

## Research Article

# Distributed Compressed Spectrum Sensing via Cooperative Support Fusion

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Spectrum sensing in wideband cognitive radio (CR) networks faces several significant practical challenges, such as extremely high sampling rates required for wideband processing, impact of frequency-selective wireless fading and shadowing, and limitation in power and computing resources of single cognitive radio. In this paper, a distributed compressed spectrum sensing scheme is proposed to overcome these challenges. To alleviate the sampling bottleneck, compressed sensing mechanism is used at each CR by utilizing the inherent sparsity of the monitored wideband spectrum. Specifically, partially known support (PKS) of the sparse spectrum is incorporated into local reconstruction procedure, which can further reduce the required sampling rate to achieve a given recovery quality or improve the quality given the same sampling rate. To mitigate the impact of fading and shadowing, multiple CRs exploit spatial diversity by exchanging local support information among them. The fused support information is used to guide local reconstruction at individual CRs. In consideration of limited power per CR, local support information percolates over the network via only one-hop local information exchange. Simulation results testify the effectiveness of the proposed scheme by comparing with several existing schemes in terms of detection performance, communication load, and computational complexity. Moreover, the impact of system parameters is also investigated through simulations.

## 1. Introduction

Spectrum sensing, whose objectives are detecting signal of licensed users (LUs) and identifying the spectrum holes for dynamic spectrum access (DSA) [1], is an important enabling technology for cognitive radio, a leading choice for efficient utilization of spectrum resource [2–4]. In wideband cognitive radio networks, cognitive radio could attain more spectrum access opportunities in wideband regime. On the other hand, the task of wideband spectrum sensing entails several major challenges, such as very high signal acquisition cost in wideband scenario, uncertain channel fading and random shadowing, and limitation in power and computational capability per CR.

To alleviate the heavy pressure on the conventional analog to digital converter (ADC) technology, compressed sensing (CS) theory [5–7] has been introduced into the application of wideband spectrum sensing by utilizing the low percentage of spectrum occupancy, a fact that motivates dynamic spectrum

access [8–10]. CS theory states that sparse signal can be reconstructed from much fewer samples than suggested by the Shannon-Nyquist sampling theorem. However, a single CR may fail to detect hidden terminals or LUs due to shadowing or deep fading. In order to alleviate this problem, cooperative spectrum sensing (CSS) [3, 4] that exploits the built-in spatial diversity among multiple CRs has been proposed for CR networks. In addition, cooperation is especially useful for CS-based approaches since compressed reconstruction is quite susceptible to noise and the performance degrades severely when the signal to noise ratio (SNR) is low. Based on how cooperative CRs share their sensing data in the network, CSS can be classified into two categories: centralized CSS and distributed CSS. In centralized CSS scheme, a fusion center (FC) is required to collect measurements from all CRs and make centralized sensing decisions. Centralized CSS schemes using CS theory are presented in [11, 12]. The performance achieves global optimization; however, the incurred power cost and communication load in transmitting local information to the FC and conveying centralized sensing decisions

back to CRs are significantly high. Besides, the network is sensitive to node failure. Unlike centralized CSS, distributed CSS does not rely on a FC for making global decision. In this case, CRs communicate among themselves and converge to a unified decision on the spectrum occupancy by iterations. Distributed scheme reduces the communication workload significantly and is robust to the failure of node and reporting channel. CS-based distributed CSS schemes have been studied in [13, 14]. In [13], each CR makes local decision on spectrum occupancy based on the local reconstruction, and then the global decision, which is the average value of local decisions, is computed in a distributed manner using the consensus averaging technique [15]. This scheme is simple to implement but does not take full advantage of the joint sparsity structure [16, 17] that local spectrum of different CRs shares the same sparsity pattern. In [14], the CR networks iteratively solve the centralized formulation using distributed implementation. Upon convergence, each CR achieves near-optimal performance. However, due to the strict consensus-enforcing constraint, this scheme converges slowly, which makes the communication load and computational complexity very high.

In this paper, a new distributed compressed spectrum sensing scheme, called support fusion-based distributed compressed spectrum sensing (SF-DCSS), is proposed. This approach alternates between local compressed reconstruction at each CR and adaptive learning of support knowledge in distributed manner among all CRs. During each iteration, each CR reconstructs local sparse spectrum through truncated  $l_1$  minimization by incorporating the support information obtained from the previous iteration. Different from traditional CS in which sparsity is the only prior information on signal characteristic, further exploitation of signal structure, such as partial support knowledge [18–23], would enhance recovery performance. To collect spatial diversity and obtain reliable support information, each CR obtains local support detection via thresholding the local reconstruction, then exchanges local support information with its one-hop neighbors, and finally obtains the same fused support information after several rounds. Using these basic operations, all CRs update their support information to be used in the next iteration. In the proposed SF-DCSS scheme, fused support information is used to guide local reconstructions at all CRs, instead of imposing strict consensus-enforcing constraint such as [14], and it is expected to expedite convergence and consequently reduce the computational complexity and communication workload. The efficiency of the proposed SF-DCSS scheme is testified by comparing with the scheme in [13] and CS-based centralized CSS scheme and CS-based distributed CSS scheme reported in [14], in terms of detection performance of the CR network, communication load, and computational complexity per CR. Moreover, the impacts of system parameters such as SNR, the number of cooperating CRs, and the compression ratio are also studied through simulations.

