Research Article

Rational-Like Solutions of a Differential-Difference Equation Related to Ablowitz-Ladik Spectral Problem

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By using the Casoratian technique, we construct the double Casoratian solutions whose entries satisfy matrix equation of a differential-difference equation related to the Ablowitz-Ladik spectral problem. Soliton solutions and rational-like solutions are obtained from taking special cases in general solutions.

1. Introduction

In recent decades, soliton equations have attracted much attention for both their physical and mathematical significance. For the soliton solutions of soliton equations, they scatter elastically with phase-shifts or amplitude changes in uniform or nonuniform media in dynamics. Many soliton equations such as KdV, mKdV, nonlinear Schrödinger equation, and sine-Gordon equation are viewed as the compatibility condition of the Ablowitz-Kaup-Newell-Segur (AKNS) spectral problem. Recently, the nonlinear Schrödinger equation attracts much attention in many fields such as oceanics, nonlinear optics, Bose-Einstein condensations [1], and atmospheric, for it interprets some freak wave (rogue wave) [2]. The propagation of the optical field complex envelope in a single-mode fiber, accounting for group velocity dispersion and Kerr nonlinearity, is governed by the nonlinear Schrödinger equation (see [3]). As the discrete version of the AKNS spectral problem, the Ablowitz-Ladik (AL) spectral problem has been studied for a few decades. The nonlinear self-dual network and Toda lattice equations are important equations related to the AL spectral problem. The Toda lattice is a system of unit masses, connected by nonlinear springs subject to an exponential restoring force [4]. The corresponding Ablowitz-Ladik hierarchy is researched for its

symmetries [5] and its decompositions [6] in mathematical sense. It is interesting to study the differential-difference equation in the Ablowitz-Ladik hierarchy and find their soliton and rational solutions (see [5, 6] and references therein).

Many methods such as the inverse scattering transformation [7], Hirota method [8], Bäcklund transformation (e.g., [9]), dressing method (e.g., [10]), and Wronskian/Casoratian technique [11] can be used for finding their solutions of continuous [12] and discrete soliton equations [13] and soliton equations with self-consistent sources [14]. It is well known that the Wronskian/Casoratian technique has been used to construct various types of exact solutions of soliton equations, such as soliton solutions (e.g., [15]) and rational solutions [16]. N-soliton solution of the discrete-time relativistic Toda lattice equation is explicitly constructed in the form of the Casorati determinant [17]. The resulting solutions can be verified by direct substitution into the corresponding linear equation. By using the technique, the solitons and the rational solutions are introduced and constructed. In the paper, we would like to consider the differential-difference equation related to the Ablowitz-Ladik spectral problem [18]:

$$\begin{pmatrix} Q_n \\ R_n \end{pmatrix}_t = \begin{pmatrix} Q_{n+1} (1 - Q_n R_n) \\ -R_{n+1} (1 - Q_n R_n) \end{pmatrix}, \quad (1)$$

which corresponding Lax pair is

$$\begin{pmatrix} \phi_{1,n+1} \\ \phi_{2,n+1} \end{pmatrix} = \begin{pmatrix} z & Q_n \\ R_n & \frac{1}{z} \end{pmatrix} \begin{pmatrix} \phi_{1,n} \\ \phi_{2,n} \end{pmatrix}, \quad (2a)$$

$$\begin{pmatrix} \phi_{1,n} \\ \phi_{2,n} \end{pmatrix}_t = \begin{pmatrix} \frac{1}{2}z^2 - Q_n R_{n-1} & zQ_n \\ zR_{n-1} & -\frac{1}{2}z^2 \end{pmatrix} \begin{pmatrix} \phi_{1,n} \\ \phi_{2,n} \end{pmatrix}, \quad (2b)$$

where $Q_n = Q(t, n)$, $R_n = R(t, n)$, $\phi_{1,n} = \phi_1(t, n)$, and $\phi_{2,n} = \phi_2(t, n)$ are functions dependent on variable t, n ($n \in \mathbb{Z}$) and decrease rapidly when $|n|$ tend to ∞ and z is a spectral parameter. In general, the distinct discrete eigenvalues $\{z_k\}$ are included in the scattering data of direct scattering problem in the frame of the inverse scattering transform. $\{z_k\}$ usually influence the shape and propagation of the multisolitons.

The paper is organized as follows. Section 2 gives the double Casoratian solution of (1). In Section 3 soliton solutions and “rational-like” solutions are derived from general solutions directly.

