Research Article

Using Different Approaches to Approximate a Pareto Front for a Multiobjective Evolutionary Algorithm: Optimal Thinning Regimes for *Eucalyptus fastigata*

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A stand-level, multiobjective evolutionary algorithm (MOEA) for determining a set of efficient thinning regimes satisfying two objectives, that is, value production for sawlog harvesting and volume production for a pulpwood market, was successfully demonstrated for a *Eucalyptus fastigata* trial in Kaingaroa Forest, New Zealand. The MOEA approximated the set of efficient thinning regimes (with a discontinuous Pareto front) by employing a ranking scheme developed by Fonseca and Fleming (1993), which was a Pareto-based ranking (a.k.a Multiobjective Genetic Algorithm—MOGA). In this paper we solve the same problem using an improved version of a fitness sharing Pareto ranking algorithm (a.k.a Nondominated Sorting Genetic Algorithm—NSGA II) originally developed by Srinivas and Deb (1994) and examine the results. Our findings indicate that NSGA II approximates the entire Pareto front whereas MOGA only determines a subdomain of the Pareto points.

1. Introduction

Forest estate planning requires the ability to have a suite of possible thinning regimes for each management unit where the estate is an amalgam of the many management units. Satisfying any kinds of goals for the forest estate is determined by choosing a mosaic of thinning regimes for all the management units. The preferred choices are normally achieved through some form of optimisation [1]. Chikumbo and Nicholas [2] demonstrated how a suite of thinning regimes at a management unit level might be approximated.

Chikumbo and Nicholas [2] formulated a stand-level optimisation problem to determine a set of efficient thinning regimes. Each regime defined an initial planting stocking, frequency of thinning, timing of thinning, intensity of thinning, final crop number prior to clearfelling, and rotation length. The problem was solved as a multiobjective optimisation to simultaneously maximise sawlog (or value) production and pulpwood (or volume) production, which are conflicting objectives. Traditionally, thinning regimes for most tree crops are determined either for value or volume

production but not for both. This normally means an initially high tree stocking density, a short rotation with no thinning operations for volume production, and a more complex regime with thinning operations, pruning, and longer rotations for value production. The emphasis for value production is to maximise volume of clear wood per tree, rather than maximising biomass growth on as many trees as possible. Such an approach to forest management does not give the forester the flexibility to deal with volatile markets where the demands for sawlog and pulpwood tend to change rapidly. An ability to provide all products at the markets with the flexibility of meeting an increased supply of the product most in demand requires a different tact to forest management. Chikumbo and Nicholas [2] demonstrated how to determine a set of efficient thinning regimes that are suitable for both volume and value production in different ratios or “tradeoff,” where the choice of a single regime was made on the basis of the product most in demand.

As such, the critical part of approximating the set of efficient thinning regimes was to find trade-off solutions (i.e., nondominated solutions) where for each solution an

improvement in one objective did not lead to worsening in the other [3]. For the mathematically savvy, nondominated solutions can be defined as follows.

Assume two solutions $(\mathbf{x}, \mathbf{u}), (\mathbf{x}', \mathbf{y}') \in \Omega$, where Ω is denoted as the solution space. Then $(\mathbf{x}, \mathbf{u})$ is said to dominate $(\mathbf{x}', \mathbf{y}')$ (also written $(\mathbf{x}, \mathbf{u}) \succ (\mathbf{x}', \mathbf{y}')$) if and only if

$$\begin{aligned} \forall i \in \{1, 2, \dots, n\} : J_i(\mathbf{x}, \mathbf{u}) &\geq J_i(\mathbf{x}', \mathbf{u}') \wedge, \\ \exists m \in \{1, 2, \dots, n\} : J_m(\mathbf{x}, \mathbf{u}) &> J_m(\mathbf{x}', \mathbf{u}'). \end{aligned} \quad (1)$$

However, $(\mathbf{x}, \mathbf{u})$ is said to cover $(\mathbf{x}', \mathbf{u}')$ ($(\mathbf{x}, \mathbf{u}) \succeq (\mathbf{x}', \mathbf{u}')$) if and only if

$$\begin{aligned} \forall i \in \{1, 2, \dots, n\} : J_i(\mathbf{x}, \mathbf{u}) &\leq J_i(\mathbf{x}', \mathbf{u}') \wedge, \\ \exists m \in \{1, 2, \dots, n\} : J_m(\mathbf{x}, \mathbf{u}) &< J_m(\mathbf{x}', \mathbf{u}'). \end{aligned} \quad (2)$$

In this case $(\mathbf{x}, \mathbf{u})$ is nondominated by $(\mathbf{x}', \mathbf{y}')$.

Therefore, the set of nondominated solutions, Ω^N , within the entire search space, Ω , is called the Pareto-optimal set.

The set of solutions to the multiobjective problem was determined using a competitive co-evolutionary genetic algorithm [4] with five subpopulations of 100 individuals each, computed over 1000 generations. This is an island model characterised by multiple sub-populations instead of a single large population. These subpopulations evolve independently for a certain number of generations (isolation time). After the isolation time a number of individuals are distributed between subpopulations (a process called migration). Each subpopulation exerts selective pressure on the other, thereby maintaining diversity a lot longer than each subpopulation would do solitarily, thereby guarding against premature convergence. When competition is superimposed between the subpopulations, the ones with higher mean fitness values will maintain larger subpopulation sizes and receive more capable individuals, because they have more chances of finding the global optimum [5].

The Fonseca and Fleming ranking scheme [6] was used to determine the nondominated solutions. Only six nondominated thinning regimes were obtained, which was far less than expected because there should be as many nondominated solutions as possible, up to the size of the population [7]. A close inspection revealed that the Fonseca and Fleming ranking scheme quickly lost diversity resulting in “genetic drift” where convergence only occurs in one “niche” of the search space. The island model formulation is not sufficient to offset this genetic drift or avoid loss of diversity. Chikumbo and Nicholas [2] had no notion of the inability of MOGA to prevent “crowding” (i.e., inability to maintain diversity and encourage a good spread of solutions), in their first attempt at estimating efficient thinning regimes for *E. fastigata*. The more generations the algorithm was run, the more the Pareto front points exhibited crowding resulting in points superimposed on each other on certain subregions of the Pareto front. The workings of MOGA are explained later. This paper explores an alternative ranking scheme that is better at maintaining diversity such that not only a few points on a limited number of sub-regions of the Pareto front are estimated, rather than all the points on the Pareto front are determined.

The concept of applying evolutionary algorithms (or population-based search algorithms that mimic the evolutionary process) to multiobjective optimisation first appeared in the literature in the late 1960s, and it was not until 1985 when Schaffer [8] published the first algorithm. Figure 1 shows a diagrammatic representation of an evolutionary algorithm and how it works. Evolutionary algorithms work on populations of individuals instead of single solutions. In this way the search is performed in a parallel manner [9].

At the beginning of the computation, a number of individuals (the population) determined by the user are randomly initialised. The objective function(s), based on the “fitness value(s),” is/are then evaluated for these individuals; thus, the first/initial generation is produced. If the optimisation criteria are not met, the creation of a new generation starts, which is guided by a process of “selection,” “crossover/recombination,” and “mutation.” Individuals are selected according to their fitness for the production of offspring. Parents are recombined to produce offspring. All offspring will be mutated with a certain probability. The fitness of the offspring is then computed. The offspring are inserted into the population replacing the parents, producing a new generation. This cycle is performed until the optimisation criteria are reached [9].

Today there exist many variants of evolutionary algorithms, namely, genetic algorithms [10], evolutionary strategies [11, 12], and evolutionary programming [13].

The subject of multiobjective optimisation problems is extremely complex and mathematically difficult, with many underresearched areas and outstanding problems [14]. Evolutionary algorithms have become established as the method at hand for exploring the search space in multi-objective optimisation problems [15].

In the absence of preference information among the objectives, there is no single solution for a multi-objective optimisation problem [16]. This is because under these circumstances, optimality takes a whole new meaning. This was first proposed by Edgeworth [17] and later generalised by Pareto [18]. Determining a set of nondominated solutions is what is now commonly known as Pareto optimality. Evolutionary algorithms have become the default method for approximating the Pareto front for multiobjective optimisation problems. This is because there are hardly any alternative methods. Also, evolutionary algorithms have the capability to search many Pareto-optimal solutions in parallel and exploit similarities of solutions by the recombination/crossover operation [15]. For multiobjective evolutionary algorithms (MOEAs), the Pareto-optimal solutions are approximated in a single optimisation run, for efficiency.

Various implementations of MOEA have appeared in the literature each with a different approach and algorithm for estimating the Pareto front. Some of the more advanced algorithms currently in use are as follows:

- (a) Schaeffer’s Vector Evaluated Genetic Algorithm (VEGA)—switching objective type [8];
- (b) Hajela and Lin’s Weighting-Based Genetic Algorithm (HLGA)—parameter variation type [19];

FIGURE 1: Evolutionary algorithm.

- (c) Fonseca and Fleming's Multiobjective Genetic Algorithm (MOGA)—Pareto-based goal interaction [6];
- (d) Horn, Nafpliotis, and Goldberg's Niche Pareto Genetic Algorithm (NPGA)—cooperative sharing [20];
- (e) Srinivas and Deb's Nondominated Sorting Genetic Algorithm (NSGA)—fitness sharing Pareto [16];
- (f) Coello's Min-Max Optimisation (CMMO)—ideal non-Pareto feasibility vetting [21];
- (g) Zitzler and Thiele's Strength Pareto Evolutionary Algorithm (SPEA)—elitist external niching type [22];
- (h) Corne, Knowles, and Oates' Pareto Envelope-Based selection Algorithm (PESA)—integrated selection and diversity maintenance via a hypergrid crowding strategy [23];
- (i) Knowles and Corne's Pareto Archived Evolution Strategy (PAES)—single parent, local search [24].

In this paper the optimal thinning regime problem for *Eucalyptus fastigata* [2] is revisited and solved using single population MOGA and NSGA II algorithms. The GEATbx software that runs in MATLAB was used in the analysis, and it employs MOGA, which became our default. The reason NSGA II was chosen as a comparison is because of its wide use and reputation in the field of evolutionary computation. The results are compared, not to find which is better of the two but rather to determine a greater depth of the Pareto front in order to provide more thinning regimes to choose from. A brief description of the data and model formulation are given in the next section followed by a concise account of the MOGA and NSGA II algorithms. Sections 4 and 5 present in some detail the findings, and a conclusion on the extent of the Pareto front is presented.

2. Data, Objective Functions, and State Equations

The data for approximating the Pareto front using the MOGA and NSGA II algorithms came from a Nelder trial [25] of *E. fastigata* established in 1979 in Kaingaroa Forest, New Zealand. The details can be found in Chikumbo and Nicholas [2].

The emphasis of this paper is to approximate the solutions, or vectors, of decision variables of MOEA that satisfy constraints and optimise an objective vector function whose elements represent two objective functions. The decision variables, $\vec{x} = [x_1, x_2, \dots, x_9]$, form the 9 genes of each chromosome or individual, which represent a potential solution with their bound constraints, for both lower bound (lb) and upper bound (ub) as shown in Figure 2. Therefore, an evolutionary algorithm is initiated with a set of randomly generated potential solutions that are driven towards optimality over generations, where each generation is defined by selection, recombination/crossover, and mutation for a set of potential solutions or population.

These objective functions or fitness functions form a mathematical description of performance, which are in conflict with each other:

$$\text{maximise } [f_1(\vec{x}), f_2(\vec{x})] \quad (3)$$

subject to m inequality constraints,

$$g_i(\vec{x}) \leq 0, \quad i = 1, 2, \dots, m, \quad (4)$$

and r equality constraints,

$$h_i(\vec{x}) = 0, \quad i = 1, 2, \dots, r, \quad (5)$$

where the objective functions, $f_1(\vec{x})$ or $f_2(\vec{x})$: $\mathbb{R}^n \rightarrow \mathbb{R}$ and the vector of decision variables, $\vec{x} = [x_1, x_2, \dots, x_9]$.

