Research Article

Propagation of Rayleigh Wave in a Thermoelastic Solid Half-Space with Microtemperatures

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The Rayleigh surface wave is studied at a stress-free thermally insulated surface of an isotropic, linear, and homogeneous thermoelastic solid half-space with microtemperatures. The governing equations of the thermoelastic medium with microtemperatures are solved for surface wave solutions. The particular solutions in the half-space are applied to the required boundary conditions at stress-free thermally insulated surface to obtain the frequency equation of the Rayleigh wave. Some special cases are also derived. The non-dimensional speed of Rayleigh wave is computed numerically and presented graphically to reveal the dependence on the frequency and microtemperature constants.

1. Introduction

The theory of materials with microstructures has been a subject of intensive study in the literature since E. Cosserat and F. Cosserat [1]. The microtemperature and/or microdeformation of the nanoparticles could be considered very important in future technologies. The thermoelasticity with microtemperatures considers the microstructure of the body, in which each microelement possesses a microtemperature. The theory of thermodynamics for elastic material with innerstructures was developed by Grot [2] according to which the molecules possess microtemperatures along with macrodeformation of the body. The experimental data for the silicone rubber containing spherical aluminum particles and for human blood presented by Říha [3] conform closely to the predicted theoretical model of thermoelasticity with microtemperatures.

Iesan and Quintanilla [4] developed the linear theory for elastic materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures. Ieşan [5] proposed a theory of thermoelastic bodies with microstructures and microtemperatures, where the microelements of the material possess microtemperatures and can undergo microrotation,

microstretch and translation. Ieşan and Quintanilla [6] presented the theory of thermoelastic bodies with inner structure and microtemperatures, which permits the transmission of heat as thermal waves at finite speed. Several papers based on the theory of thermoelasticity with microtemperatures have been published such as Iesan [7], Svanadze [8, 9], Casas and Quintanilla [10], Scalia and Svanadze [11, 12], Aoudai [13], Iesan [14], Scalia et al. [15], Ieşan and Scalia [16], Yang and Huang [17], Quintanilla [18], Svanadze and Tracinà [19], Chirita et al. [20], and Steeb et al. [21].

Due to increasing interest in nanomaterials, the significance of microtemperature and/or microdeformation of the nanoparticles cannot be ignored. The studies related to wave propagation in the theory of thermoelastic materials with microtemperature may be important in future technologies. The theory of thermoelasticity with microtemperatures (Iesan and Quintanilla [4]) is applied to study the Rayleigh wave at the thermally insulated stress-free surface of an isotropic, homogeneous thermoelastic solid half-space with microtemperature. The frequency equation of the Rayleigh wave is obtained. The dependence of numerical values of the speed of the Rayleigh wave on material parameters, frequency, and microtemperature constants is shown graphically for a particular material of the model.

2. Basic Equations

Following Iesan and Quintanilla [4], the constitutive relations for homogeneous and isotropic thermoelastic medium with microtemperatures are

$$\begin{aligned}\sigma_{ij} &= \lambda \delta_{ij} e_{rr} + 2\mu e_{ij} - \beta T \delta_{ij}, \\ q_i &= K T_{,i} + K_1 w_i, \\ q_{ij} &= -K_4 w_{r,r} \delta_{ij} - K_5 w_{i,j} - K_6 w_{j,i}, \\ Q_i &= (K_1 - K_2) w_i + (K - K_3) T_{,i}, \\ \rho \eta^* &= \beta e_{rr} + a T, \\ \rho \epsilon_i &= -b w_i,\end{aligned}\quad (1)$$

where

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad (2)$$

and $\lambda, \mu, \beta, a, b, K$ and K_j ($j = 1, 2, \dots, 6$) are constitutive coefficients. σ_{ij} are the components of the stress tensor. e_{ij} are the components of the strain tensor. ρ is the reference mass density of the medium. η^* is entropy per unit mass. ϵ_i are the components of the first moment of energy vector. q_{ij} are the components of the first heat flux moment vector. Q_i are the components of the mean heat flux vector. q_i are components of the heat flux vector. u_i are the components of the displacement vector $\vec{u}$. w_i are the components of the microtemperature vector $\vec{w}$. $T = \Theta - T_0$, where Θ is the temperature at time t . T_0 is the temperature of the medium in its natural state and assumed to be such that $|T/T_0| \ll 1$. A comma in the subscript denotes the spatial derivative and δ_{ij} is the Kronecker delta.

