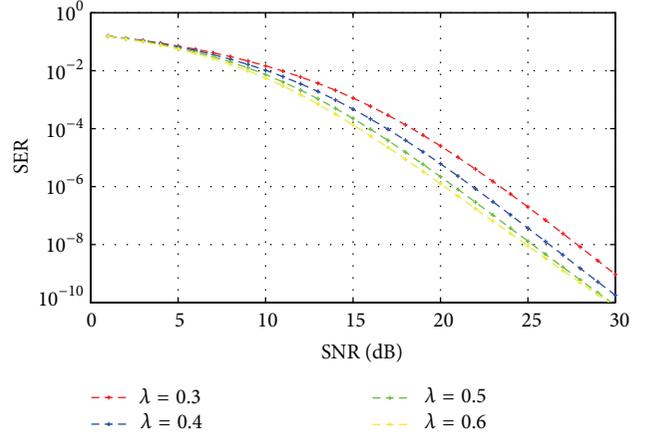


FIGURE 8: SER performance comparison of the proposed QO-DF systems with various values of λ ($\delta_{sr}^2 = 10, \delta_{rd}^2 = 1$).

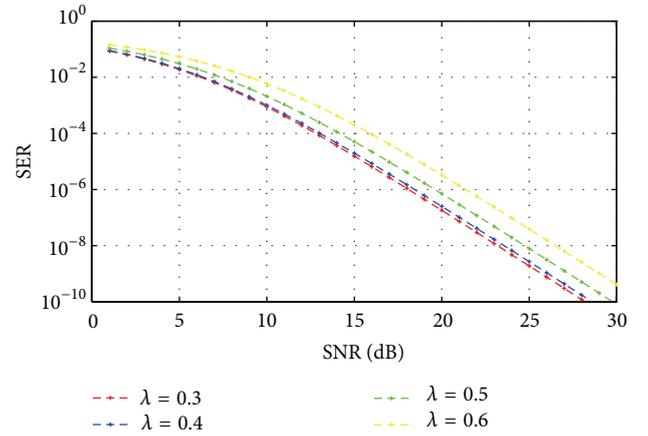


FIGURE 9: SER performance comparison of the proposed QO-DF systems with various values of λ ($\delta_{sr}^2 = 1, \delta_{rd}^2 = 10$).

of power allocation is that the quality of the decoded symbols $\tilde{s}_1, \tilde{s}_2, \tilde{s}_3$, and \tilde{s}_4 greatly depends on the channel variances of both phase I and phase II. If we define the transmit power P_d for the source and P_q is the transmit power for the relays, for a fixed total transmission power of $P_d + P_q = P_t$, where P_t is the total transmit power, then we can write the power allocation condition as

$$P_t = (1 - \lambda) P_d + \lambda P_q, \quad \text{at } 0 < \lambda < 1, \quad (32)$$

where λ is a power allocation factor between P_d and P_q .

According to the SER expression in (30), we are able to heuristically search for optimum power allocation by replacing P_q with λP_d and P_d with $(1 - \lambda) P_d$. In our study, we use computer simulations to validate the optimum power allocation concept.