2. The Double Casoratian Solution

By the dependent variable transformation

$$\begin{aligned} Q_n &= \frac{g_n}{f_n}, \\ R_n &= \frac{h_n}{f_n}, \end{aligned} \quad (3)$$

(1) can be transformed to bilinear form

$$(e^{D_n} - 1) f_n \cdot f_n = -g_n h_n, \quad (4a)$$

$$D_t g_n \cdot f_n = g_{n+1} f_{n-1}, \quad (4b)$$

$$D_t h_n \cdot f_n = -h_{n-1} f_{n+1}, \quad (4c)$$

where the operators D_t and e^{D_n} are defined by

$$\begin{aligned} D_t f(t, n) \cdot g(t, n) &= (\partial_t - \partial_{t'}) f(t, n) g(t, n)|_{t=t'}, \\ \partial_t &= \frac{\partial}{\partial t}, \end{aligned} \quad (5a)$$

$$e^{D_n} f(t, n) \cdot g(t, n) = f(t, n+1) g(t, n-1). \quad (5b)$$

To use the Casoratian technique, we give the compact notation by Freeman and Nimmo [11]:

$$\begin{aligned} \text{Cas}^{N,M}(\phi_n, \psi_n) &= |\phi_n, E\phi_n, \dots, E^{N-1}\phi_n; \psi_n, E\psi_n, \dots, E^{M-1}\psi_n| \\ &= |\widehat{N-1}; \widehat{M-1}|, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \phi_n &= (\phi_{1,n}, \phi_{2,n}, \dots, \phi_{N+M,n})^T, \\ \psi_n &= (\psi_{1,n}, \psi_{2,n}, \dots, \psi_{N+M,n})^T. \end{aligned} \quad (7)$$

E is shift operator defined by $E\phi_n = \phi_{n+1}$. We also denote that

$$|-1, \widehat{N-1}; -1, \widehat{M-1}| = |E^{-1}\phi_n, \phi_n, E\phi_n, \dots, E^{N-1}\phi_n; E^{-1}\psi_n, \psi_n, E\psi_n, \dots, E^{M-1}\psi_n|, \quad (8a)$$

$$|\widehat{2N-2}; \widehat{2M-2}| = |\phi_n, E^2\phi_n, \dots, E^{2N-2}\phi_n; \psi_n, E^2\psi_n, \dots, E^{2M-2}\psi_n|, \quad (8b)$$

$$|\widehat{2N-1}; \widehat{2M-1}| = |E\phi_n, E^3\phi_n, \dots, E^{2N-1}\phi_n; E\psi_n, E^3\psi_n, \dots, E^{2M-1}\psi_n|, \quad (8c)$$

where $E^{-1}\phi_n = \phi_{n-1}$. It is easy to get the following lemma.

Lemma 1. Consider the following:

$$\begin{aligned} |Q, a, b| |Q, c, d| - |Q, a, c| |Q, b, d| + |Q, a, d| |Q, b, c| \\ = 0, \end{aligned} \quad (9)$$

where Q is an $N \times (N-2)$ matrix and a, b, c , and d represent N -dimensional column vectors.

Theorem 2. Equation (1) has the double Casoratian solution,

$$f_n = |\widehat{2N-2}; \widehat{2M-2}|, \quad (10a)$$

$$g_n = |-1, \widehat{2N-1}; \widehat{2M-3}|, \quad (10b)$$

$$h_n = -|\widehat{2N-3}; -1, \widehat{2M-1}|, \quad (10c)$$

for the condition equations,

$$\phi_{j,n+1} = e^{k_j} \phi_{j,n}, \quad (11a)$$

$$\psi_{j,n-1} = e^{k_j} \psi_{j,n},$$

$$\begin{aligned} \phi_{j,n,t} &= \frac{1}{2} \phi_{j,n+2}, \\ \psi_{j,n,t} &= -\frac{1}{2} \psi_{j,n-2} \end{aligned} \quad (11b)$$

$$(j = 1, 2, \dots, N+M),$$

and in general

$$\begin{aligned} \phi_{n+1} &= A\phi_n, \\ \psi_{n-1} &= A\psi_n, \end{aligned} \quad (12a)$$

$$\begin{aligned} \phi_{n,t} &= \frac{1}{2} \phi_{n+2}, \\ \psi_{n,t} &= -\frac{1}{2} \psi_{n-2}, \end{aligned} \quad (12b)$$

where $A = (a_{ij})_{(N+M) \times (N+M)}$ is an arbitrary matrix independent of t and n . Thus the corresponding solution of (1) can be expressed as

$$Q_n = \frac{|-1, \widehat{2N-1}; \widehat{2M-3}|}{|\widehat{2N-2}; \widehat{2M-2}|}, \quad (13)$$

$$R_n = -\frac{|\widehat{2N-3}; -1, \widehat{2M-1}|}{|\widehat{2N-2}; \widehat{2M-2}|}.$$