FIGURE 2: Decision variables with their ub and lb for each chromosome in the MOEA formulation.

The objective functions were as follows:

$$f_1(\vec{x}) = \max_{u(t)} \sum_{t=T-(n-1)}^T \frac{u(t)}{\text{sph}(t)} * \text{sba}(t) * \text{mht}(t) * \text{fhv}_{\text{value}}(t) * \frac{\text{vol}(t)}{\text{sph}(t)} * p_{\vec{x}}, \quad (6)$$

for sawlog production (NZ\$m⁶ stems⁻¹ ha⁻²), and

$$f_2(\vec{x}) = \max_{u(t)} \sum_{t=T-(n-1)}^T \frac{u(t)}{\text{sph}(t)} * \text{sba}(t) * \text{mht}(t) * \text{fhv}_{\text{vol}}(t) * p_{\vec{x}}, \quad (7)$$

for pulpwood production (NZ\$m³ ha⁻²), where t is time in years, $\text{fhv}_{\text{value}}$ and fhv_{vol} are forest holding values [26] for sawlog production and pulpwood production, respectively,

$$\text{sph}(t) = \text{sph}(t-1) - u(t), \quad (8)$$

$$\begin{aligned} \text{sba}(t+1) = & a_1(\text{sph}(t))\text{sba}(t) + a_2(\text{sph}(t))\text{sba}(t-1) \\ & + b_1(\text{sph}(t)), \end{aligned} \quad (9)$$

where

$$\begin{aligned} a_1(t) &= a_{-1} * \exp(b_{-1} * \text{sph}(t)) + c_{-1} * \exp(d_{-1} * \text{sph}(t)); \\ a_2(t) &= a_{-2} * \exp(b_{-2} * \text{sph}(t)) + c_{-2} * \exp(d_{-2} * \text{sph}(t)); \text{ and} \\ b_1(t) &= a_{-3} * \text{sph}(t)^{b_{-3}} + c_{-3}. \end{aligned}$$

$$\begin{aligned} \text{mht}(t+1) &= a_3(\text{sph}(t))\text{mht}(t) + a_4(\text{sph}(t))\text{mht}(t-1) \\ &+ b_2, \end{aligned} \quad (10)$$

where

$$\begin{aligned} a_3(t) &= -1.055 * a_4(t) + 0.9548; \\ a_4(t) &= -7510/(\text{sph}(t) + 1.13e + 04); \text{ and} \\ b_2(t) &= 0.6579. \end{aligned}$$

$$\begin{aligned} \text{vol}(t+1) &= a_5(\text{sph}(t)) \text{vol}(t) + a_6(\text{sph}(t))\text{vol}(t-1) \\ &+ b_3(\text{sph}(t)), \end{aligned} \quad (11)$$

where,

$$\begin{aligned} a_5(t) &= a_{-6} * \exp(b_{-4} * \text{sph}(t)) + c_{-6} * \exp(d_{-3} * \text{sph}(t)); \\ a_6(t) &= a_{-7} * \exp(b_{-5} * \text{sph}(t)) + c_{-7} * \exp(d_{-4} * \text{sph}(t)); \text{ and} \\ b_3(t) &= a_{-8} * \text{sph}(t)^{b_{-6}} + c_{-8}. \end{aligned}$$

$$p_{\vec{x}} = \begin{cases} 1, & \text{if } x_6 + (x_7 + x_8 + x_9) \geq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

The relationship $\text{vol}(t)/\text{sph}(t)$ in objective function (6) concentrates volume production per tree, which is the goal for sawlog production. State equation (8) tracks the number of trees per hectare (sph) harvested (i.e., u or control) subject to the two objectives, and growth dynamics in state equations (9)–(11), which are stand basal area in square metres per hectare (sba), mean stand height of the 100 largest trees per hectare (mht), and volume in cubic metres per hectare (vol), respectively. The state equations (9)–(11) are discrete-time dynamical equations [27], which have been demonstrated to accurately determine growth dynamics influenced by changes in tree spacing, initial planting stocking, and thinning treatment [28]. The parameters of these state equations are shown in Table 1.

The constraint

$$x_6 - (x_7 + x_8 + x_9) \geq 0 \quad (13)$$

ensures that the total number of trees thinned over the complete rotation will not exceed the initial number of trees planted and is handled by a penalty function, $p_{\vec{x}}$, in the objective functions (3) and (4), which is equal to one when constraint (13) is not violated and zero otherwise.

The thinning regime problem for *E. fastigata* is a non-linear optimal control problem of dynamical equations (9)–(11). These dynamical equations experience state changes in time under some rules, that is, the growth dynamics of *E. fastigata*. These equations are mathematical “black box” representations of the underlying physical growth processes of *E. fastigata*. The state is described by a collection of variables that fully characterise growth. These parameters are time

TABLE 1: Parameters of the state equations (7)–(9) [2].

Parameter	Value	95% confidence bounds	
		Lower bound	Upper bound
a_1	0.1467	0.1088	0.1847
b_1	−000337	−0.005411	−0.001328
c_1	1.871	1.844	1.898
d_1	$−4.817e − 06$	$−1.013e − 05$	$4.922e − 07$
a_2	−0.1428	−0.1788	−0.1069
b_2	−0.00358	−0.005614	−0.001547
c_2	−0.8829	−0.9068	−0.859
d_2	$−8.491e − 06$	$−1.852e − 05$	$1.536e − 06$
a_3	0.001906	0.0001262	0.003685
b_3	0.8226	0.7127	0.9326
c_3	0.1109	0.03341	0.1885
a_4	−1.055	−1.135	−0.9745
c_4	0.9548	0.9056	1.004
a_5	−7510	$−1.051e + 04$	−4508
c_5	$1.13e + 04$	6447	$1.615e + 04$
a_6	0.1103	0.07602	0.1446
b_4	−0.001873	−0.003263	−0.000483
c_6	1.973	1.933	2.013
d_3	$−7.167e − 06$	$−1.331e − 05$	$−1.02e − 06$
a_7	−0.1119	−0.1447	−0.07914
b_5	−0.001911	−0.00325	−0.0005712
c_7	−0.9798	−1.018	−0.9417
d_4	$−1.365e − 05$	$−2.558e − 05$	$−1.725e − 06$
a_8	0.004516	−0.002179	0.01121
b_6	0.8976	0.722	1.073
c_8	0.5459	0.07415	1.018

TABLE 2: MOGA and NSGA II specifications for the stand-level optimal thinning regime problem for *E. fastigata*.

	MOGA	NSGA II
Population size	500	500
Maximum generations	500	500
Number of objectives	2	2
Number of variables	9	9
Number of constraints	1	1
Mutation	Random	Gaussian
Recombination/crossover	Discrete	Intermediate
Selection	Stochastic universal sampling	Tournament

variant as they vary with the changing number of stems per hectare. The state is a vector of values in a Euclidean state space and evolves continuously with time along a continuous trajectory through the state space. The value of the state at any time is governed by its past values, as well as by the past values of all external influences on the system. The external influences that are manipulated to regulate the state constitute the control inputs, which are the decision variables.

The aim in the *E. fastigata* control problem is to determine the control inputs that produce a desired outcome, represented by the two objective functions, (4) and (5). Finding analytical solutions to nonlinear optimal control problems is a nontrivial task. It has only been achieved satisfactorily for limited classes of systems. Even more difficult a task is to analytically solve the optimal control problem. However, we employ evolutionary algorithms for solving the *E. fastigata* control problem as first demonstrated by Chikumbo [29] for a single objective control problem. A summary of the genetic algorithms for the *E. fastigata* control problem is shown in Table 2 for MOGA and NSGA II algorithms that are explained in more detail in the following section.

3. Ranking Schemes for Estimating the Pareto Front

This section explains the basis of the algorithms used to estimate the Pareto front, particularly MOGA and NSGA II. The explanation here is in sink with the findings in this paper.

Many approaches are now in use for MOEA (with different ranking schemes that alter the “fitness landscape” of the search space by adding more “ordering information”), and the key to their success lies in how they accomplish fitness

assignment and selection and how they maintain diversity to achieve a well-distributed Pareto (trade-off) front [15]. For fitness assignment and selection, there exist two groups of ranking schemes, that is, non-Pareto [8, 19] and Pareto-based [30] schemes. The MOGA and NSGA II considered here are both Pareto-based approaches.

Goldberg [30] made a key observation that, for a multimodal search space with multiple peaks, different subdomains (which he called niches) exist with unique stable subgroups of individuals (analogous to species). A conventional evolutionary algorithm will tend to converge its individuals to only one niche (a phenomenon known as genetic drift). However, important information that could have been determined from the other peaks or sub-domains of importance is missed. This is the reason why Chikumbo and Nicholas [2] could only find 6 nondominated solutions for the *E. fastigata* problem, using the MOGA algorithm. The use of ranking schemes encourages the formation of artificial niches, thereby preventing genetic drift and predicting nondominated solutions from most of these important subdomains of a search space.

To maintain diversity in the population, fitness sharing may be used [31]. The idea of fitness sharing is to subdivide the population into several subpopulations based on the similarity among individuals. Note that “similarity” in the context of MOEA can be measured in the space of the decision variables or in the space of the objective functions. Fitness sharing is defined in the following way:

$$\phi(d_{ij}) = \begin{cases} 1 - \left(\frac{d_{ij}}{\sigma_{\text{share}}}\right)^\alpha, & d_{ij} < \sigma_{\text{share}} \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

In expression (14), $\alpha = 1$, d_{ij} indicates the distance between solutions/individuals i and j (in any space defined), and σ_{share} is a parameter (or threshold) that defines the size of a niche or neighbourhood. Any solutions within this distance will be considered as part of the same niche. The fitness, f_s , of an individual i is modified as follows:

$$f_{s_i}^{\text{modified}} = \frac{f_{s_i}}{\sum_{j=1}^M \phi(d_{ij})}, \quad (15)$$

where M is the number of individuals located in the neighborhood (or niche) of the i th individual.

Several other schemes are possible to maintain diversity and to encourage a good spread of solutions, such as crowding. In a crowding model, crowding of solutions anywhere in the search space is discouraged, thereby providing the diversity needed to maintain multiple nondominated solutions. The crowding distance in the model is determined from the sum of distances between solution’s neighbours in Euclidean space that is defined by objective function values.

3.1. Fonseca and Fleming’s Multiobjective Genetic Algorithm (MOGA). The ranking scheme proposed by Fonseca and Fleming [6] involved assigning an individual’s rank (in the objective function space) equal to the number of population individuals that dominated that individual; that is, the ranking was done according to the degree of domination; the

more members of the current population that dominate a particular individual, the lower its rank. It, therefore, uses fitness sharing in the objective function space and mating is also restricted. Reproduction probabilities were determined by means of exponential ranking. Afterwards the fitness values were averaged and shared among individuals having identical ranks [15]. Finally, stochastic universal sampling, which provides zero bias and minimum spread (i.e., the range of possible values for the number of offspring of an individual) was used to fill the sampling pool.

The main strength of MOGA is that it is efficient and relatively easy to implement. It has also been successful in solving optimal control problems where it has exhibited very good overall performance [32].

3.2. Srinivas and Deb’s Nondominated Sorting Genetic Algorithm (NSGA II). NSGA [16] is a nondomination-based genetic algorithm for multi-objective optimisation. It is an effective algorithm that was initially popular. However, it was later criticised for its computational complexity, lack of elitism (i.e., a mechanism for retaining in the population the best individual hitherto), and for choosing the optimal value for the sharing parameter [33]. Consequently Deb et al. [34] developed a second-generation algorithm, NSGA II. This improved version is a better sorting algorithm that incorporates elitism and does not choose the sharing parameter *a priori*. NSGA II is one of the most successful evolutionary multiobjective optimisation algorithms.