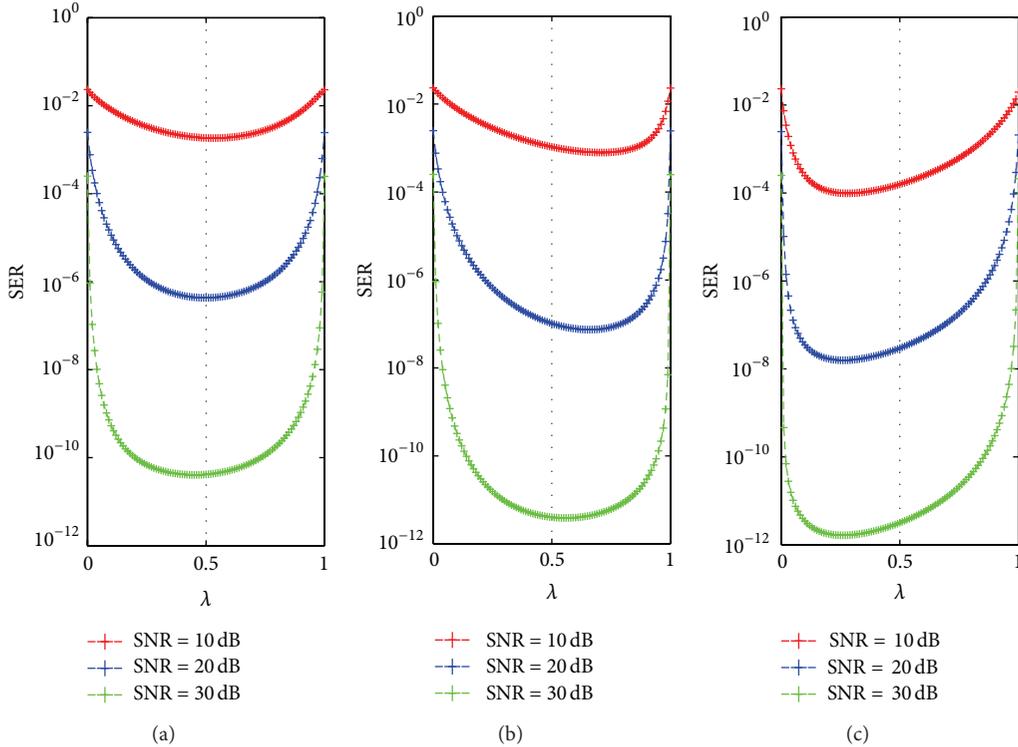


FIGURE 10: SER performance comparison of QO-DF system with various values of λ (a) $\delta_{sr}^2 = 1, \delta_{rd}^2 = 1$ (b) $\delta_{sr}^2 = 10, \delta_{rd}^2 = 1$ (c) $\delta_{sr}^2 = 1, \delta_{rd}^2 = 10$.

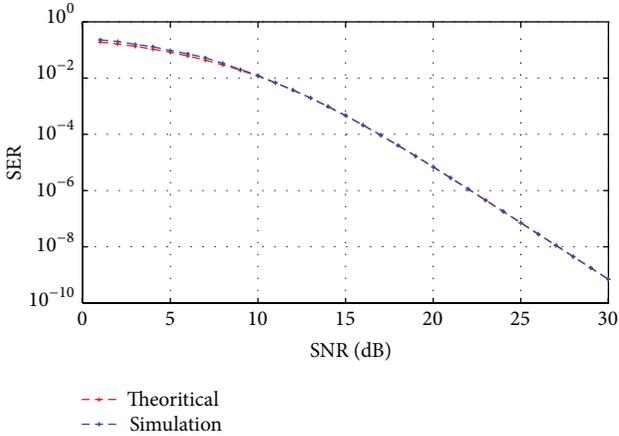


FIGURE 11: SER comparison of the proposed QO-DF system between theoretical analysis and simulation result.

6. Simulation and Results

In this section, based on computer simulations by MATLAB software, performance evaluations of the proposed QO-DF protocols are examined. For the sake of comparison, the conventional DF protocol for four communication nodes is also tested. The BPSK modulation with a total transmitted power has average energy 1, a variance of a noise is N_0 and bandwidth efficiency is 1 bit/s/Hz. In addition, Jake’s model [13] is employed with a normalize doppler shift of 5,000 Hz

TABLE 2: Cooperative protocol code rate.

Cooperation protocol	Code rate
Direct signal only (noncooperative)	1
1-relay cooperative DF	1/2
2-relay cooperative DF	1/3
3-relay cooperative QO-DF	1/2

for simulating Rayleigh fading channels, and we also assume all channel link variances in the system as appropriately balanced, that is, $\delta_{sr}^2 = \delta_{sd}^2 = \delta_{rd}^2 = 1$.