Proof. Here we give two notations. $(\widehat{2N-2}; \widehat{2M-2})$ denotes the columns $(E^2\phi_n, E^4\phi_n, \dots, E^{2N-2}\phi_n; E^2\psi_n, E^4\psi_n, \dots, E^{2M-2}\psi_n)$, which is related to the even-order derivatives. $(\widehat{2N-1}; \widehat{2M-1})$ denotes the columns $(E^3\phi_n, E^5\phi_n, \dots, E^{2N-1}\phi_n; E^3\psi_n, E^5\psi_n, \dots, E^{2M-1}\psi_n)$, which is related to the odd-order derivatives. First, by use of the Casoratian technique, it is easy to get

$$f_{n+1} = |\widehat{2N-1}; \widehat{2M-1}|, \quad (14a)$$

$$f_{n-1} = |-1, \widehat{2N-3}; -1, \widehat{2M-3}|,$$

$$g_{n+1} = |\widehat{2N}; \widehat{2M-2}|, \quad (14b)$$

$$h_{n-1} = -|\widehat{2N-4}; -2, \widehat{2M-2}|,$$

$$f_{n,t} = \frac{1}{2} (|\widehat{2N-4}, 2N; \widehat{2M-2}| - |\widehat{2N-2}; -2, \widehat{2M-2}|), \quad (14c)$$

$$g_{n,t} = \frac{1}{2} (|-1, \widehat{2N-3}, 2N+1; \widehat{2M-3}| - |-1, \widehat{2N-1}; -1, \widehat{2M-3}|), \quad (14d)$$

$$h_{n,t} = \frac{1}{2} (|\widehat{2N-3}; -3, \widehat{2M-1}| - |\widehat{2N-5}, 2N-1; -1, \widehat{2M-1}|). \quad (14e)$$

For (4a), note that

$$|\widehat{2N-2}; \widehat{2M-2}|^2 = \prod_{j=1}^{N+M} e^{k_j} |\widehat{2N-2}; \widehat{2M-2}|$$

$$\cdot \prod_{j=1}^{N+M} e^{-k_j} |\widehat{2N-2}; \widehat{2M-2}| \quad (15)$$

$$= |\widehat{2N-1}; -1, \widehat{2M-3}| |-1, \widehat{2N-3}; \widehat{2M-1}|$$

and taking $Q = (\widehat{2N-3}, \widehat{2M-3})$ in (9) yields

$$\begin{aligned} & |\widehat{2N-3}, 2N-1; \widehat{2M-3}, 2M-1| \\ & \cdot |-1, \widehat{2N-3}; -1, \widehat{2M-3}| \\ & - |\widehat{2N-3}, 2N-1; -1, \widehat{2M-3}| \\ & \cdot |-1, \widehat{2N-3}; \widehat{2M-3}, 2M-1| \\ & - |-1, \widehat{2N-3}, 2N-1; \widehat{2M-3}| \\ & \cdot |\widehat{2N-3}; -1, \widehat{2M-3}, 2M-1| = 0. \end{aligned} \quad (16)$$

It follows that

$$\begin{aligned} & f_n^2 - g_n h_n - f_{n+1} f_{n-1} \\ & = |\widehat{2N-1}; -1, \widehat{2N-3}| |-1, \widehat{2N-3}; \widehat{2M-1}| \\ & + |-1, \widehat{2N-1}; \widehat{2M-3}| |-1, \widehat{2N-3}; \widehat{2M-1}| \\ & - |\widehat{2N-1}; \widehat{2M-1}| |-1, \widehat{2N-3}; -1, \widehat{2M-3}| \\ & = 0. \end{aligned} \quad (17)$$

Thus, ((10a), (10b), (10c)) solves (4a).

For (4b), from (9) and noting the following identities,

$$\prod_{j=1}^{N+M} e^{k_j} |\widehat{2N-2}; \widehat{2M-2}| = |\widehat{2N-1}; -1, \widehat{2M-3}|, \quad (18a)$$

$$\prod_{j=1}^{N+M} e^{k_j} |\widehat{2N-4}, 2N; \widehat{2M-2}| \quad (18b)$$

$$= |\widehat{2N-3}, 2N+1; -1, \widehat{2M-3}|,$$

$$\prod_{j=1}^{N+M} e^{k_j} |\widehat{2N}; \widehat{2M-2}| = |\widehat{2N+1}; \widehat{2M-3}|, \quad (18c)$$

we have

$$\begin{aligned} & |-1, \widehat{2N-3}, 2N+1; \widehat{2M-3}| |\widehat{2N-2}; \widehat{2M-2}| \\ & - |-1, \widehat{2N-1}; \widehat{2M-3}| |\widehat{2N-4}, 2N; \widehat{2M-2}| \\ & - |\widehat{2N}; \widehat{2M-2}| |-1, \widehat{2N-3}; -1, \widehat{2M-3}| = 0. \end{aligned} \quad (19)$$