Convergence to the Pareto front is made possible by non-dominated sorting. This sorting ranks individuals by iteratively determining the nondominated solutions in the population, assigning those individuals the next best rank and removing them from the population. Favouring individuals with large crowding distances maintain diversity within one rank. NSGA II, being elitist, keeps as many nondominated solutions as possible, up to the size of the population [7]. The elitist selection is as follows. After evaluating the offspring’s fitness (nondominated rank and crowding distance), parents and offspring fight for survival as Pareto dominance is applied to the combined population of parents and offspring. Then, the least dominated N solutions survive to make the population of the next generation. The sampling pool is filled using the tournament selection process, where a number of individuals (ranging from 2 to N) are chosen randomly and the “best” individual from that group is selected as parent for the next generation.

4. Results

The MOGA for the optimal thinning regime optimisation problem ran for 25 minutes using the MATLAB [35] “SPMD” parallel computing for the objective function evaluations on a MacBook Pro, with a 2.93 GHz Intel Core 2 Duo processor. The SPMD is a block of MATLAB statements that denote a single program to be executed by the available processors, and multiple data are assigned to the processors for computation in parallel. In this case it was more of “MPSD,” that is, multiple programs single data, since each

TABLE 3: Summary of the approaches for the *E. fastigata* optimal thinning regime problem using MOGA, NSGA II, and ccGA as computed on a MacBook Pro with a 2.93 GHz Intel Core 2 Duo processor.

Model	Turnaround time (mins)	MATLAB parallel computing	Population size	Max generations	No. of nondominated solutions
MOGA	25	SPMD	500	500	153
NSGA II	82	PARFOR	500	500	500
ccGA-MOGA [2]	39	n/a	500	1000	6

TABLE 4: Incomplete set of the Pareto front estimated using MOGA.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1-value prodn	Obj2-volume prodn
1	6	13	21	35	930	298	101	183	4.79E + 24	2.79E + 26
2	6	13	21	35	930	298	101	183	4.79E + 24	2.79E + 26
3	6	13	21	35	930	298	101	183	4.79E + 24	2.79E + 26
4	6	13	21	35	930	298	101	183	4.79E + 24	2.79E + 26
5	6	13	21	35	930	298	101	183	4.79E + 24	2.79E + 26
6	6	13	21	35	930	298	101	183	4.79E + 24	2.79E + 26
...	...	...	...	...	...	...	...	...	...	...
153	5	13	21	34	902	275	135	190	1.01E + 24	1.18E + 26

processor computed a different objective function each from the same decision variable vector. The NSGA II had a longer turnaround time of 82 minutes, and it also utilised parallel computing using the MATLAB “PARFOR” for the objective function evaluations. PARFOR is an extension of the normal FOR loop statement, where the iterations of statements in the loop are executed in parallel on separate processors.

The MOGA had a total of 153 nondominated solutions whereas NSGA II had 500. Chikumbo and Nicholas [2] ran their competitive coevolutionary genetic algorithm (ccGA) with 5 subpopulations, using MOGA for Pareto front estimation and the same hardware but with no parallel computing. The computation ran over twice as many generations and in 39 minutes, but approximated only 6 nondominated solutions, because of genetic drift. Table 3 shows a summary of these results.

Many of the MOGA nondominated solutions were replicates and superimposed on each other when plotted on graph as shown in Figure 3. It is also clear from the graph that the Pareto front was disjointed, a reminder of the difficulty of solving such kinds of problems using conventional techniques as experienced by Chikumbo and Mareels [36]. Table 4 shows initial nondominated solutions and the complete solutions are found in Table 6.

NSGA II nondominated solutions were more widely spread. This revealed a greater extent of the Pareto front than that was not obvious with MOGA as shown in Figure 4. The Pareto front was even more disjointed being composed of a set of disjoint continuous sets in the objective function space. The disjointed parts gave rise to isolated nondominated solution subsets, while the continuous parts (or topologies) were associated with the actual isolated solutions. Each topology was associated with a specific network of decision

FIGURE 3: Disjointed Pareto front as predicted by the MOGA.

variables or thinning regimes. A snippet of the isolated nondominated solutions is shown in Table 5, and the complete results are found in Table 7.

When the MOGA solutions are superimposed on the NSGA II ones it becomes clear that these solutions are originating from the same niches in the search space as shown in Figure 5.

The 6 nondominated solutions predicted by Chikumbo and Nicholas [2] occupied the niche close to the origin but were not included in the graphical plot in Figure 5 because they were not possible to see clearly.

TABLE 5: Incomplete set of the Pareto front estimated using NSGA II.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1-value prodn	Obj2-volume prodn
1	10	15	20	25	2000	0	0	0	0	0
2	10	15	20	25	2000	0	0	200	$-2.8E+08$	$2.18E+09$
3	5	12	18	35	900	300	200	200	$6.34E+24$	$7.16E+26$
4	5	13	18	35	928	268	187	187	$5.51E+24$	$6.30E+26$
5	5	15	18	35	900	161	191	187	$3.39E+24$	$3.84E+26$
6	6	14	19	34	900	226	179	193	$6.52E+23$	$3.76E+25$
...	...	...	...	...	...	...	...	...	...	...
500	5	13	18	35	902	226	200	193	$4.76E+24$	$5.38E+26$

FIGURE 4: Disjointed Pareto front as predicted by the NSGA II.

FIGURE 5: Superimposed Pareto fronts from MOGA and NSGA II.

Investigating the implications of the overlaps, or lack thereof, of the Pareto sets in the objective function domain involved the comparison in the decision variables domain (or phenotype) using the Kruskal-Willis test [37]. This is a nonparametric version of the classical one-way analysis of variance and an extension of the Wilcoxon rank sum test [38], specifically for testing equality of population medians among groups (which in this case were the three Pareto sets, ccGA-MOGA, MOGA, and NSGA II). Ordering the data from smallest to largest across all groups and taking the numeric index of this ordering determined the ranks.

4.1. Initial Planting Stocking. A P value of $3.8626e-17$ which is practically zero means that the null hypothesis is rejected, suggesting that at least one sample median of the initial planting stockings from the different Pareto sets is significantly different. The box plots in Figure 6 show that the NSGA II Pareto set is different from the other two, which are significantly similar within the 25th and 75th percentiles. The NSGA II shows other initial planting densities (shown as “+” signs in Figure 6) that are higher than the range that overlaps

those from the ccGA-MOGA and MOGA Pareto sets. The Wilcoxon rank sum test, plotted in Figure 7, shows more clearly the difference between the population medians of these Pareto sets for the initial planting stockings.

4.2. Final Crop Number. A P value of 0.1542 and the box plots in Figure 8 suggest some overlap between the final crop numbers from the three Pareto sets, where the MOGA and NSGA II sets are lower than and higher than the common overlapping range of final crop numbers, respectively (as shown by the “+” signs in Figure 8). The mean ranking in Figure 9 emphasizes the degree of overlap of the final crop numbers from the three Pareto sets.

4.3. Rotation Length. There was a difference in the rotation length medians amongst the Pareto sets because of the P value of $1.0275e-04$. The NSGA II Pareto set showed a much greater range of 25–35 years, whereas ccGA-MOGA was consistently 35 years and MOGA only ranged between 34 and 35 years. The box plots are shown in Figure 10. The summary of the mean ranks of the rotation lengths in Figure 11 clarifies

TABLE 6: A Pareto set estimated using MOGA.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1-value prodn	Obj2-volume prodn
1	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
2	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
3	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
4	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
5	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
6	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
7	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
8	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
9	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
10	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
11	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
12	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
13	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
14	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
15	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
16	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
17	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
18	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
19	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
20	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
21	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
22	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
23	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
24	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
25	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
26	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
27	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
28	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
29	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
30	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
31	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
32	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
33	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
34	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
35	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
36	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
37	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
38	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
39	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
40	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
41	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
42	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
43	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
44	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
45	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26

TABLE 6: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1-value prodn	Obj2-volume prodn
46	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
47	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
48	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
49	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
50	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
51	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
52	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
53	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
54	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
55	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
56	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
57	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
58	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
59	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
60	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
61	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
62	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
63	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
64	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
65	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
66	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
67	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
68	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
69	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
70	6	13	21	35	930	298	101	183	4.7873E + 24	2.793E + 26
71	6	15	20	35	921	295	175	196	4.7768E + 24	2.778E + 26
72	6	14	18	35	901	279	197	122	4.601E + 24	2.6556E + 26
73	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
74	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
75	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
76	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
77	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
78	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
79	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
80	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
81	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
82	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
83	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
84	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
85	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
86	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
87	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
88	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
89	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
90	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26

TABLE 6: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1-value prodn	Obj2-volume prodn
91	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
92	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
93	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
94	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
95	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
96	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
97	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
98	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
99	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
100	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
101	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
102	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
103	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
104	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
105	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
106	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
107	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
108	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
109	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
110	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
111	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
112	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
113	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
114	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
115	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
116	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
117	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
118	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
119	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
120	6	12	18	35	979	287	171	187	4.4224E + 24	2.6228E + 26
121	5	14	21	35	920	194	171	200	4.0137E + 24	4.5726E + 26
122	6	13	21	35	930	245	192	158	3.9339E + 24	2.2947E + 26
123	5	14	20	35	901	290	74	190	1.0703E + 24	1.2109E + 26
124	5	13	18	35	900	280	118	160	1.0343E + 24	1.1696E + 26
125	5	14	21	34	908	279	60	175	1.0227E + 24	1.1601E + 26
126	5	15	18	34	998	299	165	171	1.0132E + 24	1.1855E + 26
127	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
128	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
129	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
130	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
131	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
132	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
133	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
134	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
135	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26

TABLE 6: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1-value prodn	Obj2-volume prodn
136	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
137	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
138	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
139	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
140	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
141	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
142	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
143	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
144	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
145	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
146	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
147	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
148	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
149	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
150	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
151	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
152	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26
153	5	13	21	34	902	275	135	190	1.0139E + 24	1.1473E + 26

FIGURE 6: Box plots of the initial planting stockings from the ccGA-MOGA, MOGA, and NSGA II Pareto sets. The edges of each box represent the 25th and 75th percentiles; the ends of the whiskers are the minimum and maximum; outliers are plotted individually with the “+” sign.

the overlap between ccGA-MOGA and MOGA population medians for the rotational lengths, and also between ccGA-MOGA and NSGA II.

4.4. Frequency of Thinning. The NSGA II Pareto set was the only one that had a regime with no thinnings at all where the initial planting stocking was 2000 stems ha⁻¹ with a final harvesting at age 25 years. Also in the set was included a regime with a single thinning at age 20 of 200 stems ha⁻¹ and

FIGURE 7: Wilcoxon rank sum test for the initial planting stockings from the ccGA-MOGA, MOGA, and NSGA II Pareto sets.

final harvesting at age 25 years. As for the rest of the thinning regimes from all the Pareto sets, the frequency of thinning at a total of three in one rotation was the norm with small differences in the timings and intensities.

4.5. Timing and Intensity of Thinning. For brevity only the description (without the box plots and the plotted mean ranks) is given here for the timing and intensity of thinnings.