Following Iesan and Quintanilla [4], the constitutive equations (1) combined with the reduced Clausius-Duhem inequality in context of the linear theory of thermoelasticity with microtemperature imply the following inequalities:

$$\begin{aligned}3K_4 + K_5 + K_6 &\geq 0, & K_6 + K_5 &\geq 0, \\ K_6 - K_5 &\geq 0, & K &\geq 0, \\ (K_1 + T_0 K_3)^2 - 4T_0 K K_2 &\leq 0.\end{aligned}\quad (3)$$

Following Iesan and Quintanilla [4], the fundamental system of field equations of the linear theory of thermoelasticity with microtemperatures

(i) the equations of motion

$$\sigma_{ji,j} + \rho f_i = \rho \ddot{u}_i, \quad (4)$$

(ii) the balance energy

$$\rho T_0 \dot{\eta}^* = q_{i,i} + \rho S, \quad (5)$$

(iii) the first moment of energy

$$\rho \dot{\epsilon}_i = q_{ji,j} + q_i - Q_i + \rho M_i, \quad (6)$$

where f_i are the components of the body force vector, M_i are the components of the first heat source moment vector, and S is the heat supply. Superposed dot represents the temporal derivative and other symbols are described previously.

Using (1) and (2) in (4) to (6), the following system of linear partial differential equations is obtained:

$$\begin{aligned}\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta T_{,i} + \rho f_i &= \rho \ddot{u}_i, \\ K T_{,ii} - \beta T_0 \dot{u}_{i,i} + K_1 w_{i,i} + \rho S &= a T_0 \dot{T}, \\ K_6 w_{i,jj} + (K_4 + K_5) w_{j,ji} - K_3 T_{,i} - K_2 w_i - \rho M_i &= b \dot{w}_i.\end{aligned}\quad (7)$$

The field equations (7) in term- of displacement, macro- and microtemperatures for a linear homogeneous elastic solid in the absence of body force, heat source, and first-heat source moment vector are written in the following form:

$$\begin{aligned}\mu \nabla^2 \vec{u} + (\lambda + 2\mu) \nabla (\nabla \cdot \vec{u}) - \beta \nabla T &= \rho \ddot{\vec{u}}, \\ K \nabla^2 T - \beta T_0 \nabla \cdot \dot{\vec{u}} + K_1 (\nabla \cdot \vec{w}) - a T_0 \dot{T} &= 0, \\ K_6 \nabla^2 \vec{w} + (K_4 + K_5) \nabla (\nabla \cdot \vec{w}) - K_3 \nabla T - K_2 \vec{w} - b \dot{\vec{w}} &= 0.\end{aligned}\quad (8)$$

3. Analytical 2D Solution

We consider a homogeneous and isotropic thermoelastic medium of an infinite extent with Cartesian coordinates system (x, y, z) , which is previously at uniform temperature. The origin is taken on the plane surface and the z -axis is taken normally into the medium ($z \geq 0$). The surface $z = 0$ is assumed stress free and thermally insulated. The present study is restricted to the plane strain parallel to the x - z plane, with the displacement vector $\vec{u} = (u_1, 0, u_3)$. Introducing the scalar potentials ϕ and ξ , and vector potential $\vec{\psi}$ through Helmholtz representation of a vector field as

$$\vec{u} = \nabla \phi + \nabla \times \vec{\psi}, \quad \nabla \cdot \vec{\psi} = 0, \quad \vec{w} = \nabla \xi. \quad (9)$$