Similarly, we can get

$$\begin{aligned} & -|-1, \widehat{2N-1}; -1, \widehat{2M-3}| |\widehat{2N-2}; \widehat{2M-2}| \\ & + |-1, \widehat{2N-1}; \widehat{2M-3}| |\widehat{2N-2}; -2, \widehat{2M-2}| \\ & - |\widehat{2N}; \widehat{2M-2}| |-1, \widehat{2N-3}; -1, \widehat{2M-3}| = 0. \end{aligned} \quad (20)$$

By combining (19) with (20) and from ((14a), (14b), (14c), (14d), (14e)), one can have that ((10a), (10b), (10c)) solves (4b). In a similar way, we can also verify that ((10a), (10b), (10c)) solves (4c).

Now from ((10a), (10b), (10c)) and using the Casoratian technique, we show that ((10a), (10b), (10c)) solves ((4a), (4b), (4c)) for the condition equations ((12a), (12b)). In fact, we only need to verify (15), which can be obtained by the following identities:

$$\left| \widehat{2N-1; -1, 2M-3} \right| = |A| \left| \widehat{2N-2; 2M-2} \right|, \quad (21a)$$

$$\left| -1, \widehat{2N-3; 2M-1} \right| = |A^{-1}| \left| \widehat{2N-2; 2M-2} \right|, \quad (21b)$$

where A^{-1} is the inverse matrix of A . $\square$

3. Soliton Solutions and Rational-Like Solutions

From ((12a), (12b)), we can get the general solution

$$\begin{aligned} \phi_n &= C e^{(1/2)e^{2B}t+nB}, \\ \psi_n &= D e^{-(1/2)e^{2B}t-nB}, \end{aligned} \quad (22)$$

where $A = e^B$, $C = (c_1, c_2, \dots, c_{N+M})^T$, and $D = (d_1, d_2, \dots, d_{N+M})^T$ are real constant vectors. In order to get the soliton solutions and rational-like solutions, we take matrix B in canonical form Γ in (22) and expand the functions ϕ_n and ψ_n :

$$\phi_n = C e^{(1/2)e^{2\Gamma}t+n\Gamma} = \sum_{s=0}^{\infty} P_s(n, t) e^{(1/2)tI} \Gamma^s C, \quad (23a)$$

$$\psi_n = D e^{-(1/2)e^{2\Gamma}t-n\Gamma} = \sum_{s=0}^{\infty} P_s(-n, -t) e^{-(1/2)tI} \Gamma^s D, \quad (23b)$$

where

$$P_s(n, t) = \sum_{j=0}^s \frac{n^j}{j!} p_{s-j}(\tilde{t}), \quad (24a)$$

$$\begin{aligned} P_s(\tilde{t}) &= \sum_{\|\gamma\|=s} \frac{2^{\|\gamma\|}}{\gamma! \tilde{t}^\gamma}, \\ \gamma &= (\gamma_1, \gamma_2, \gamma_3, \dots), \quad \gamma_j \geq 0, \quad (j = 1, 2, 3, \dots), \end{aligned} \quad (24b)$$

$$\|\gamma\| = \gamma_1 + 2\gamma_2 + 3\gamma_3 + \dots,$$

$$\gamma! = \gamma_1! \gamma_2! \gamma_3! \dots,$$

$$\tilde{t} = \left(\frac{t_1}{2}, \frac{t_2}{2}, \frac{t_3}{2}, \dots \right), \quad (24c)$$

$$t_j = \frac{t}{j!},$$

$$\tilde{t}^\gamma = \left(\frac{t_1}{2} \right)^{\gamma_1} \left(\frac{t_2}{2} \right)^{\gamma_2} \left(\frac{t_3}{2} \right)^{\gamma_3} \dots$$

and I is the unit matrix. If

$$\Gamma = \begin{pmatrix} k_1 & & 0 \\ & k_2 & \\ & & \ddots \\ 0 & & & k_{N+M} \end{pmatrix}, \quad k_i \neq k_j \quad (i \neq j), \quad (25)$$

then we can obtain the soliton solutions (13) from (22) (substituting B for Γ), where

$$\begin{aligned} \phi_{j,n} &= c_j e^{(1/2)e^{2k_j}t+nk_j}, \\ \psi_{j,n} &= d_j e^{-(1/2)e^{2k_j}t-nk_j} \end{aligned} \quad (26)$$

($j = 1, 2, \dots, N + M$).