The remaining part of this paper is organized as follows. The signal model and the spectrum sensing problem of interest are introduced in Section 2. Section 3 presents the details of the proposed scheme of distributed compressed spectrum

sensing. Numerical experiments are given to demonstrate the performance of the proposed approach in Section 4 and some conclusions are drawn in Section 5.

## 2. Signal Modeling and Problem Statement

Consider a licensed communication system operating over a wideband spectrum that is divided into  $N$  nonoverlapping narrowband subbands (also known as slots [13, 14]). Suppose that  $I$  spatially distributed CRs collaboratively sense the wideband spectrum and the division scheme of the monitored wideband spectrum is known to all CRs, which are ideally synchronized. However, the power spectral density (PSD) of each subband is dynamically varying due to its occupancy status caused by the LUs. Without loss of generality, we assume that the licensed system uses frequency division multiple access (FDMA) technology such that there is at most one active LU on each slot.

Suppose that the Nyquist-rate discrete form of received signal at  $i$ th CR is denoted by an  $N \times 1$  vector  $\mathbf{r}_t^{(i)}$  and its corresponding frequency-domain discrete version is denoted by  $\mathbf{r}_f^{(i)}$ , which is given by

$$\mathbf{r}_f^{(i)} = \mathbf{G}^{(i)} \mathbf{s}_f, \quad (1)$$

where the  $N \times 1$  vector  $\mathbf{s}_f$  denotes transmitted spectrum of LUs and  $\mathbf{G}^{(i)} = \text{diag}(g^{(i)})$  is an  $N \times N$  channel gain matrix whose  $n$ th diagonal element is determined by distance and channel fading gain between the LU transmitter using  $n$ th subband and the  $i$ th CR receiver.

It is shown by many investigations that the radio spectrum is in a very low utilization ratio [1–3]. Therefore, it is reasonable, by using sparse structure, to model the received spectrum of individual CR in a particular time and geographical area, which suggests that  $\mathbf{r}_f^{(i)}$  ( $i = 1, \dots, I$ ) is sparse. This fact is exactly the motivation for introducing CS theory into wideband spectrum sensing. Moreover, the nonzero entries of received spectrum at each CR are usually in a large dynamic range [10, 12] due to the fading wireless environment. For the same frequency component, it may be too small to detect at some CRs but relatively large at others. These signal characteristics may cause high miss detection probability for single CR, which is the reason that cooperation among multiple spatially distributed CRs is useful in combating many random factors, such as ambient noise and uncertain channel fading/shadowing.

In addition to sparsity at individual CR user, it is worth noting that different CRs share the same nonzero support (known as joint sparsity structure [16, 17]), provided that the channel does not experience deep fades [13], even though the nonzero values may be different at individual CRs. This is due to the fact that all CRs are affected by the same LUs as stated in (1).

Aiming at breaking the sampling bottleneck, CS theory has been introduced into the application of spectrum sensing. The first step in CS-based spectrum sensing is to collect time-domain compressed measurements at individual CR. The

mathematical representation of compressed measurements at  $i$ th CR can be expressed as follows:

$$y_t^{(i)} = \Phi^{(i)} \mathbf{r}_t^{(i)} = \Phi^{(i)} \mathbf{F}_N^{-1} \mathbf{r}_f^{(i)}, \quad (2)$$

where  $\mathbf{F}_N^{-1}$  denotes the inverse DFT matrix of size  $N$  and  $\Phi$  is an  $M \times N$  measurement matrix with  $M < N$ . It makes sense that only  $M$  samples need to be measured instead of  $N$  samples.

It has been demonstrated theoretically that the problem of recovering sparse  $\mathbf{r}_f^{(i)}$  from low-dimensional measurements  $y_t^{(i)}$  will stop being underdetermined if the matrix  $\Phi \mathbf{F}_N^{-1}$  satisfies restricted isometry property (RIP) [6, 7]. It also has been proved that Gaussian and Bernoulli random matrices will satisfy the RIP with high probability provided  $M$  is sufficiently large.

According to the hierarchical access model [1], overlay spectrum sharing protocol is adopted in which CR avoids transmitting at any occupied subbands. In this case, the goal of CS-based CSS scheme is to make a global decision  $\hat{\mathbf{d}}$  on spectrum occupancy status of LUs by jointly processing all local compressed measurements  $\{y_t^{(i)}\}_{i=1}^I$ . Specifically, entries in  $\hat{\mathbf{d}}$  are 1 or 0, indicating whether the corresponding subband is occupied or not.

In consideration of the limitation power per CR, we aim to design a distributed compressed spectrum sensing scheme without using FC in the following section.

### 3. Distributed Compressed Spectrum Sensing

In CS recovery, the existence of noise or insufficient measurements will result in inexact reconstruction. However, the above-mentioned characteristic of large dynamic range makes it possible to identify the locations of large entries (i.e., incomplete support information) from inexact reconstruction and it may be achieved even without recovering the locations of small entries. In this section, we first study how to exploit this support information of the sparse spectrum to improve the sensing performance. Following the guideline of iterative support detection algorithm [22], we then develop a distributed approach to adaptively obtain reliable support knowledge. Finally, a distributed scheme to wideband spectrum sensing is presented based on these studies.