Timings for the first thinnings from the three Pareto sets showed similarities between NSGA II and ccGA-MOGA with overlaps of the timing medians at 5 years. MOGA had a higher median at 6 years. The intensities of the first thinnings

TABLE 7: A Pareto set estimated using NSGA II.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
1	10	15	20	25	2000	0	0	0	0	0
2	10	15	20	25	2000	0	0	200	-2.8E+08	2.18E+09
3	5	12	18	35	900	300	200	200	6.34E+24	7.16E+26
4	5	13	18	35	928	268	187	187	5.51E+24	6.30E+26
5	5	15	18	35	900	161	191	187	3.39E+24	3.84E+26
6	6	14	19	34	900	226	179	193	6.52E+23	3.76E+25
7	6	15	18	35	903	138	197	196	2.27E+24	1.31E+26
8	5	13	18	35	912	300	189	187	6.26E+24	7.11E+26
9	7	14	19	35	900	214	197	200	1.89E+24	6.64E+25
10	5	14	18	35	900	164	192	190	3.46E+24	3.91E+26
11	7	14	19	35	901	136	199	195	1.20E+24	4.22E+25
12	6	14	19	35	904	123	199	200	2.02E+24	1.17E+26
13	5	15	18	35	901	123	200	199	2.59E+24	2.93E+26
14	5	13	18	35	923	300	191	169	6.20E+24	7.07E+26
15	5	13	18	35	911	263	194	196	5.49E+24	6.24E+26
16	5	13	19	35	923	248	156	198	5.12E+24	5.84E+26
17	5	12	18	35	926	253	187	190	5.22E+24	5.95E+26
18	7	14	19	34	925	152	158	168	4.02E+22	1.42E+24
19	5	13	18	35	906	300	111	188	6.30E+24	7.14E+26
20	5	12	18	35	939	272	195	188	5.54E+24	6.35E+26
21	5	13	18	35	911	209	197	194	4.36E+24	4.95E+26
22	5	14	18	34	945	223	185	185	7.90E+23	9.08E+25
23	6	13	19	35	908	287	177	189	4.70E+24	2.72E+26
24	5	15	18	35	911	186	196	195	3.88E+24	4.41E+26
25	5	13	18	34	907	272	181	174	9.98E+23	1.13E+26
26	6	14	19	35	900	300	174	199	4.95E+24	2.86E+26
27	5	14	18	35	913	190	195	197	3.96E+24	4.50E+26
28	5	15	18	35	908	215	194	194	4.50E+24	5.10E+26
29	5	12	18	35	918	260	189	193	5.40E+24	6.14E+26
30	6	15	18	35	902	168	196	196	2.76E+24	1.60E+26
31	5	14	18	35	915	191	194	191	3.97E+24	4.51E+26
32	6	14	19	35	924	286	176	190	4.62E+24	2.69E+26
33	5	13	18	34	1009	300	198	184	1.01E+24	1.18E+26
34	5	14	18	35	900	215	198	190	4.53E+24	5.13E+26
35	5	14	18	35	906	166	196	197	3.48E+24	3.94E+26
36	5	13	18	35	911	207	197	194	4.32E+24	4.90E+26
37	5	15	18	35	911	176	196	197	3.67E+24	4.17E+26
38	5	15	18	35	909	160	198	198	3.34E+24	3.79E+26
39	6	15	18	35	1067	145	189	200	6.36E+22	3.88E+24
40	5	13	18	35	901	252	186	185	5.31E+24	6.01E+26
41	6	15	18	35	905	143	196	197	2.35E+24	1.36E+26
42	7	14	18	35	903	210	194	182	1.85E+24	6.51E+25
43	5	12	18	35	900	285	181	185	6.02E+24	6.80E+26
44	5	15	18	35	908	141	197	197	2.95E+24	3.34E+26
45	5	13	18	35	918	268	175	181	5.56E+24	6.33E+26
46	5	13	18	35	901	202	200	186	4.26E+24	4.81E+26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
47	5	13	18	35	930	222	194	193	4.55E + 24	5.21E + 26
48	6	13	18	35	901	255	193	178	4.21E + 24	2.43E + 26
49	5	14	18	35	910	234	191	189	4.89E + 24	5.55E + 26
50	5	15	18	35	905	131	198	197	2.75E + 24	3.11E + 26
51	6	14	19	35	976	300	200	199	4.63E + 24	2.75E + 26
52	5	15	18	35	903	178	196	190	3.74E + 24	4.23E + 26
53	5	15	18	34	1019	191	200	182	6.35E + 23	7.48E + 25
54	5	12	18	35	915	282	184	198	5.87E + 24	6.68E + 26
55	5	15	18	35	900	157	199	200	3.31E + 24	3.74E + 26
56	6	15	18	35	902	144	196	197	2.37E + 24	1.37E + 26
57	5	15	18	35	900	121	200	200	2.55E + 24	2.88E + 26
58	7	13	18	35	903	217	195	178	1.91E + 24	6.73E + 25
59	6	13	18	35	937	264	186	189	4.21E + 24	2.46E + 26
60	5	15	18	35	929	161	199	197	3.30E + 24	3.77E + 26
61	5	15	18	35	910	177	192	193	3.69E + 24	4.19E + 26
62	5	14	18	35	918	250	188	186	5.19E + 24	5.90E + 26
63	5	15	18	35	908	161	197	197	3.37E + 24	3.82E + 26
64	7	14	19	35	901	129	199	197	1.14E + 24	4.00E + 25
65	5	13	18	35	900	279	191	182	5.89E + 24	6.66E + 26
66	6	14	18	35	904	156	196	190	2.56E + 24	1.48E + 26
67	7	14	19	35	923	202	194	192	1.75E + 24	6.19E + 25
68	6	14	19	35	906	179	197	194	2.93E + 24	1.70E + 26
69	5	14	18	35	919	206	195	193	4.27E + 24	4.86E + 26
70	5	13	18	35	931	299	183	181	6.14E + 24	7.02E + 26
71	6	14	19	35	927	284	176	189	4.57E + 24	2.67E + 26
72	6	14	19	35	901	150	191	197	2.47E + 24	1.43E + 26
73	5	12	18	35	929	250	186	191	5.14E + 24	5.87E + 26
74	7	13	18	35	902	237	193	171	2.09E + 24	7.35E + 25
75	7	14	19	35	904	141	198	195	1.24E + 24	4.36E + 25
76	5	13	18	35	908	283	120	187	5.93E + 24	6.72E + 26
77	5	13	18	35	903	265	190	195	5.58E + 24	6.31E + 26
78	5	15	18	35	903	136	199	196	2.86E + 24	3.23E + 26
79	6	13	18	35	935	255	193	191	4.08E + 24	2.38E + 26
80	6	14	19	35	929	167	198	195	2.68E + 24	1.56E + 26
81	5	13	18	35	914	221	194	196	4.60E + 24	5.23E + 26
82	5	14	18	35	913	223	191	192	4.65E + 24	5.28E + 26
83	6	13	18	35	900	248	197	170	4.09E + 24	2.36E + 26
84	5	13	18	35	1010	285	192	192	5.47E + 24	6.42E + 26
85	7	14	19	35	919	235	183	200	2.04E + 24	7.22E + 25
86	5	15	18	35	903	140	200	199	2.94E + 24	3.33E + 26
87	5	13	18	35	961	300	194	188	6.00E + 24	6.93E + 26
88	5	13	18	35	900	244	181	182	5.15E + 24	5.82E + 26
89	5	13	18	35	934	246	200	194	5.03E + 24	5.76E + 26
90	5	13	18	35	921	228	195	194	4.72E + 24	5.37E + 26
91	5	15	18	35	903	130	199	199	2.73E + 24	3.09E + 26
92	5	13	18	35	910	268	191	193	5.60E + 24	6.36E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
93	7	13	18	35	902	235	196	168	2.07E + 24	7.29E + 25
94	5	13	18	35	916	237	190	195	4.93E + 24	5.60E + 26
95	6	14	19	35	900	148	194	187	2.44E + 24	1.41E + 26
96	5	13	18	33	900	271	192	191	1.75E + 23	1.98E + 25
97	5	12	18	35	944	300	171	189	6.09E + 24	6.99E + 26
98	5	13	18	35	900	275	187	194	5.81E + 24	6.56E + 26
99	5	15	18	35	903	128	200	200	2.69E + 24	3.04E + 26
100	6	14	19	35	975	217	178	168	5.86E + 23	3.47E + 25
101	5	15	18	35	900	165	177	183	6.08E + 23	6.88E + 25
102	5	15	18	35	913	180	193	192	3.75E + 24	4.26E + 26
103	5	13	18	35	910	296	177	194	6.19E + 24	7.03E + 26
104	6	14	19	35	1096	199	200	177	8.56E + 22	5.27E + 24
105	5	12	18	34	900	300	187	186	1.11E + 24	1.25E + 26
106	5	12	18	35	900	282	200	182	5.96E + 24	6.73E + 26
107	5	13	18	35	961	275	200	200	5.49E + 24	6.35E + 26
108	5	15	18	35	904	142	195	195	2.98E + 24	3.37E + 26
109	7	14	19	35	921	230	183	199	1.99E + 24	7.06E + 25
110	5	13	18	35	922	295	184	181	6.10E + 24	6.96E + 26
111	5	13	18	35	938	222	192	194	4.52E + 24	5.18E + 26
112	6	13	19	35	900	208	197	188	3.43E + 24	1.98E + 26
113	5	14	18	34	958	220	179	182	7.70E + 23	8.89E + 25
114	5	13	18	35	917	281	188	187	5.84E + 24	6.64E + 26
115	5	14	18	35	919	200	199	182	4.14E + 24	4.72E + 26