If

$$\Gamma = \begin{pmatrix} 0 & & 0 \\ 1 & 0 & \\ & \ddots & \ddots \\ 0 & & 1 & 0 \end{pmatrix}_{N+M}, \quad (27)$$

then it follows from ((23a), (23b)) and $\Gamma^{N+M} = 0$ that

$$\phi_n = \sum_{s=0}^{N+M-1} P_s(n, t) e^{(1/2)tI} \Gamma^s C, \quad (28a)$$

$$\psi_n = \sum_{s=0}^{N+M-1} P_s(-n, -t) e^{-(1/2)tI} \Gamma^s D. \quad (28b)$$

We specially give some rational-like solutions in the following. Taking $(N, M) = (1, 1)$, $\Gamma = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, and $C = D = (1, 0)^T$ in ((28a), (28b)) leads to

$$\begin{aligned} \phi_n &= e^{(1/2)t} \begin{pmatrix} 1 \\ n+t \end{pmatrix}, \\ \psi_n &= e^{-(1/2)t} \begin{pmatrix} 1 \\ -n-t \end{pmatrix}. \end{aligned} \quad (29)$$

Hence, the solution ((10a), (10b), (10c)) reads

$$\begin{aligned} Q_n &= -\frac{e^t}{n+t}, \\ R_n &= -\frac{e^{-t}}{n+t}. \end{aligned} \quad (30)$$

Similarly, we can calculate the following rational-like solutions with respect to n from ((28a), (28b)),

$$Q_n = \frac{2e^t}{n + n^2 + 2nt + t^2}, \quad (31a)$$

$$R_n = -\frac{e^{-t}(n + n^2 + 2nt + 2t + t^2)}{n + n^2 + 2nt + t^2};$$

$$Q_n = -\frac{e^t(n + n^2 + 2nt + t^2)}{n + n^2 + 2nt + 2t + t^2}, \quad (31b)$$

$$R_n = -\frac{2e^{-t}}{n + n^2 + 2nt + 2t + t^2};$$

$$Q_n = -\frac{2e^t[n^3 + 3n^2 + 3n^2t + t^3 + n(2 + 3t + 3t^2)]}{n^4 + 4n^3(t + 1) + t^2(6 + 4t + t^2) + n^2(5 + 12t + 6t^2) + 2n(1 + 4t + 6t^2 + 2t^3)}, \quad (31c)$$

$$R_n = \frac{2e^{-t}[n^3 + 3n^2(t + 1) + t(6 + 6t + t^2) + n(2 + 9t + 3t^2)]}{n^4 + 4n^3(t + 1) + t^2(6 + 4t + t^2) + n^2(5 + 12t + 6t^2) + 2n(1 + 4t + 6t^2 + 2t^3)};$$

$$Q_n = -\frac{6e^t}{n^3 + t^3 + 3n^2(t + 1) + n(2 + 3t + 3t^2)}, \quad (31d)$$

$$R_n = -\frac{e^{-t}[n^4 + n^3(t + 1) + n^2(5 + 12t + 6t^2) + t^2(6 + 4t + t^2) + 2n(1 + 4t + 6t^2 + 2t^3)]}{2[n^3 + t^3 + 3n^2(t + 1) + n(2 + 3t + 3t^2)]};$$

$$Q_n = -\frac{e^t[n^4 + n^3(t + 1) + n^2(5 + 12t + 6t^2) + t^2(6 + 4t + t^2) + 2n(1 + 4t + 6t^2 + 2t^3)]}{n^3 + t(6 + 6t + t^2) + 3n^2(t + 1) + n(2 + 9t + 3t^2)}, \quad (31e)$$

$$R_n = -\frac{6e^{-t}}{n^3 + t(6 + 6t + t^2) + 3n^2(t + 1) + n(2 + 9t + 3t^2)}$$

for $(N, M) = (2, 1)$, $(N, M) = (1, 2)$, $(N, M) = (2, 2)$, $(N, M) = (3, 1)$, and $(N, M) = (1, 3)$, respectively.

4. Conclusion

The differential-difference equation related to the Ablowitz-Ladik spectral problem, which has mathematical and physical significance, is solved by the Casoratian technique. The soliton and rational-like solutions are obtained through some condition equations in general case, which can be examined by substituting the solutions into the Casoratian solutions directly.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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