*3.1. Compressed Spectrum Reconstruction with Partially Known Support.* In traditional CS, the solution of local reconstruction from local compressed measurements is the one with minimum number of nonzeros among infinite data-consistency candidates [6, 7]. For notational simplicity, we drop the index  $i$  in this subsection. The corresponding procedure can be formulated as follows:

$$\begin{aligned} \hat{\mathbf{r}}_f &= \arg \min_{\mathbf{r}_f} \|\mathbf{r}_f\|_0 \\ \text{s.t. } y_t &= \Phi \mathbf{F}_N^{-1} \mathbf{r}_f. \end{aligned} \quad (3)$$

Recently, theoretical analysis and numerical study have demonstrated that incorporating partially known support

into the CS reconstruction can effectively reduce the number of measurements required by traditional CS to achieve a given reconstruction quality or improve the quality given the same number of measurements.

Assume that partial support (denoted by  $S$ ) is known; the candidates for the sparse  $\mathbf{r}_f$  are restricted in a signal space, which is smaller than that in traditional CS. Here we give an explanation for the difference between traditional CS and CS with PKS (CS-PKS) from a qualitative point of view. Suppose that the size of  $\text{supp}(\mathbf{r}_f)$  is  $K$ . Traditional CS needs to search for solutions in  $C(N, K)$  possible  $K$ -dimensional subspaces, where  $C(N, K)$  denotes the number of combinations of  $N$  things selected  $K$  at a time. If the support is partially known such that a set  $S$  with size  $s$  is known to belong to support set, we only need to search for solutions in  $C(N-s, K-s)$  possible  $(K-s)$ -dimensional subspaces. Apparently, the search space is reduced when the support is partially known. In this case, the reconstruction procedure can be formulated as

$$\begin{aligned} \hat{\mathbf{r}}_f &= \arg \min_{\mathbf{r}_f} \|\mathbf{r}_f\|_{S^c} \\ \text{s.t. } y_t &= \Phi \mathbf{F}_N^{-1} \mathbf{r}_f, \end{aligned} \quad (4)$$

where  $S^c$  denotes the complement of the set  $S$  with respect to  $\{1, 2, \dots, N\}$  and  $(\mathbf{r}_f)_{S^c}$  refers to a subvector of  $\mathbf{r}_f$  that contains the elements with indexes in  $S^c$ .

In order to decrease computational complexity and improve the robustness, an approximate solution of (4) can be obtained by solving

$$\begin{aligned} \hat{\mathbf{r}}_f &= \arg \min_{\mathbf{r}_f} \|\mathbf{r}_f\|_{S^c} \\ \text{s.t. } y_t &= \Phi \mathbf{F}_N^{-1} \mathbf{r}_f. \end{aligned} \quad (5)$$

This problem is referred to as the truncated minimization since the objective function is related to a truncated version of the signal, instead of the entire signal. This formulation encourages a solution with more zeros outside  $S$ , and thus it may recover the signal more accurately than traditional CS does for signals whose support includes  $S$ . Based on sufficient condition [19, 22, 23] for CS-PKS, the number of measurements required for CS-PKS is less than that required for traditional CS, and the more the support is known, the fewer the measurements needed are. The robustness of truncated minimization in noisy case has been discussed in [20–22].

In practice, the PKS  $S$  may not exactly belong to the true support. There may be some false nonzeros (assumed nonzero but actually zero) in  $S$ . In this case, we separate PKS  $S$  into two parts, true nonzeros  $S_c$  with size  $|S_c|$  and false nonzeros  $S_f$  with size  $|S_f|$ . Numerical simulations [18] and theoretical analysis [22] show that truncated minimization with some false nonzeros can still improve the recovery performance, provided that the true nonzeros are more than the false ones by a certain factor. In addition, the more accurate the PKS is, the fewer the measurements needed are.

In the application of wideband spectrum sensing, the received signal is inevitably polluted by noise. In this case, a conic constraint is required and (5) should be modified to

$$\begin{aligned} \hat{\mathbf{r}}_f &= \arg \min_{\mathbf{r}_f} \left\| (\mathbf{r}_f)_S \right\|_1 \\ \text{s.t. } & \left\| \mathbf{y}_t - \Phi \mathbf{F}_N^{-1} \mathbf{r}_f \right\|_2^2 \leq \varepsilon, \end{aligned} \quad (6)$$

where  $\varepsilon$  bounds the amount of noise energy in the compressed measurement. Note that (6) is still a convex optimization problem [24] and a number of solvers can be used to efficiently solve this problem.

*3.2. Adaptive Learning of Support Knowledge in a Distributed Manner.* Now, let us turn to each of the CR  $i$ ,  $i = 1, 2, \dots, I$ . After compressed reconstruction, the indicator of local support information  $\mathbf{T}^{(i)} \in \{0, 1\}^{N \times 1}$  can be obtained by thresholding the reconstructed spectrum  $\hat{\mathbf{r}}_f^{(i)}$ , following the guideline of iterative support detection algorithm [22]. This procedure can be formulated as

$$\mathbf{T}^{(i)} = \left( \hat{\mathbf{r}}_f^{(i)} \geq \tau^{(i)} \right), \quad (7)$$

where  $\tau_i$  is adaptive to the reconstructed signal and is elaborated in next section.

However, the existence of ambient noise results in sharp increase in the number of false nonzeros in local support information. Too many false nonzeros can severely degrade the recovery performance of truncated minimization. Recognizing the joint sparsity structure that different CRs share the same sparsity pattern, it is an effective approach to obtain reliable support information via cooperative support fusion among multiple CRs. Network-wide, the optimal fused result is the average value  $\bar{\mathbf{T}} = (1/I) \sum_{i=1}^I \mathbf{T}^{(i)}$ .