116	5	12	18	35	908	297	200	200	6.23E + 24	7.06E + 26
117	7	14	19	35	900	139	195	200	1.23E + 24	4.31E + 25
118	7	14	19	35	900	155	186	191	1.37E + 24	4.81E + 25
119	5	12	18	35	902	285	195	193	6.01E + 24	6.80E + 26
120	5	15	18	35	902	129	200	199	2.71E + 24	3.07E + 26
121	5	13	18	35	915	277	188	189	5.77E + 24	6.56E + 26
122	7	14	18	35	967	201	196	193	1.67E + 24	6.02E + 25
123	5	13	18	35	900	288	199	196	6.08E + 24	6.88E + 26
124	5	15	18	35	910	182	195	194	3.80E + 24	4.31E + 26
125	5	12	18	35	916	278	194	198	5.79E + 24	6.58E + 26
126	5	14	18	35	907	202	193	194	4.23E + 24	4.80E + 26
127	5	13	18	35	991	265	194	179	5.16E + 24	6.02E + 26
128	7	14	19	35	922	151	200	200	1.31E + 24	4.63E + 25
129	5	14	18	35	953	213	199	195	4.28E + 24	4.93E + 26
130	8	14	19	35	916	161	189	193	5.95E + 23	1.42E + 25
131	5	12	18	35	934	282	187	193	5.77E + 24	6.61E + 26
132	5	15	18	35	907	149	196	196	3.12E + 24	3.54E + 26
133	5	13	18	35	928	252	166	200	5.18E + 24	5.92E + 26
134	5	12	18	35	922	281	191	190	5.82E + 24	6.63E + 26
135	6	14	19	35	902	162	191	199	2.67E + 24	1.54E + 26
136	5	15	18	35	900	144	200	191	3.03E + 24	3.43E + 26
137	7	14	18	35	906	208	194	195	1.83E + 24	6.44E + 25
138	6	14	19	35	913	148	196	197	2.41E + 24	1.40E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
139	5	15	18	35	911	147	195	195	3.06E + 24	3.48E + 26
140	5	15	18	35	903	150	200	199	3.15E + 24	3.57E + 26
141	6	15	18	35	948	101	199	190	2.79E + 23	1.64E + 25
142	5	13	18	35	911	240	200	198	5.01E + 24	5.69E + 26
143	6	14	18	35	900	196	200	199	3.23E + 24	1.86E + 26
144	5	15	18	35	904	133	198	197	2.79E + 24	3.16E + 26
145	5	15	18	35	905	167	200	183	3.50E + 24	3.97E + 26
146	5	14	18	35	911	195	199	199	4.07E + 24	4.62E + 26
147	5	14	18	35	910	200	196	195	4.18E + 24	4.74E + 26
148	5	15	18	35	904	156	195	191	3.27E + 24	3.71E + 26
149	5	15	18	35	904	135	200	198	2.83E + 24	3.21E + 26
150	5	13	18	35	916	259	188	188	5.38E + 24	6.12E + 26
151	7	15	18	35	1000	168	198	196	2.38E + 23	8.66E + 24
152	7	14	19	35	900	150	196	180	1.32E + 24	4.65E + 25
153	5	13	18	34	943	238	178	200	8.45E + 23	9.70E + 25
154	7	14	19	35	902	133	199	198	1.17E + 24	4.12E + 25
155	5	15	18	35	904	191	200	194	4.01E + 24	4.54E + 26
156	5	12	18	35	907	262	192	187	5.50E + 24	6.23E + 26
157	5	15	18	35	919	176	183	199	3.64E + 24	4.15E + 26
158	5	13	18	35	903	244	187	186	5.13E + 24	5.81E + 26
159	5	15	18	35	900	119	200	200	2.51E + 24	2.83E + 26
160	5	12	18	35	901	286	199	200	6.04E + 24	6.82E + 26
161	5	15	18	35	908	146	197	197	3.05E + 24	3.46E + 26
162	5	14	18	35	936	191	196	191	3.89E + 24	4.46E + 26
163	5	15	18	35	900	212	192	200	4.47E + 24	5.05E + 26
164	6	14	19	35	906	182	198	195	2.98E + 24	1.73E + 26
165	5	12	18	35	915	300	197	200	6.25E + 24	7.10E + 26
166	7	14	19	35	920	222	187	196	1.93E + 24	6.82E + 25
167	5	13	18	35	936	265	179	196	5.41E + 24	6.20E + 26
168	5	14	18	35	999	208	188	181	7.03E + 23	8.23E + 25
169	7	15	18	35	985	174	198	195	2.49E + 23	9.04E + 24
170	5	12	18	35	929	257	189	194	5.28E + 24	6.03E + 26
171	5	14	18	35	916	195	187	197	4.05E + 24	4.61E + 26
172	5	12	18	35	900	287	189	195	6.06E + 24	6.85E + 26
173	7	13	18	35	900	252	200	178	2.23E + 24	7.83E + 25
174	5	12	18	34	968	298	199	196	1.04E + 24	1.20E + 26
175	7	14	19	35	900	201	200	200	1.78E + 24	6.24E + 25
176	5	13	18	35	900	222	178	196	4.68E + 24	5.29E + 26
177	5	15	18	35	900	132	200	200	2.78E + 24	3.14E + 26
178	6	14	19	35	900	113	190	200	3.26E + 23	1.88E + 25
179	5	15	18	35	903	148	194	195	3.11E + 24	3.52E + 26
180	5	13	18	35	900	300	138	171	6.33E + 24	7.16E + 26
181	5	13	18	35	900	249	200	189	5.25E + 24	5.94E + 26
182	6	13	18	35	920	238	191	193	3.86E + 24	2.24E + 26
183	5	13	18	35	900	195	200	186	4.11E + 24	4.65E + 26
184	6	14	19	35	929	268	167	183	4.31E + 24	2.51E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
185	5	15	18	35	905	147	194	194	3.08E + 24	3.49E + 26
186	6	15	18	35	903	148	200	194	2.43E + 24	1.40E + 26
187	5	15	18	35	903	126	200	199	2.65E + 24	3.00E + 26
188	6	13	18	35	905	274	185	191	4.50E + 24	2.60E + 26
189	6	15	18	34	1034	123	176	175	9.69E + 21	5.85E + 23
190	5	14	18	35	937	188	199	199	3.83E + 24	4.39E + 26
191	7	14	19	35	910	171	186	197	1.50E + 24	5.28E + 25
192	5	13	18	35	936	234	185	188	4.78E + 24	5.47E + 26
193	5	13	18	35	923	258	195	191	5.33E + 24	6.08E + 26
194	5	13	18	35	917	300	124	186	6.24E + 24	7.09E + 26
195	5	15	18	35	900	134	200	199	2.82E + 24	3.19E + 26
196	5	13	18	35	915	258	186	197	5.37E + 24	6.10E + 26
197	5	12	18	35	904	238	200	200	5.01E + 24	5.66E + 26
198	6	14	19	35	900	237	199	193	3.91E + 24	2.26E + 26
199	5	14	18	35	900	206	200	188	4.34E + 24	4.91E + 26
200	5	14	18	35	910	183	196	194	3.82E + 24	4.34E + 26
201	6	14	19	35	901	278	172	177	4.58E + 24	2.65E + 26
202	5	15	18	35	904	186	200	194	3.91E + 24	4.42E + 26
203	6	14	19	35	900	146	186	195	2.41E + 24	1.39E + 26
204	6	13	19	35	928	207	196	194	3.33E + 24	1.94E + 26
205	6	14	18	35	904	158	196	191	2.59E + 24	1.50E + 26
206	5	15	18	35	909	142	197	197	2.97E + 24	3.36E + 26
207	6	13	18	35	910	244	195	187	3.99E + 24	2.31E + 26
208	6	14	19	35	913	173	197	196	2.82E + 24	1.63E + 26
209	5	12	18	35	923	287	191	190	5.94E + 24	6.76E + 26
210	5	13	18	34	914	269	191	182	9.81E + 23	1.11E + 26
211	5	14	18	35	914	182	193	196	3.79E + 24	4.30E + 26
212	5	12	18	35	900	268	200	162	5.66E + 24	6.40E + 26
213	5	14	18	34	972	248	196	170	8.58E + 23	9.96E + 25
214	5	13	18	34	900	286	188	180	1.06E + 24	1.19E + 26
215	5	12	18	35	1055	255	178	196	8.26E + 23	9.83E + 25
216	5	14	18	34	1027	209	197	200	6.91E + 23	8.16E + 25
217	6	15	18	35	901	135	199	198	2.22E + 24	1.28E + 26
218	5	15	18	35	903	129	200	198	2.71E + 24	3.07E + 26
219	6	13	18	35	913	294	190	197	4.80E + 24	2.78E + 26
220	6	14	19	35	901	271	173	178	4.47E + 24	2.58E + 26
221	5	13	18	35	1033	185	181	195	1.07E + 23	1.26E + 25
222	5	14	18	35	922	215	188	196	4.44E + 24	5.06E + 26
223	6	13	18	35	900	282	178	200	4.66E + 24	2.69E + 26
224	5	12	18	35	936	244	186	188	4.98E + 24	5.71E + 26
225	7	14	19	35	903	172	199	199	1.51E + 24	5.33E + 25
226	7	14	19	35	918	154	199	199	1.34E + 24	4.73E + 25
227	5	13	18	34	913	300	190	184	1.10E + 24	1.24E + 26
228	5	12	18	35	993	272	192	197	5.29E + 24	6.18E + 26
229	7	14	19	35	919	233	183	200	2.02E + 24	7.16E + 25
230	5	13	18	35	924	206	187	190	4.25E + 24	4.85E + 26
231	7	13	18	35	900	278	192	153	2.46E + 24	8.64E + 25