Inserting (9) in (8), we obtain

$$\mu \nabla^2 \vec{\psi} = \rho \ddot{\vec{\psi}}, \quad (10)$$

$$(\lambda + 2\mu) \nabla^2 \phi - \beta T = \rho \ddot{\phi}, \quad (11)$$

$$K \nabla^2 T - \beta T_0 \nabla^2 \dot{\phi} - a T_0 \dot{T} + K_1 \nabla^2 \xi = 0, \quad (12)$$

$$(K_4 + K_5 + K_6) \nabla^2 \xi - K_2 \xi - b \ddot{\xi} - K_3 T = 0. \quad (13)$$

For thermoelastic surface waves in the half-space propagating in x -direction, the potential functions ϕ , T , and ξ are taken in the following form:

$$\{\phi, T, \xi, \psi\} = \{\hat{\phi}(z), \hat{T}(z), \hat{\xi}(z), \hat{\psi}(z)\} \exp \{i(\eta x - \chi t)\}, \quad (14)$$

where $\chi^2 = \eta^2 c^2$, η is the wave number, c is the phase velocity, and $\psi = (\vec{\psi})_y$. Substituting (14) in (11) to (13) and eliminating $\hat{\phi}$, $\hat{T}$, and $\hat{\xi}$, we obtain the following auxiliary equation:

$$D^6 - \eta^2 A_0 D^4 + \eta^4 A_1 D^2 - \eta^6 A_2 = 0, \quad (15)$$

where $D = d/dz$ and

$$\begin{aligned}
 A_0 &= 3 - (a_1 + a_2 + a_3 + a_4 + a_5) c^2, \\
 A_1 &= 3 - 2(a_1 + a_2 + a_3 + a_4 + a_5) c^2 \\
 &\quad + (a_1 a_2 + a_2 a_3 + a_2 a_4 + a_3 a_4 + a_4 a_5) c^4, \\
 A_2 &= 1 - (a_1 + a_2 + a_3 + a_4 + a_5) c^2 \\
 &\quad + (a_1 a_2 + a_2 a_3 + a_2 a_4 + a_3 a_4 + a_4 a_5) c^4 - (a_2 a_3 a_4) c^6, \\
 a_1 &= \frac{\beta \beta_0}{K \mu_0 \chi^2}, \quad a_2 = \frac{K_8}{K_7 \chi^2}, \quad a_3 = \frac{a_0}{K \chi^2}, \\
 a_4 &= \frac{\rho}{\mu_0}, \quad a_5 = \frac{K_1 K_3}{K K_7 \chi^2}, \\
 a_0 &= i \chi a T_0, \quad \beta_0 = i \chi \beta T_0, \\
 K_7 &= K_4 + K_5 + K_6, \quad K_8 = i b \chi - K_2.
 \end{aligned} \tag{16}$$

Taking into account (15) and keeping in mind that $\hat{\phi}, \hat{T}, \hat{\xi} \rightarrow 0$ as $z \rightarrow \infty$ for surface waves, the solutions ϕ , T , and ξ are written as

$$\begin{aligned}
 \phi &= [A \exp(-\eta \lambda_1 z) + B \exp(-\eta \lambda_2 z) \\
 &\quad + C \exp(-\eta \lambda_3 z)] \exp\{\iota(\eta x - \chi t)\}, \\
 T &= [p_1 A \exp(-\eta \lambda_1 z) + p_2 B \exp(-\eta \lambda_2 z) \\
 &\quad + p_3 C \exp(-\eta \lambda_3 z)] \exp\{\iota(\eta x - \chi t)\}, \\
 \xi &= [q_1 A \exp(-\eta \lambda_1 z) + q_2 B \exp(-\eta \lambda_2 z) \\
 &\quad + q_3 C \exp(-\eta \lambda_3 z)] \exp\{\iota(\eta x - \chi t)\},
 \end{aligned} \tag{17}$$

where

$$\begin{aligned}
 \lambda_1^2 + \lambda_2^2 + \lambda_3^2 &= A_0, \\
 \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_1^2 \lambda_3^2 &= A_1, \\
 \lambda_1^2 \lambda_2^2 \lambda_3^2 &= A_2, \\
 p_i &= \frac{\eta^2 \{ \rho c^2 - (\lambda + 2\mu)(1 - \lambda_i^2) \}}{\beta}, \quad (i = 1, 2, 3), \\
 q_i &= \frac{K_3 p_i}{K_8 - \eta^2 (1 - \lambda_i^2) K_7}, \quad (i = 1, 2, 3).
 \end{aligned} \tag{18}$$