In consideration of limited power per CR, we aim to compute  $\bar{\mathbf{T}}$  in a distributed manner. Here, the average-consensus technique [15] is adopted to iteratively compute  $\bar{\mathbf{T}}$  via one-hop local communications [13, 14], which can significantly reduce the transmission power consumed during sensing. To represent this distributed implementation, we assume that  $G = (V, \zeta)$  is an undirected connected graph depicting the connectivity of different CRs in the networks. CRs form the set of vertices  $V = \{1, 2, \dots, I\}$ . Each edge  $(i, j) \in \zeta$  indicates that  $i$ th CR and  $j$ th CR are one-hop neighbors to each other, between which one-hop communication is allowed. Let  $V^{(i)}$  denote the set of one-hop neighbors of  $i$ th CR and let  $\mathbf{T}^{(i)}(1) := \mathbf{T}^{(i)}$  be a vector associated with  $i$ th CR at 1th iteration. At  $k$ th iteration, each CR  $i$  broadcasts  $\mathbf{T}^{(i)}(k)$  to its neighbors  $V^{(i)}$  and updates itself as follows:

$$\begin{aligned} & \mathbf{T}^{(i)}(k+1) \\ &= \mathbf{T}^{(i)}(k) + \sum_{j \in V^{(i)}} w_{ij} \left( \mathbf{T}^{(j)}(k) - \mathbf{T}^{(i)}(k) \right) \quad \forall i, k, \end{aligned} \quad (8)$$

where  $w_{ij}$  is a weight associated with the edge  $(i, j)$ . Here, the Metropolis-Hastings weights [15] are adopted and they are defined as follows:

$$w_{ij} = \begin{cases} \frac{1}{(\max\{|V^{(i)}|, |V^{(j)}|\} + 1)}, & (i, j) \in \zeta \\ 1 - \sum_{j \in V^{(i)}} \frac{1}{(\max\{|V^{(i)}|, |V^{(j)}|\} + 1)}, & i = j \\ 0, & \text{otherwise,} \end{cases} \quad (9)$$

where  $|V^{(i)}|$  denotes the size of set  $V^{(i)}$  and it is also known as degree of  $i$ th CR.

Using weights stated in (9), (8) can guarantee that

$$\mathbf{T}^{(i)}(k) \longrightarrow \frac{1}{I} \sum_{i=1}^I \mathbf{T}^{(i)}(1) = \bar{\mathbf{T}} \quad \forall i = 1, 2, \dots, I. \quad (10)$$

Note that each CR does not have to converge to the average value  $\bar{\mathbf{T}}$ , since it is only interested in identifying the locations of entries in  $\bar{\mathbf{T}}$  whose values are larger than a predefined threshold  $\gamma$ . Apparently, much less iterations are needed to identify those locations than those required for convergence. The fused support information is constituted by those locations and can be formulated as

$$S := \{n : |\bar{\mathbf{T}}[n]| > \gamma\}. \quad (11)$$

Thus, through several local one-hop communications, each CR obtains more reliable support information  $S$ . Subsequently, each CR can update its local reconstruction through truncated  $l_1$  minimization by incorporating this support information. Note that support information obtained through (11) is the same for all CRs. Using the same fusion support information to guide local reconstructions can encourage the same sparsity pattern of reconstructed spectrum at different CRs. Through this way of exploiting spatial diversity among all CRs, the impact of wireless fading and ambient noise are expected to be eliminated significantly.

*3.3. Support Fusion-Based Distributed Compressed Spectrum Sensing.* Based on the above studies, we propose a SF-DCSS scheme that adaptively and iteratively learns and uses the support information in CS such that the sensing performance of the CR network can be improved.

During iterations of the proposed SF-DCSS scheme, it is obvious that fused support information  $S$  needs to change from one iteration to the next. Otherwise, two iterations will result in identical local reconstructions for all CRs and thus the stagnation of proposed scheme. Therefore, we terminate the iterations when iteration posts no change in the set  $S$  or the iterations reach predefined maximum iteration times  $t_{\max}$ .

When iteration terminates, local decision on spectrum occupancy status  $\hat{\mathbf{d}}^{(i)}$  at each CR  $i$  can be made by comparing the local spectral estimate  $\hat{\mathbf{r}}_f^{(i)}$  in the last iteration with a decision threshold  $\eta^{(i)}$ :

$$\hat{\mathbf{d}}^{(i)} = \left( |\hat{\mathbf{r}}_f^{(i)}| \geq \eta^{(i)} \right) \quad \forall i = 1, 2, \dots, I. \quad (12)$$

**Initialization:** Each CR  $i$  collects local compressed measurements  $y_i^{(i)}$  via (2), and initializes iteration number  $t = 1$  and fusion support information  $S(1) = \emptyset$ . Set the maximum number of iterations  $t_{\max}$ .

**Iteration:**

**repeat**

(1) *Local Compressed Reconstruction:* Each CR  $i$  recovers local reconstructions  $\hat{\mathbf{r}}_f^{(i)}(t)$  via (6);

(2) *Adaptive Learning of Support Information:*

(2a) Each CR obtains local support information  $\mathbf{T}^{(i)}$  via (7);

(2b) Each CR broadcasts  $\mathbf{T}^{(i)}$  to its one-hop neighbors in  $V^{(i)}$ , iteratively updates  $\mathbf{T}^{(i)}$  via (8) using weights in (9), and finally obtains fusion support information  $S(t)$  via (11);

(3) *Update Iteration Number:*  $t = t + 1$

**until**  $t > t_{\max}$  or  $S(t) = S(t - 1)$ .

**Decision:** Upon convergence, each CR  $i$  obtains local decision  $\hat{\mathbf{d}}^{(i)}$  by thresholding  $\hat{\mathbf{r}}_f^{(i)}(t)$  via (12), broadcasts  $\hat{\mathbf{d}}^{(i)}$  to its one-hop neighbors in  $V^{(i)}$ , iteratively updates in the same manner as Step (2b), and finally makes global decision  $\hat{\mathbf{d}}$  on spectrum occupancy status via (13).