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
232	5	15	18	35	903	139	199	199	2.92E + 24	3.30E + 26
233	5	12	18	35	968	297	187	180	5.90E + 24	6.83E + 26
234	5	14	18	35	948	225	182	200	4.54E + 24	5.23E + 26
235	6	14	19	35	910	225	197	192	3.68E + 24	2.13E + 26
236	5	13	18	35	914	236	195	196	4.91E + 24	5.59E + 26
237	5	15	18	35	905	126	199	199	2.64E + 24	2.99E + 26
238	6	13	18	35	907	275	192	187	4.51E + 24	2.61E + 26
239	5	15	18	35	901	125	199	200	2.63E + 24	2.97E + 26
240	5	13	18	35	907	233	197	198	4.88E + 24	5.54E + 26
241	5	13	18	34	900	293	187	180	1.08E + 24	1.22E + 26
242	5	13	18	35	918	243	189	195	5.04E + 24	5.74E + 26
243	6	13	18	35	911	245	197	196	4.00E + 24	2.32E + 26
244	7	13	18	35	903	205	198	174	1.81E + 24	6.36E + 25
245	5	15	18	35	901	130	199	200	2.73E + 24	3.09E + 26
246	5	14	18	35	903	217	189	198	4.56E + 24	5.17E + 26
247	5	13	18	35	903	221	195	192	4.65E + 24	5.26E + 26
248	5	13	18	35	911	294	188	187	6.14E + 24	6.98E + 26
249	5	13	18	35	916	215	195	190	4.47E + 24	5.08E + 26
250	6	14	19	35	951	250	192	191	3.94E + 24	2.32E + 26
251	6	14	19	35	926	243	200	193	3.91E + 24	2.28E + 26
252	5	12	18	34	971	300	200	197	1.04E + 24	1.21E + 26
253	6	15	18	35	952	108	188	189	5.20E + 22	3.06E + 24
254	5	13	18	35	937	248	189	192	5.06E + 24	5.80E + 26
255	6	15	18	35	949	128	194	188	3.53E + 23	2.07E + 25
256	7	14	19	35	924	199	194	192	1.72E + 24	6.10E + 25
257	5	12	18	35	900	292	194	185	6.17E + 24	6.97E + 26
258	5	12	18	35	977	237	192	199	4.67E + 24	5.42E + 26
259	5	14	18	35	914	183	192	197	3.81E + 24	4.33E + 26
260	7	14	19	35	900	190	200	200	1.68E + 24	5.90E + 25
261	6	14	18	35	922	218	191	188	3.52E + 24	2.05E + 26
262	7	14	19	35	922	190	195	193	1.64E + 24	5.83E + 25
263	5	12	18	35	904	300	178	196	6.32E + 24	7.15E + 26
264	6	15	18	35	906	142	196	197	2.33E + 24	1.35E + 26
265	5	12	18	35	901	246	199	179	5.19E + 24	5.87E + 26
266	5	13	18	35	919	242	190	196	5.02E + 24	5.71E + 26
267	5	13	18	34	931	249	181	181	8.94E + 23	1.02E + 26
268	5	13	18	35	1031	224	189	200	7.38E + 23	8.73E + 25
269	6	15	18	35	949	158	200	194	4.36E + 23	2.56E + 25
270	5	13	18	35	938	245	200	181	4.99E + 24	5.72E + 26
271	5	15	18	35	910	170	197	197	3.55E + 24	4.03E + 26
272	5	13	18	35	905	255	196	192	5.36E + 24	6.07E + 26
273	7	14	19	35	919	162	198	198	1.41E + 24	4.98E + 25
274	6	13	18	35	908	246	197	191	4.03E + 24	2.33E + 26
275	6	13	18	35	900	252	197	168	4.16E + 24	2.40E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
276	5	15	18	35	908	169	193	194	3.53E + 24	4.01E + 26
277	7	14	19	35	903	178	199	195	1.57E + 24	5.52E + 25
278	5	13	18	35	923	239	196	191	4.94E + 24	5.63E + 26
279	7	14	19	35	903	173	196	195	1.52E + 24	5.36E + 25
280	7	14	19	35	911	177	184	197	1.55E + 24	5.46E + 25
281	5	13	18	35	904	251	182	184	5.28E + 24	5.98E + 26
282	5	12	18	35	914	292	197	199	6.09E + 24	6.92E + 26
283	6	14	19	35	903	233	194	196	3.83E + 24	2.21E + 26
284	5	15	18	35	900	155	200	199	3.26E + 24	3.69E + 26
285	5	15	18	35	900	156	195	196	3.29E + 24	3.72E + 26
286	6	13	18	35	907	275	192	187	4.51E + 24	2.61E + 26
287	5	15	18	35	901	125	199	200	2.63E + 24	2.97E + 26
288	5	13	18	35	907	233	197	198	4.88E + 24	5.54E + 26
289	5	13	18	34	900	293	187	180	1.08E + 24	1.22E + 26
290	5	13	18	35	918	243	189	195	5.04E + 24	5.74E + 26
291	6	13	18	35	911	245	197	196	4.00E + 24	2.32E + 26
292	7	13	18	35	903	205	198	174	1.81E + 24	6.36E + 25
293	5	15	18	35	901	130	199	200	2.73E + 24	3.09E + 26
294	5	14	18	35	903	217	189	198	4.56E + 24	5.17E + 26
295	5	13	18	35	903	221	195	192	4.65E + 24	5.26E + 26
296	5	13	18	35	911	294	188	187	6.14E + 24	6.98E + 26
297	5	13	18	35	916	215	195	190	4.47E + 24	5.08E + 26
298	6	14	19	35	951	250	192	191	3.94E + 24	2.32E + 26
299	6	14	19	35	926	243	200	193	3.91E + 24	2.28E + 26
300	5	12	18	34	971	300	200	197	1.04E + 24	1.21E + 26
301	6	15	18	35	952	108	188	189	5.20E + 22	3.06E + 24
302	5	13	18	35	937	248	189	192	5.06E + 24	5.80E + 26
303	6	15	18	35	949	128	194	188	3.53E + 23	2.07E + 25
304	7	14	19	35	924	199	194	192	1.72E + 24	6.10E + 25
305	5	12	18	35	900	292	194	185	6.17E + 24	6.97E + 26
306	5	12	18	35	977	237	192	199	4.67E + 24	5.42E + 26
307	5	14	18	35	914	183	192	197	3.81E + 24	4.33E + 26
308	7	14	19	35	900	190	200	200	1.68E + 24	5.90E + 25
309	6	14	18	35	922	218	191	188	3.52E + 24	2.05E + 26
310	7	14	19	35	922	190	195	193	1.64E + 24	5.83E + 25
311	5	12	18	35	904	300	178	196	6.32E + 24	7.15E + 26
312	6	15	18	35	906	142	196	197	2.33E + 24	1.35E + 26
313	5	12	18	35	901	246	199	179	5.19E + 24	5.87E + 26
314	5	13	18	35	919	242	190	196	5.02E + 24	5.71E + 26
315	5	13	18	34	931	249	181	181	8.94E + 23	1.02E + 26
316	5	13	18	35	1031	224	189	200	7.38E + 23	8.73E + 25
317	6	15	18	35	949	158	200	194	4.36E + 23	2.56E + 25
318	5	13	18	35	938	245	200	181	4.99E + 24	5.72E + 26
319	5	15	18	35	910	170	197	197	3.55E + 24	4.03E + 26
320	5	13	18	35	905	255	196	192	5.36E + 24	6.07E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
321	7	14	19	35	919	162	198	198	1.41E + 24	4.98E + 25
322	6	13	18	35	908	246	197	191	4.03E + 24	2.33E + 26
323	6	13	18	35	900	252	197	168	4.16E + 24	2.40E + 26
324	5	15	18	35	908	169	193	194	3.53E + 24	4.01E + 26
325	7	14	19	35	903	178	199	195	1.57E + 24	5.52E + 25
326	5	13	18	35	923	239	196	191	4.94E + 24	5.63E + 26
327	7	14	19	35	903	173	196	195	1.52E + 24	5.36E + 25
328	7	14	19	35	911	177	184	197	1.55E + 24	5.46E + 25
329	5	13	18	35	904	251	182	184	5.28E + 24	5.98E + 26
330	5	12	18	35	914	292	197	199	6.09E + 24	6.92E + 26
331	6	14	19	35	903	233	194	196	3.83E + 24	2.21E + 26
332	5	15	18	35	900	155	200	199	3.26E + 24	3.69E + 26
333	5	15	18	35	900	156	195	196	3.29E + 24	3.72E + 26
334	5	15	18	35	904	183	196	193	3.84E + 24	4.35E + 26
335	5	13	18	33	1035	256	200	186	1.47E + 23	1.74E + 25
336	6	14	18	35	907	223	194	198	3.65E + 24	2.11E + 26
337	5	15	18	35	905	173	200	182	3.63E + 24	4.11E + 26
338	5	12	18	35	900	269	193	188	5.68E + 24	6.42E + 26
339	7	14	19	34	962	137	190	196	2.00E + 23	7.20E + 24
340	5	13	18	35	920	254	173	197	5.26E + 24	5.99E + 26
341	7	14	19	35	900	163	200	200	1.44E + 24	5.06E + 25
342	5	15	18	35	906	157	196	195	3.29E + 24	3.73E + 26
343	6	13	18	35	914	267	186	181	4.35E + 24	2.52E + 26
344	6	14	19	35	963	194	200	188	3.02E + 24	1.78E + 26
345	6	14	19	35	903	130	198	199	2.14E + 24	1.23E + 26
346	7	14	19	35	900	157	200	200	1.39E + 24	4.87E + 25
347	5	13	18	35	966	257	176	176	5.11E + 24	5.92E + 26
348	6	14	19	35	904	178	189	193	2.92E + 24	1.69E + 26
349	5	14	18	35	959	216	199	195	4.32E + 24	4.99E + 26
350	7	14	19	35	956	247	188	197	2.07E + 24	7.45E + 25
351	5	15	18	35	906	138	200	198	2.89E + 24	3.28E + 26
352	6	14	19	35	912	236	191	193	3.85E + 24	2.23E + 26
353	5	14	18	35	908	224	192	192	4.69E + 24	5.32E + 26
354	5	15	18	35	900	158	189	178	5.82E + 23	6.59E + 25
355	7	14	19	35	900	116	200	196	1.79E + 23	6.30E + 24
356	5	13	18	35	911	273	188	187	5.70E + 24	6.48E + 26
357	5	15	18	35	900	152	199	198	3.20E + 24	3.62E + 26
358	6	13	18	35	905	288	183	191	4.73E + 24	2.74E + 26
359	5	15	18	35	904	165	194	192	3.46E + 24	3.92E + 26
360	6	14	19	35	905	269	176	182	4.42E + 24	2.55E + 26
361	5	15	18	35	907	152	196	196	3.18E + 24	3.61E + 26
362	7	14	19	35	900	142	181	188	2.19E + 23	7.71E + 24
363	5	12	18	35	900	286	194	185	6.04E + 24	6.83E + 26
364	7	14	19	34	961	206	196	198	3.01E + 23	1.08E + 25
365	5	12	18	35	913	251	190	196	5.24E + 24	5.95E + 26
366	5	12	18	35	1023	263	189	200	8.74E + 23	1.03E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
367	5	14	18	35	910	193	193	192	4.03E + 24	4.57E + 26
368	5	13	18	35	1009	227	192	191	7.62E + 23	8.94E + 25
369	5	15	18	35	903	125	200	199	2.62E + 24	2.97E + 26
370	6	14	19	35	900	299	191	194	4.94E + 24	2.85E + 26
371	5	13	18	35	934	254	172	196	5.19E + 24	5.95E + 26
372	5	13	18	35	917	284	186	186	5.90E + 24	6.71E + 26
373	6	14	19	34	900	157	185	200	4.53E + 23	2.61E + 25
374	6	14	19	35	939	228	188	188	3.63E + 24	2.12E + 26
375	6	14	18	35	900	201	199	195	3.31E + 24	1.91E + 26
376	6	13	18	35	907	237	193	185	3.89E + 24	2.25E + 26
377	5	13	18	35	905	291	183	186	6.12E + 24	6.93E + 26
378	6	14	19	35	929	209	187	191	3.36E + 24	1.96E + 26
379	5	15	18	35	910	178	191	194	3.72E + 24	4.22E + 26
380	5	13	18	35	910	290	178	194	6.07E + 24	6.88E + 26
381	6	14	19	35	900	145	194	186	2.39E + 24	1.38E + 26
382	5	14	18	35	916	199	198	184	4.13E + 24	4.70E + 26
383	5	14	18	35	906	171	197	196	3.58E + 24	4.06E + 26
384	5	13	18	34	950	239	195	194	8.43E + 23	9.71E + 25
385	5	12	18	35	901	283	199	200	5.97E + 24	6.75E + 26
386	5	13	18	35	924	299	191	170	6.17E + 24	7.04E + 26
387	6	13	18	35	945	260	196	185	4.12E + 24	2.42E + 26
388	6	15	18	35	900	153	200	187	2.52E + 24	1.45E + 26
389	6	14	19	35	903	139	198	199	2.28E + 24	1.32E + 26
390	5	13	18	34	932	253	177	178	9.07E + 23	1.04E + 26
391	5	15	18	35	903	124	200	199	2.60E + 24	2.95E + 26
392	6	14	19	35	901	140	199	200	2.30E + 24	1.33E + 26
393	5	14	18	35	907	168	197	194	3.52E + 24	3.99E + 26
394	6	14	19	35	962	286	197	199	4.47E + 24	2.64E + 26
395	7	14	19	35	1016	154	195	200	2.15E + 23	7.88E + 24
396	5	14	18	35	922	223	200	199	4.61E + 24	5.25E + 26
397	6	14	19	35	992	275	200	187	4.19E + 24	2.50E + 26
398	6	15	18	35	903	142	197	196	2.33E + 24	1.35E + 26
399	5	13	18	35	916	245	194	197	5.09E + 24	5.79E + 26
400	5	14	18	35	903	172	189	190	3.61E + 24	4.09E + 26
401	6	14	18	35	942	246	199	180	3.91E + 24	2.29E + 26
402	5	14	18	35	905	170	196	194	3.57E + 24	4.04E + 26
403	5	15	18	35	900	170	200	193	3.58E + 24	4.05E + 26
404	5	14	18	35	910	244	191	186	5.10E + 24	5.79E + 26
405	6	14	19	35	923	271	183	190	4.38E + 24	2.55E + 26
406	5	15	18	35	900	127	200	200	2.67E + 24	3.02E + 26
407	5	15	18	35	909	181	194	195	3.78E + 24	4.29E + 26
408	6	14	19	35	969	209	199	200	3.24E + 24	1.92E + 26
409	5	12	18	35	930	280	169	191	5.75E + 24	6.57E + 26
410	5	14	18	35	900	176	198	200	3.71E + 24	4.19E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
411	5	13	18	34	907	259	189	176	9.50E + 23	1.08E + 26
412	7	14	19	35	901	193	199	180	1.70E + 24	5.99E + 25
413	5	14	18	35	913	197	195	188	4.10E + 24	4.66E + 26
414	7	15	18	35	973	185	197	195	2.68E + 23	9.67E + 24
415	5	12	18	35	900	277	199	200	5.85E + 24	6.61E + 26
416	5	15	18	35	904	143	199	197	3.00E + 24	3.40E + 26
417	6	13	18	35	911	268	193	191	4.38E + 24	2.54E + 26
418	5	13	18	34	902	261	193	191	9.62E + 23	1.09E + 26
419	5	15	18	35	913	174	198	194	3.62E + 24	4.12E + 26
420	5	13	18	35	914	253	190	190	5.27E + 24	5.99E + 26
421	5	14	18	34	908	224	192	184	8.20E + 23	9.31E + 25
422	5	12	18	35	901	273	190	195	5.76E + 24	6.51E + 26
423	5	13	18	35	926	176	193	194	3.62E + 24	4.13E + 26
424	6	14	19	35	915	191	195	194	3.11E + 24	1.80E + 26
425	5	15	18	35	900	124	200	200	2.61E + 24	2.95E + 26
426	7	13	18	35	901	220	194	174	1.94E + 24	6.83E + 25
427	6	14	19	35	900	128	200	200	2.11E + 24	1.22E + 26
428	5	13	18	35	915	232	196	195	4.83E + 24	5.49E + 26
429	5	15	18	35	911	154	199	193	3.21E + 24	3.65E + 26
430	5	13	18	35	901	199	200	186	4.19E + 24	4.74E + 26
431	5	14	18	35	906	194	196	194	4.07E + 24	4.61E + 26
432	5	12	18	35	918	276	186	198	5.73E + 24	6.52E + 26
433	7	14	19	35	923	135	200	200	1.17E + 24	4.14E + 25
434	5	14	18	35	910	232	189	189	4.85E + 24	5.50E + 26
435	7	14	19	35	938	211	191	197	1.80E + 24	6.42E + 25
436	6	14	18	34	935	185	187	188	5.17E + 23	3.02E + 25
437	5	13	18	35	994	242	188	200	4.70E + 24	5.49E + 26
438	6	15	18	35	946	108	197	190	2.99E + 23	1.75E + 25
439	5	15	18	35	911	153	199	197	3.19E + 24	3.62E + 26
440	6	13	18	35	947	224	199	199	3.54E + 24	2.08E + 26
441	5	13	18	34	951	195	192	196	6.87E + 23	7.91E + 25
442	7	13	18	35	901	258	198	163	2.28E + 24	8.01E + 25
443	5	15	18	35	913	155	194	195	3.23E + 24	3.67E + 26
444	7	14	19	35	903	161	196	181	1.42E + 24	4.99E + 25
445	6	14	18	35	919	222	194	188	3.60E + 24	2.09E + 26
446	5	15	18	35	933	215	199	181	4.40E + 24	5.03E + 26
447	6	13	18	35	906	206	195	190	3.38E + 24	1.95E + 26
448	5	15	18	35	909	132	199	200	2.76E + 24	3.13E + 26
449	5	15	18	35	910	180	193	196	3.76E + 24	4.27E + 26
450	6	13	18	35	914	280	194	199	4.56E + 24	2.65E + 26
451	5	12	18	35	953	284	200	194	5.72E + 24	6.58E + 26
452	5	13	18	35	935	241	191	191	4.92E + 24	5.64E + 26
453	6	15	18	35	904	176	194	194	2.89E + 24	1.67E + 26
454	6	14	19	35	931	213	187	191	3.41E + 24	1.99E + 26
455	5	15	18	35	900	123	200	200	2.59E + 24	2.93E + 26