Figure 6 shows that SER performance of the proposed QO-DF system is better than the conventional four communication nodes DF system. Both of the proposed QO-DF systems, with relay-selection and phase-rotation techniques, have the SNR difference in comparison with the conventional DF system, specifically, 0.8 dB and 0.9 dB at BER of 10^{-5} , and then achieve 2.7 dB and 2.8 dB SNR difference at BER of 10^{-7} , respectively. The code rate of the proposed QO-DF system is shown in Table 2. In addition, the proposed QO-DF system will achieve a code rate two times higher than that of conventional DF in the case when no relay incorrectly decodes the symbols.

Next, we studied the effect of the channel qualities between source to relay and relay to destination for the optimum power allocation strategy. Figures 7–9 show the SER simulation results of the proposed QO-DF system with λ changing in the range of 0.3 to 0.6.

Figure 7 shows that, when channel variances of source to relay are equal to channel variances of relay to destination, that is, $\delta_{sr}^2 = \delta_{rd}^2$, at SNR < 7 dB, $\lambda = 0.3$ gives the lowest results of SER, and, at SNR > 7 dB, $\lambda = 0.5$ gives the lowest results of SER. The best average SER result for whole SNR range is at $\lambda = 0.5$.

Figure 8 shows that, when channel variances of source to relay are higher than channel variances of relay to destination, that is, $\delta_{sr}^2 \gg \delta_{rd}^2$, $\lambda = 0.6$ gives the lowest results of SER.

Figure 9 shows that, when channel variances of source to relay are lower than channel variances of relay to destination, that is, $\delta_{sr}^2 \ll \delta_{rd}^2$, $\lambda = 0.3$ gives the lowest results of SER.

We can summarize the strategy of power allocation to the proposed QO-DF system as follows. If the link qualities of source to relay are higher than the relay to destination, in (32), P_d goes to 0, and P_q goes to P_t . This implies that we should put more power at the relay nodes and less power at the source node. On the other hand, if the link qualities of source to relay are lower than those of the relay to destination link, P_d goes to P_t and P_q goes to 0. This implies that we should use almost all the power P_t at the source node, and use less power at the relay nodes. In addition, when the link qualities are approximately equal, we should put almost equal power at the source and the relay nodes.

For a high SNR case, we can observe the effect of the channel qualities on the power allocation strategy, as in Figure 10, by plotting the exact SER as a function of λ at SNR = 10 dB, 20 dB, and 30 dB with (a) $\delta_{sr}^2 = 1, \delta_{rd}^2 = 1$ (b) $\delta_{sr}^2 = 10, \delta_{rd}^2 = 10$, and (c) $\delta_{sr}^2 = 1, \delta_{rd}^2 = 10$. They show that the optimum BER results of all the different channel variances are not much different at $\lambda = 0.5$. Therefore, it is reasonable to adopt the equal power allocation scheme, that is, $\lambda = 0.5$, as a suboptimum power allocation, which in turn results in a simple power allocation strategy in the case of no available channel feedback.

In Figure 11, we present an SER comparison of the proposed QO-DF with phase-rotation technique between the theoretical SER and the simulation SER. These curves show that the theoretical result performs close to the simulation result.

7. Conclusion

In this paper, we have proposed a QO-DF cooperative communication system for four communication nodes in wireless cooperative communications, which could be well applied to wireless ad hoc networks. We also derived the theoretical SER, and compared the results with the simulation results. The theoretical SER shows a closed result to the simulated results. Furthermore, the proposed system achieves the full diversity of four by virtue of increasing several signal transmissions in the relaying phase. The optimum power allocation has also been investigated. In addition, it turns out that an equal power allocation could be used as a suboptimum power allocation for a slight SER degradation penalty. From simulation results, we can observe that the performance of the proposed schemes is significantly better than the conventional DF protocol. Another advantage of

the proposed scheme is that it uses less time for signal transmission in the relaying phase so that the code rate is two times higher than the conventional DF system. Hence, this proposed protocol is suitable for future multimedia wireless communication.

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