ALGORITHM 1: Support fusion-based distributed compressed spectrum sensing.

Globally at the network level, the sufficient statistic for optimal decision fusion is the average value  $\bar{\mathbf{c}} = (1/I) \sum_{i=1}^I \hat{\mathbf{d}}^{(i)}$ . Using the average-consensus algorithm stated above, each CR can obtain the unified fusion decision  $\hat{\mathbf{d}}$  through several local one-hop communications. The unified fusion decision  $\hat{\mathbf{d}}$  can be formulated as follows:

$$\hat{\mathbf{d}} = (\bar{\mathbf{c}} \geq c_{th}) = \left( \left( \frac{1}{I} \sum_{i=1}^I \hat{\mathbf{d}}^{(i)} \right) \geq c_{th} \right). \quad (13)$$

Based on fusion decision  $\hat{\mathbf{d}}$ , spectrum holes for DSA can be clearly given.

To recap, the complete SF-DCSS scheme is summarized in Algorithm 1.

**3.4. Choice of Thresholds.** In the proposed SF-DCSS scheme, there are four thresholds in (7), (11), (12), and (13), respectively. Those thresholds significantly affect the performance of SF-DCSS scheme. In this subsection, we elaborate the choice of thresholds.

*Choice of Threshold in (7).* Equation (7) is used to obtain local support information at each CR. For each CR  $i$ , small  $\tau^{(i)}$  will result in too many false detections in  $\mathbf{T}^{(i)}$  and thereby make the reliability of fusion support information worse and then degrade recovery performance. On the other hand, large  $\tau^{(i)}$  will result in too small size of  $\mathbf{T}^{(i)}$ ; hence, the fusion support set  $S$  might be empty or a large number of iterations are required for convergence.

Different strategies on choice of  $\tau^{(i)}$  have been studied in [22]. One is based on locating the “first significant jump” in the ascending sequence  $|\hat{\mathbf{r}}_f^{(i)}|$  ( $\hat{\mathbf{r}}_f^{(i)}|_{[n]}$  denotes the  $n$ th largest component of  $\hat{\mathbf{r}}_f^{(i)}$  by magnitude) and this strategy looks for the smallest  $n$  such that

$$|\hat{\mathbf{r}}_f^{(i)}|_{[n+1]} - |\hat{\mathbf{r}}_f^{(i)}|_{[n]} > \tau_{\text{jump}}^{(i)}, \quad (14)$$

where the corresponding  $|\hat{\mathbf{r}}_f^{(i)}|_{[n]}$  is the threshold what we find for  $\tau^{(i)}$ . Following the guideline of [22], we set  $\tau_{\text{jump}}^{(i)}$  to  $(4/t)M^{-1}\|\hat{\mathbf{r}}_f^{(i)}\|_{\infty}$ ,  $t = 1, 2, 3, 4$  for the first 4 iterations in the simulations of the next section. Although this setting is certainly not necessarily optimal, it works well in the simulations.

*Choice of Threshold in (11).* Thresholds in (11) are used to reduce the false detection caused by ambient noise and obtain reliable fusion support information at each CR. If  $\gamma$  is set to be zero, false detections at different CRs will be accumulated in  $S$ . Obviously, larger  $\gamma$  will result in more reliable  $S$ , but the cardinality of PKS  $|S|$  will be smaller at the same time. Numerical experiments in [18] suggest that it is better to have  $|S_c| - |S_f|$  true detections and  $|S_f|$  false detections than to have  $|S_c| - |S_f|$  true detections only. Following this guideline, we set  $\gamma = 1/I$  in the following simulations. When there are a large number of CRs in the CR network, we should properly increase the value of  $\gamma$  to exclude false detections in fusion support set.

*Choice of Threshold in (12).* Equation (12) is used to make local decision on spectrum occupancy status at individual CR. Large  $\eta^{(i)}$  will result in large miss detection probability and harmful interference with the LU, whilst small  $\eta^{(i)}$  tends to cause large false alarm probability and loss of spectrum access opportunities. Usually, the value of  $\eta^{(i)}$  is chosen according to the desired probability of false alarm, using the well-known Neyman-Pearson binary hypothesis test rule [13].

*Choice of Threshold in (13).* Equation (13) is used to obtain unified fusion decision at each CR. The choice of  $c_{th}$  reflects how conservative the CR network is in taking spectrum access opportunities. A conservative choice is  $c_{th} = 1/I$ ; it suggests that CR network decides the presence of a LU as long as one of  $I$  CRs claims presence. A more greedy choice is  $c_{th} = 1/2$  (a.k.a. majority vote).