TABLE 7: Continued.

Regime	Age at 1st thinning (years)	Age at 2nd thinning (years)	Age at 3rd thinning (years)	Clear felling age (years)	Initial planting density (stems ha ⁻¹)	Stems thinned at 1st thinning (stems ha ⁻¹)	Stems thinned at 2nd thinning (stems ha ⁻¹)	Stems thinned at 3rd thinning (stems ha ⁻¹)	Obj1- value prodn	Obj2-volume prodn
456	5	12	18	35	946	265	200	194	5.37E + 24	6.17E + 26
457	5	13	18	35	918	243	189	195	5.04E + 24	5.74E + 26
458	5	14	18	35	935	216	187	188	4.41E + 24	5.05E + 26
459	5	13	18	35	920	253	174	197	5.24E + 24	5.97E + 26
460	5	13	18	35	929	237	200	195	4.87E + 24	5.56E + 26
461	5	15	18	35	902	136	182	200	5.00E + 23	5.66E + 25
462	5	15	18	35	904	135	198	198	2.83E + 24	3.21E + 26
463	5	13	18	35	911	210	197	194	4.38E + 24	4.98E + 26
464	5	13	18	35	920	229	196	192	4.74E + 24	5.40E + 26
465	5	12	18	35	919	292	185	200	6.06E + 24	6.90E + 26
466	5	13	18	35	913	260	192	192	5.42E + 24	6.16E + 26
467	6	14	19	35	921	164	197	196	2.65E + 24	1.54E + 26
468	6	15	18	35	901	133	199	198	2.19E + 24	1.26E + 26
469	5	14	18	35	915	194	191	196	4.03E + 24	4.59E + 26
470	5	13	18	35	921	240	199	193	4.97E + 24	5.66E + 26
471	5	14	18	35	906	218	194	197	4.57E + 24	5.18E + 26
472	7	13	18	35	901	249	197	168	2.20E + 24	7.73E + 25
473	6	14	19	35	916	184	189	197	2.99E + 24	1.73E + 26
474	6	15	18	35	906	141	196	197	2.31E + 24	1.34E + 26
475	6	14	19	34	901	142	192	200	4.09E + 23	2.36E + 25
476	6	15	18	35	952	126	194	187	3.47E + 23	2.04E + 25
477	6	14	19	35	903	155	195	190	2.55E + 24	1.47E + 26
478	5	15	18	35	913	146	196	197	3.04E + 24	3.45E + 26
479	6	14	19	35	906	164	185	193	2.69E + 24	1.55E + 26
480	5	13	18	35	969	274	200	200	5.44E + 24	6.30E + 26
481	5	14	18	35	909	212	193	187	4.43E + 24	5.03E + 26
482	5	12	18	35	936	285	192	187	5.83E + 24	6.67E + 26
483	5	14	18	35	903	211	191	192	4.44E + 24	5.02E + 26
484	5	14	18	35	904	174	193	192	3.65E + 24	4.14E + 26
485	5	13	18	35	1001	263	190	200	5.08E + 24	5.95E + 26
486	5	12	18	35	903	292	187	196	6.15E + 24	6.96E + 26
487	5	15	18	35	912	181	194	195	3.77E + 24	4.28E + 26
488	5	12	18	35	900	284	189	184	6.00E + 24	6.78E + 26
489	6	14	19	35	903	175	198	198	2.88E + 24	1.66E + 26
490	5	15	18	35	900	171	200	193	3.60E + 24	4.07E + 26
491	6	14	19	35	906	186	186	194	3.05E + 24	1.76E + 26
492	5	15	18	35	908	137	199	198	2.86E + 24	3.25E + 26
493	5	14	18	35	910	197	190	192	4.11E + 24	4.67E + 26
494	5	13	18	35	912	272	185	187	5.68E + 24	6.45E + 26
495	5	12	18	35	906	300	181	197	6.30E + 24	7.14E + 26
496	6	14	19	35	900	212	197	195	3.50E + 24	2.02E + 26
497	6	14	18	35	916	228	193	182	3.71E + 24	2.15E + 26
498	7	14	19	35	929	150	198	197	1.29E + 24	4.58E + 25
499	5	13	18	35	913	274	149	190	5.71E + 24	6.49E + 26
500	5	13	18	35	902	226	200	193	4.76E + 24	5.38E + 26

FIGURE 8: Final crop number box plots from the ccGA-MOGA, MOGA, and NSGA II Pareto sets.

FIGURE 10: Rotation length box plots from the ccGA-MOGA, MOGA, and NSGA II Pareto sets.

FIGURE 9: Wilcoxon rank sum test for the final crop numbers from the ccGA-MOGA, MOGA, and NSGA II Pareto sets.

FIGURE 11: Wilcoxon rank sum test for the rotation lengths from the ccGA-MOGA, MOGA and NSGA II Pareto sets.

also showed some similarities from all the Pareto sets although the NSGA II intensities covered a wider range than the other two.

The timings for the second thinnings were similar for the MOGA and NSGA II, whereas the intensities from all the Pareto sets were also the same, with NSGA II, again, showing a wider range of intensities.

The timings for the third thinnings showed dissimilarities between MOGA and NSGA II but showing thinning intensity similarities from all the three Pareto sets, with both MOGA and NSGA II showing a wider range of thinning intensities.

5. Discussion

The previous results suggest that the nondominated solutions from the three Pareto sets belong to the same family of solutions. It is clear that the Pareto front is disjointed with many topologies and the differences observed in the decision variables (i.e., initial planting stocking; final crop number; rotation length; frequency, timing, and intensity of thinning) are mere reflections of the different topologies of the Pareto

front or sub-domains (niches) of the nondominated solutions. It also seems that the computational efficiency of MOGA or ccGA-MOGA is due to the fact that the Fonseca and Fleming ranking scheme attempts to only predict part of the Pareto front, whereas NSGA II seems to approximate all the Pareto points, hence the differences in the turnaround times as shown in Table 3. This is despite employing parallel computing capability for the MOGA and NSGA II algorithms. It is also important to note that, with just an Intel Core 2 Duo processor, it may not be possible to experience appreciable improvements in the computational efficiencies.

The question that lingers with the Fonseca and Fleming ranking scheme is how it determines the subdomain of the Pareto front to approximate and whether it is possible for the user to have control on which subdomain of the Pareto front to estimate. The same question can be raised about the NSGA II algorithm, whether it can be made to concentrate on estimating Pareto points on a specific subdomain rather than all the Pareto points, making the algorithm more computationally efficient. Although the answer to the above question is beyond the scope of this paper, it does seem obvious that MOGA and NSGA II can be applied for different

output and computational requirements. Where computational efficiency is of paramount importance, MOGA would be the appropriate choice but at the expense of approximating only a part of the Pareto front. The user will also not have the ability to choose the subregion on the Pareto front to estimate. Situations with large problems characterised by many objectives (greater than three) and a large set of decision variables may benefit from the application of MOGA. However, when it is vitally important to approximate the entire Pareto front, NSGA II would be ideal, but with a computational overhead. Note also that most research in evolutionary algorithms has centered on dealing with 2 or 3 objective functions and beyond that is still an area of active research [39].

A larger set of nondominated solutions from the search space implies greater flexibility in decision making in sifting through a plethora of alternatives based on prevailing preferences and values of the decision maker(s). From a silvicultural management perspective, this is good news for the forester, as such flexibility at a stand level facilitates implementing a mosaic of thinning regimes across an estate to meet sawlog and pulpwood demands from markets that are plagued with uncertainty.

The ability of NSGA II to maintain diversity in the search space to approximate an entire Pareto front lies in the way it does its fitness assignment (through fitness sharing) as alluded to earlier. The crowding model in NSGA II has been demonstrated to work, which encouraged a good spread of solutions over all the subdomains or topologies of the Pareto front. Although MOGA also has the ability to form niches and distribute the population over the Pareto-optimal region, its fitness sharing is done in the objective function domain. This maintains diversity in the objective function values and may not necessarily do so for the decision variables, which are important to the decision maker(s) [16]. An attempt to understand this further led to running MOGA and NSGA II models over 1000 generations. MOGA converged to a single point on the Pareto front, whereas NSGA II kept all its 500 Pareto points. As for the ccGA-MOGA, which is just a modified MOGA with 5 subpopulations linked through migration instead of one large population [2], diversity was maintained through the subpopulations [5] and convergence to a single Pareto point after 1000 generations was avoided. To construct a better MOGA that will not converge to a single point on the Pareto front, several niching and elitist models have to be considered, and Obayashi et al. [40] reviewed some of these models.

At this stage there is no similar study in this same problem domain of optimal silvicultural regimes that can be compared against the results obtained here. However, there is work in other disciplines that compare well with some of the findings in this paper.

- (a) Ducheyne et al. [41] found out NSGA II to produce better results than MOGA although the latter had a more evenly spread-out Pareto front, for a bi-objective forest estate management problem where the estate consisted of 295 stands (or management units). The problem was run for only 50 generations,

leaving questions as to whether the results would have been any different if the problem was run over more generations. The Pareto front was also smooth and continuous, a sharp contrast to the one in this paper;

- (b) Zitzler et al. [15] provided a systematic comparison of various approaches to biobjective function optimisation using carefully chosen test functions. MOGA and the first-generation NSGA were among some of these approaches. Some of the test functions produced smooth well-distributed nondominated sets and the most difficult ones, disjointed Pareto fronts. NSGA was ranked better than MOGA for performance.
- (c) Sağ and Çunkaş [42] also compared various approaches that included NSGA, NSGA II, MOGA, VEGA, SPEA, SPEA2 (second-generation SPEA), NPGA, and PESA, using test functions for multi-objective optimisation. Their general findings were that while the NSGA-II, SPEA2, and PESA could keep track of all the feasible solutions found during the optimisation, the number of Pareto-optimal solutions of other algorithms remained poor. Focusing on a more difficult problem that had two objective functions and two constraints where the Pareto set was discontinuous and divided into 5 regions, NSGA-II, SPEA2 and PESA kept track of all the feasible solutions. MOGA was poor at finding points at the extreme ends of the curve giving a good sampling of solutions at the midsection of the curve. However, the number of Pareto-optimal solutions remained poor for MOGA.