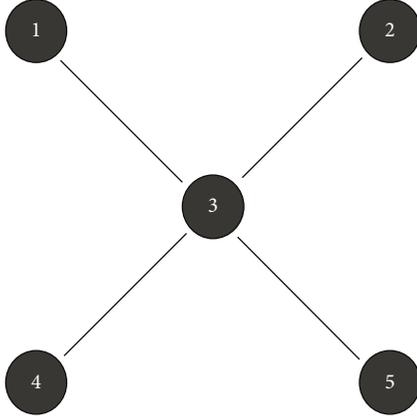


FIGURE 1: Connectivity of CR network with 5 nodes (both ends of each line segment are neighbors and local communications are allowed between them).

#### 4. Simulations

In this section, simulation results are provided to illustrate performance of the proposed SF-DCSS scheme. First, the simulation setup and relevant performance metrics are described. Then we compare the detection performance of several CS-based cooperative spectrum sensing schemes and investigate their computational complexity and communication workload. Lastly, the impact of system parameters such as SNR, the number of cooperating CRs, and the compression ratio is studied.

**4.1. Simulation Step and Performance Metrics.** Consider a monitored wideband which is equally partitioned into  $N = 128$  subbands, among which 24 subbands are randomly occupied by LUs. Each subband experiences frequency-selective fading, while the channel gain coefficient  $g^{(i)}[n]$  on each subband is time-invariant within each sensing period and is generated randomly according to Rayleigh fading distribution. Suppose that the CR network contains 5 spatially distributed CRs and its connectivity is shown in Figure 1.

The signal to noise ratio (SNR) is defined as the ratio of the average received signal to noise power over the entire wideband spectrum. Suppose that the received signal at each CR is corrupted by additive white Gaussian noise (AWGN) and all CRs have the same received SNR. The compression ratio  $\lambda = M/N$  reflects the reduced number of samples  $M$  collected at each CR receiver, with reference to the number  $N$  needed in full-rate Nyquist sampling. For the spectrum hole detection problem, performance metrics of interest are the probabilities of detection  $P_d$  and false alarm  $P_{fa}$ , which are evaluated by comparing the estimated occupancy status  $\hat{\mathbf{d}}$  with the true occupancy status  $\mathbf{d}$  over all subbands, which are given by

$$P_d = \frac{\|\hat{\mathbf{d}} \& \mathbf{d}\|_0}{\|\mathbf{d}\|_0}, \quad (15)$$

$$P_{fa} = \frac{\|\hat{\mathbf{d}} \& (\sim \mathbf{d})\|_0}{\|\sim \mathbf{d}\|_0},$$

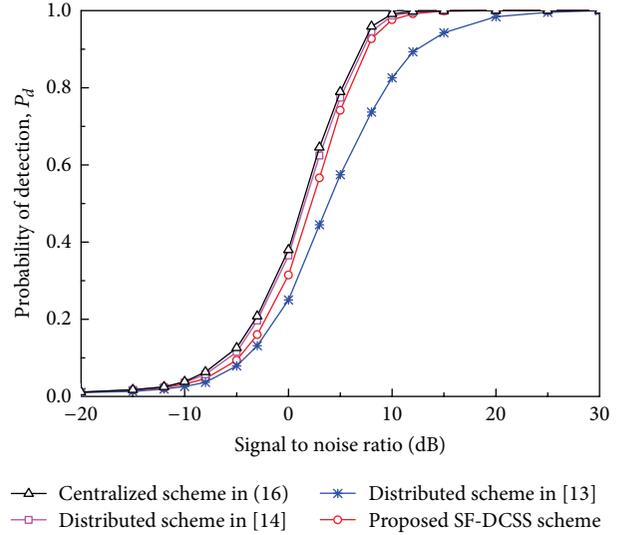


FIGURE 2: Detection probability versus SNR for  $P_{fa} = 0.01$  and  $\lambda = 0.5$ .

where  $\&$  and  $\sim$  denote bitwise AND and bitwise NOT, respectively, and here  $l_0$  norm function is used to calculate the number of nonzero elements.

**4.2. Performance Comparison among Several CS-CSS Schemes.** The proposed SF-DCSS scheme in Algorithm 1 is compared with three benchmark schemes, a centralized fusion scheme, a distributed scheme for decision fusion in [13], and a distributed fusion consensus scheme in [14]. Particularly, the centralized scheme requires a FC to collect local measurements  $\{y_t^{(i)}\}_{i=1}^I$ , local measurement matrixes  $\{\Phi^{(i)}\}_{i=1}^I$ , and local noise energy bounds  $\{\epsilon^{(i)}\}_{i=1}^I$  from all CRs. The reconstruction result is obtained through solving the minimization problem [14, 17], which can be written as

$$\{\hat{\mathbf{r}}_f^{(i)}\}_{i=1}^I = \arg \min_{\{\mathbf{r}_f^{(i)}\}_{i=1}^I} \sum_{n=1}^N \sqrt{\sum_{i=1}^I |\mathbf{r}_f^{(i)}[n]|^2} \quad (16)$$

$$\text{s.t. } \|\mathbf{y}_t^{(i)} - \Phi^{(i)} \mathbf{F}_N^{-1} \mathbf{r}_f^{(i)}\|_2^2 \leq \epsilon^{(i)}, \quad \forall i.$$

**4.2.1. Detection Performance versus SNR.** Figure 2 depicts the detection performance of the four schemes, for desired probability of false alarm  $P_{fa} = 0.01$  and compression ratio  $\lambda = 0.5$ . The centralized scheme performs the best at the expense of the highest communication workload. The distributed scheme in [13] has the worst performance since it reconstructs sparse signals separately at individual CR and computes the average value of local estimates as fused estimate by using average consensus algorithm. However it is very simple to implement. Distributed scheme in [14] has near-optimal performance at all SNRs since it iteratively solves the centralized formulation as shown in (16) by using distributed implementation. However it converges slowly and has high computational complexity and communication workload, which will be shown below.

As shown in Figure 2, there is a small gap between detection performance curve of proposed SF-DCSS scheme and that of distributed scheme in [14]. This suggests that proposed scheme can achieve comparable detection performance with the distributed scheme in [14] which has near-optimal performance. This gap is relatively large at low SNR (e.g.,  $-3, 0, 3,$  and  $5$  dB), since the ambient noise severely worsens the reliability of fusion support information and consequently degrades recovery performance. As SNR increases, the detection performance improves dramatically. At high SNR, this gap becomes trivial. When SNR =  $15$  dB, centralized scheme, distributed scheme in [14], and proposed SF-DCSS scheme achieve  $100\%$  detection probability at the same time.

**4.2.2. Computational Complexity and Communication Workload.** The exploitation of spatial diversity in cooperative sensing results in a significant improvement in detection performance. However, cooperation among CRs may also incur a variety of overheads. This overhead refers to any performance degradation caused by cooperative sensing, such as computational complexity and communication workload. To evaluate communication workload, we introduce the metric of communication step. When all CRs transmit a vector of size  $N \times 1$  to its neighbors, one communication step occurs.

Among the four aforementioned schemes, the centralized scheme apparently has the highest overhead and the distributed scheme in [13] has the lowest. In this subsection, we compare the cooperative overhead (i.e., computational complexity and communication workload) of proposed SF-DCSS scheme with that of the distributed scheme in [14]. Both of them consist of an iterative loop. At each iteration of distributed scheme in [14], one communication step occurs and each CR needs to solve an optimization problem which incurs comparable complexity with traditional compressed reconstruction.

According to the proposed SF-DCSS scheme depicted in Algorithm 1, computational complexity mainly depends on step 1 in recovering local reconstructions at individual CR. Other steps are based on basic arithmetic and logical operations which are omitted here for simplicity. Each CR needs to solve a truncated  $l_1$  minimization problem per iteration. Note that PKS has been added to the reconstruction procedure as shown in (6). It is proved in [18] that the use of PKS can also lead to a reduction in computation time in addition to the reduction in the number of required measurements. Communication workload of SF-DCSS scheme mainly depends on the implementation of average consensus technique as depicted in step (2b) and last step. Different from [14], average consensus technique in Algorithm 1 is used to make decisions in (11) and (13), other than computing the exact average value. According to the analysis in Section 3.2, much less iterations are needed here than those required for converging to the average value. For the connectivity of CR network depicted in Figure 1, 8 communication steps are sufficient to make decisions in (11) and (13), respectively.

Under the same parameter configuration with the above subsection, iterations of SF-DCSS scheme and distributed

scheme in [14] for various SNRs are given in Table 1. Each value in the table is obtained by averaging over 400 Monte Carlo trials.

Take SNR =  $-5$  dB for example, proposed SF-DCSS requires solving no more than 3 truncated  $l_1$  minimization at each CR and requires no more than 26 communication steps; however, the distributed scheme in [14] needs to solve at least 35 optimization problems at each CR and at least 35 communication steps. This suggests that proposed SF-DCSS scheme significantly reduces the computational complexity per CR, regardless of the reduction in computational complexity introduced by truncated  $l_1$  minimization. Moreover, SF-DCSS also reduces the communication workload, by comparing with the distributed scheme in [14]. Since exact average value is not required for SF-DCSS scheme, we can spend fewer bits to represent the exchanged information among neighbors.

When the number of CRs in CR network increases or the degrees of CRs in the network reduce, iterations of the distributed scheme in [14] increase dramatically. In this case, those reductions will become more considerable.

**4.3. Performance of Proposed SF-DCSS Scheme.** Parameters of CR networks, such as the number of cooperating CRs, the compression ratio, and SNR, significantly affect the performance of SF-DCSS scheme. In this subsection, we investigate the impact of those system parameters.