This paper has attempted to provide an insight into some of the well-established algorithms for estimating MOEA Pareto fronts at a stand level. The intention was not to minimise or disregard the important work already done in other disciplines. Instead, the aim was to bring forest analysts close to the MOEA community so that both can interact and mutually benefit. In the near future, if forest analysts take on board some of the findings presented in this paper, then the goals of this paper will have been accomplished.

6. Conclusion

The results clearly indicate that, for the stand-level optimal thinning regime problem, the NSGA II is capable of approximating an entire Pareto front at the expense of computational efficiency and that MOGA and ccGA-MOGA, though computationally efficient, will crowd solutions and also estimate only a part of the Pareto front. The key to NSGA II wider coverage in estimating the entire Pareto front lies in its ability of information sharing across individual potential solutions and maintaining diversity in the search space in collectively satisfying the two objectives, sawlog production and pulpwood production. For a better MOGA that will approximate an entire Pareto front, a niching and elitist model will have to be included that will provide a means for controlling the number, location, extent, and distribution

of solutions in the search space. In its current form, MOGA suffers from genetic drift and will converge to a single point on the Pareto front if run for many generations. However, with the island model version, ccGA-MOGA, where the population is divided into subpopulations, genetic drift may be minimised although not sufficiently enough to avert estimation for only a subregion of the Pareto front. The final take-home message here is that, for a MOEA, it is important to choose an approach fit for purpose as in finding the appropriate balance between computational efficiency and the extent of the Pareto front to be estimated.

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References

- [1] O. Chikumbo, R. H. Bradbury, and S. Davey, "Large-scale ecosystem management as a complex systems problem: multi-objective optimisation with spatial constraints," in *Applied Complexity—From Neural Nets to Managed Landscapes*, S. Halloy and T. Williams, Eds., pp. 124–155, Institute for Crop & Food Research Ltd, Christchurch, New Zealand, 2000.
- [2] O. Chikumbo and I. Nicholas, "Efficient thinning regimes for *Eucalyptus fastigata*: multi-objective stand-level optimisation using the island model genetic algorithm," *Ecological Modelling*, vol. 222, no. 10, pp. 1683–1695, 2011.
- [3] M. J. Osborne and A. Rubenstein, *A Course in Game Theory* (p7), MIT Press, Cambridge, MA, USA, 1994.
- [4] P. Siarry, A. Pétrowski, and M. Bessaou, "A multipopulation genetic algorithm aimed at multimodal optimization," *Advances in Engineering Software*, vol. 33, no. 4, pp. 207–213, 2002.
- [5] O. Chikumbo, "Exploration and exploitation in function optimization using stochastic generate-and-test algorithms," in *Proceedings of the International Conference on Genetic and Evolutionary Methods*, H. R. Arabnia and A. M. G. Solo, Eds., pp. 22–27, CSREA Press, Las Vegas, Nev, USA, July 2009.
- [6] C. M. Fonseca and P. J. Fleming, "Genetic algorithms for multiple objective optimization: formulation, discussion and generalization," in *Proceedings of the 5th International Conference on Genetics Algorithms*, S. Forrest, Ed., Morgan Kaufmann Publishers, San Mateo, Calif, USA, 1993.
- [7] L. T. Bui, J. Branke, and H. A. Abbass, "Multi-objective optimisation for dynamic environments," in *Proceedings of the Congress on Evolutionary Computation*, pp. 2349–2356, IEEE Press, Edinburgh, UK, 2005.
- [8] J. D. Schaffer, "Multiple objective optimisation with vector evaluated genetic algorithms," in *Genetic Algorithms and their Applications: Proceedings of the 1st International Conference on Genetic Algorithms*, L. Erlbaum, Ed., pp. 93–100, 1985.
- [9] H. Polheim, "GEATbx: introduction, evolutionary algorithms: overview, methods and operators," 2006, <http://www.geatbx.com/>.
- [10] J. H. Holland, *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann Arbor, Mich, USA, 1975.
- [11] I. Rechenberg, *Evolutionsstrategie—Optimierung Technischer Systeme Nach Prinzipien der Biologischen Evolution*, Frommann-Holzboog, Stuttgart, Germany, 1973.
- [12] H.-P. Schwefel, *Numerical Optimisation of Computer Models*, John Wiley & Sons, Chichester, UK, 1981.
- [13] L. J. Fogel, A. J. Owens, and M. J. Walsh, "Intelligent decision making through a simulation of evolution," *Behavioral Science*, vol. 11, no. 4, pp. 253–272, 1966.
- [14] C. A. Coello Coello, "20 Years of evolutionary multi-objective optimization: what has been done and what remains to be done," in *Computational Intelligence: Principles and Practice*, G. Y. Yen and D. B. Fogel, Eds., pp. 73–78, IEEE Computational Intelligence Society, 2006.
- [15] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, 2000.
- [16] N. Srinivas and K. Deb, "Multi-objective optimisation using non-dominated sorting in genetic algorithms," *Evolutionary Computation*, vol. 2, no. 3, pp. 221–248, 1994.
- [17] F. Y. Edgeworth, *Mathematical Psychics*, P. Keagan, London, UK, 1881.
- [18] V. Pareto, *Cours D'Economie Politique*, vol. 1-2, F. Rouge, Lausanne, Switzerland, 1896.
- [19] P. Hajela and C. Y. Lin, "Genetic search strategies in multicriterion optimal design," *Structural Optimization*, vol. 4, no. 2, pp. 99–107, 1992.
- [20] J. Horn, N. Nafpliotis, and D. E. Goldberg, "A niched Pareto genetic algorithm for multi-objective optimisation," in *Proceedings of the 1st IEEE Conference on Evolutionary Computation, IEEE World Congress on Computational Computation*, vol. 1, pp. 82–87, IEEE Services Centre, Piscataway, NJ, USA, 1994.
- [21] C. A. Coello Coello, "Using the min-max method to solve multi-objective optimization problems with genetic algorithms," in *Proceedings of the 6th Ibero-American Conference on AI: Progress in Artificial Intelligence (IBERAMIA '98)*, pp. 303–313, Springer, London, UK, 1998.
- [22] E. Zitzler and L. Thiele, "An evolutionary algorithm for multi-objective optimization: the strength Pareto approach," Tech. Rep. 43, Computer Engineering and Networks Laboratory, Swiss Federal Institute of Technology, Zurich, 1998.
- [23] D. W. Corne, J. D. Knowles, and M. J. Oates, "The Pareto envelope-based selection Algorithm for multi-objective optimisation," in *Proceedings of the Parallel Problem Solving from Nature 6th Conference*, M. Schoenauer, K. Deb, G. Rudolph, X. Yao, E. Lutton, and J. J. Merelo, Eds., pp. 839–849, Paris, France, 2000.
- [24] D. J. Knowles and D. Corne, "The Pareto archived evolution strategy: a new baseline algorithm for Pareto multi-objective optimisation," in *Proceedings of the Congress on Evolutionary Computation (CEC '99)*, P. J. Angeline, Z. Michalewicz, M. Schoenauer, X. Yao, and A. Zalzal, Eds., vol. 1, pp. 98–105, 1999.
- [25] J. A. Nelder, "New kinds of systematic designs for spacing experiments," *Biometrics*, vol. 18, pp. 283–307, 1962.
- [26] J. H. Mayo and T. J. Straka, "The holding value premium in standing timber valuation," *Appraisal Journal*, vol. 37, no. 1, pp. 98–106, 2005.
- [27] L. Ljung, *System Identification: Theory for the User*, Prentice Hall, Upper Saddle River, NJ, USA, 1987.
- [28] O. Chikumbo, I. M. Y. Mareels, and B. J. Turner, "Predicting stand basal area in thinned stands using a dynamical model,"

- Forest Ecology and Management*, vol. 116, no. 1–3, pp. 175–187, 1999.
- [29] O. Chikumbo, “An optimal regime model using competitive co-evolutionary genetic algorithms,” in *Proceedings of the International Joint Conference on Computational Intelligence*, A. Dourado, A. Rosa, and K. Madani, Eds., pp. 210–217, Institute for Systems and Technologies of Information, Control and Communication (ISTICC), Madeira, Portugal, October 2009.
 - [30] D. E. Goldberg, *Genetic Algorithms in Search, Optimisation and Machine Learning*, Addison-Wesley, Reading, Mass, USA, 1989.
 - [31] D. E. Goldberg and J. Richardson, “Genetic algorithms with sharing for multimodal function optimization,” in *Proceedings of the 2nd International Conference on Genetic Algorithms and their Applications*, J. Grefenstette, Ed., pp. 41–49, Lawrence Erlbaum Associates, Cambridge, Mass, USA, 1987.
 - [32] C. A. Coello Coello, *An empirical study of evolutionary techniques for multi-objective optimization in engineering design*, Ph.D. thesis, Tulane University, New Orleans, La, USA, 1996.
 - [33] C. A. Coello Coello, “A short tutorial on evolutionary multi-objective optimisation,” in *Proceedings of the 1st International Conference on Evolutionary Multi-Criterion Optimisation*, E. Zitzler, K. Deb, L. Thiele, C. A. Coello Coello, and D. Corne, Eds., Lecture Notes in Computer Science No. 1993, pp. 21–40, Springer, 2001.
 - [34] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.
 - [35] MathWorks, 2011, <http://www.mathworks.com/help/techdoc/index.html>.
 - [36] O. Chikumbo and I. M. Y. Mareels, “Predicting terminal time and final crop number for a forest plantation stand: Pontryagin’s maximum principle,” in *Ecosystems and Sustainable Development*, E. Tiezzi, C. A. Brebbia, and J. L. Uso, Eds., vol. 2 of ISBN 1-85312-834-X, pp. 1227–1237, WIT Press, 2003.
 - [37] W. Kruskal and W. A. Wallis, “Use of ranks in one-criterion analysis of variance,” *Journal of the American Statistical Association*, vol. 47, pp. 583–621, 1952.
 - [38] J. D. Gibbons, *Nonparametric Statistical Inference*, Marcel Dekker, New York, NY, USA, 2nd edition, 1985.
 - [39] J. Wallenius, J. S. Dyer, P. C. Fishburn, R. E. Steuer, S. Zionts, and K. Deb, “Multiple criteria decision making, multiattribute utility theory: recent accomplishments and what lies ahead,” *Management Science*, vol. 54, no. 7, pp. 1336–1349, 2008.
 - [40] S. Obayashi, S. Takahashi, and Y. Takeguchi, “niching and elitist models for MOGAs,” in *Proceedings of the 5th International Conference Parallel Problem Solving From Nature (PPSN ’98)*, A. E. Eiben, M. Schoenauer, and H. P. Schwefel, Eds., pp. 260–269, 1998.
 - [41] E. Ducheyne, R. R. De Wulf, and B. De Baets, “Bi-objective genetic algorithms for forest management planning: a comparative study,” in *Proceedings of the Genetic and Evolutionary Computation Conference*, L. Spector, E. D. Goodman, A. Wu et al., Eds., pp. 63–66, Morgan Kaufmann Publishers, July 2001.
 - [42] T. Sağ and M. Çunkaş, “A tool for multiobjective evolutionary algorithms,” *Advances in Engineering Software*, vol. 40, no. 9, pp. 902–912, 2009.