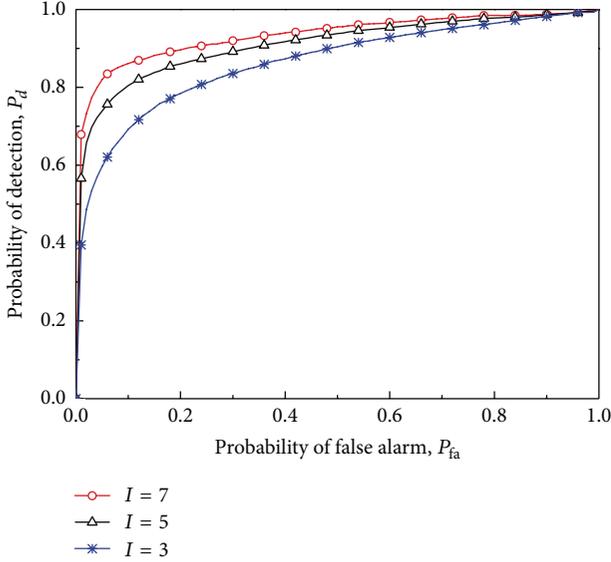
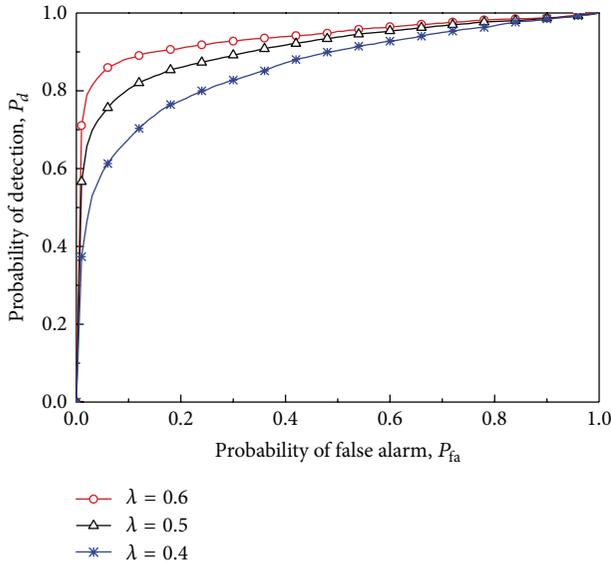
**4.3.1. Performance versus Number of CRs.** Cooperation among multiple spatially distributed CRs offers cooperative gain. It is an attractive and effective approach to combat multipath fading and shadowing and mitigate the receiver uncertainty problem. Figure 3 depicts the ROC curves for various number of collaborating CRs  $I$ , under the parameter setting:  $K = 64$ , SNR =  $3$  dB. Apparently, the probability of detection  $P_d$  improves as  $I$  increases, for the same probability of false alarm  $P_{fa}$ . This suggests that more collaborating CRs introduce more cooperative gain.

**4.3.2. Performance versus Compression Ratio.** In the proposed SF-DCSS scheme, CS is introduced to break the bottleneck of ADC technology in wideband regime, such as sampling rate, fidelity, and power consumption. However, it also incurs performance degradation, especially in the noisy case. Figure 4 depicts the ROC performance for various compression ratios, under the parameter setting:  $I = 5$ , SNR =  $3$  dB. As shown in Figure 4, for the same probability of false alarm  $P_{fa}$ , the probability of detection  $P_d$  improves as more compressed measurements are collected.

**4.3.3. Performance versus SNR.** As stated above, ambient noise can significantly worsen the reliability of fusion support information and consequently degrade the detection performance. Figure 5 depicts the ROC performance for various SNRs, under the parameter setting:  $I = 5$ ,  $K = 64$ . As shown in Figure 5, the detection performance improves significantly when the SNR increases. Ambient noise significantly affects the reliability of fusion support information and

TABLE 1: Iterations of proposed scheme and distributed scheme in [14].

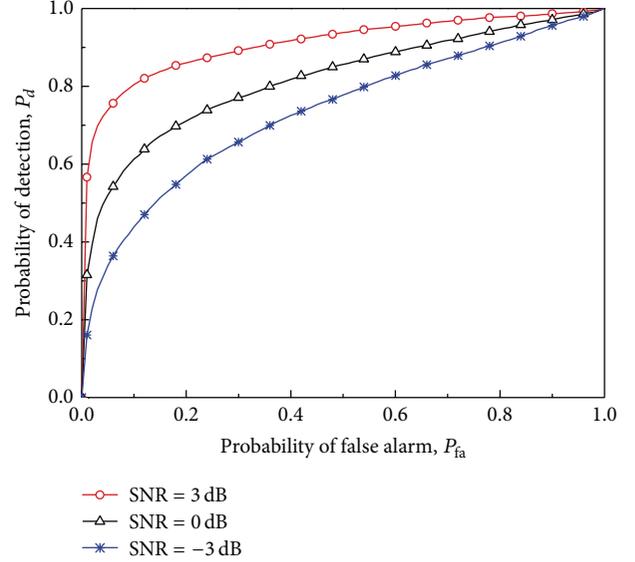
SNR	-5 dB	0 dB	5 dB	10 dB
Proposed SF-DCSS scheme	2.18	2.67	3.35	3.23
Distributed scheme in [14]	35.50	36.01	36.33	37.96

FIGURE 3: ROC performance of the proposed SF-DCSS scheme for different numbers of collaborating CRs (SNR = 3 dB,  $\lambda = 0.5$ ).FIGURE 4: ROC performance of the proposed SF-DCSS scheme for different compression ratios ( $I = 5$ , SNR = 3 dB).

consequently affects detection performance of proposed SF-DCSS scheme.

## 5. Conclusions

Recognizing the joint sparsity structure and large dynamic range of sparse spectrum at each CR, this paper has developed a distributed compressed spectrum sensing scheme via

FIGURE 5: ROC performance of the proposed SF-DCSS scheme for different SNRs ( $I = 5$ ,  $\lambda = 0.5$ ).

cooperative support fusion. The use of compressed sensing effectively reduces the signal acquisition cost and then makes it possible to simultaneously monitor a very wideband spectrum. Using fusion support information to guide local reconstructions is a valid way in exploiting spatial diversity among all CRs, and it can effectively mitigate the impact of fading and shadowing. It also expedites the convergence and lowers the computation and communication load. During spectrum sensing, only one-hop communications are used which significantly reduce the consumed transmission power per CR. Compared with those schemes with optimal or near-optimal detection performance, this proposed scheme significantly reduces the cooperative overhead such as computational complexity and communication workload, at the expense of slight degradation in detection performance. This proposed scheme is suitable for the case of large number of CRs and small degrees of nodes in CR network. In those cases, the cooperative overheads incurred by optimal or near-optimal schemes become extremely high.

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