Substituting (14) in to (10) and keeping in mind that $\hat{\psi} \rightarrow 0$ as $z \rightarrow \infty$ for surface waves, we obtain the following solution:

$$\psi = E \exp[-\eta \lambda_4 z + \iota(\eta x - \chi t)], \tag{19}$$

where

$$\lambda_4^2 = 1 - \frac{c^2}{c_2^2}, \quad c_2^2 = \frac{\mu}{\rho}. \tag{20}$$

4. Derivation of Frequency Equation

The mechanical and thermal conditions at the thermally insulated surface $z = 0$ are as follows:

(i) vanishing of the normal stress component

$$\sigma_{zz} = 0, \tag{21}$$

(ii) vanishing of the tangential stress component

$$\sigma_{zx} = 0, \tag{22}$$

(iii) vanishing of the normal heat flux component

$$q_z = 0, \tag{23}$$

(iv) vanishing of normal first heat flux moment vector component

$$q_{zz} = 0, \tag{24}$$

where

$$\begin{aligned}
 \sigma_{zz} &= \lambda \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} \right) + 2\mu \left(\frac{\partial^2 \psi}{\partial x \partial z} \right) + 2\mu \frac{\partial^2 \phi}{\partial z^2} - \beta T, \\
 \sigma_{zx} &= \mu \left[2 \frac{\partial^2 \phi}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} \right],
 \end{aligned} \tag{25}$$

$$q_z = K \frac{\partial T}{\partial z} + K_1 \frac{\partial \xi}{\partial z},$$

$$q_{zz} = K_4 \frac{\partial^2 \xi}{\partial x^2} + K_7 \frac{\partial^2 \xi}{\partial z^2}.$$

Using the solutions (17) and (19) for ϕ , T , ξ , and ψ in (21) to (24) and eliminating A , B , C , and E , the following equation is obtained:

$$X_1 Y_1 + X_2 Y_2 + X_3 Y_3 = 0, \tag{26}$$

where

$$\begin{aligned}
 X_1 &= (1 - \lambda_4^2) s_1 + 4\mu \lambda_1 \lambda_4, \\
 X_2 &= (1 - \lambda_4^2) s_2 + 4\mu \lambda_2 \lambda_4, \\
 X_3 &= (1 - \lambda_4^2) s_3 + 4\mu \lambda_3 \lambda_4, \\
 Y_1 &= \lambda_2 q_3 (K p_2 + K_1 q_2) (K_4 - K_7 \lambda_3^2) \\
 &\quad - \lambda_3 q_2 (K p_3 + K_1 q_3) (K_4 - K_7 \lambda_2^2), \\
 Y_2 &= \lambda_3 q_1 (K p_3 + K_1 q_3) (K_4 - K_7 \lambda_1^2) \\
 &\quad - \lambda_1 q_3 (K p_1 + K_1 q_1) (K_4 - K_7 \lambda_3^2), \\
 Y_3 &= \lambda_1 q_2 (K p_1 + K_1 q_1) (K_4 - K_7 \lambda_2^2) \\
 &\quad - \lambda_2 q_1 (K p_2 + K_1 q_2) (K_4 - K_7 \lambda_1^2), \\
 s_i &= \lambda (1 - \lambda_i^2) - 2\mu \lambda_i^2 + \frac{\beta p_i}{\eta^2}, \quad (i = 1, 2, 3).
 \end{aligned} \tag{27}$$

Equation (26) is the frequency equation of Rayleigh wave in thermoelastic medium with microtemperature.

5. Special Cases

5.1. Isotropic Thermoelastic Case. In the case where microtemperature is absent, that is, when $K_1 = K_2 = K_3 = K_4 = K_5 = K_6 = b = 0$, the frequency equation (26) is reduced to

$$(1 - \lambda_4^{*2})(\lambda_2^* p_2^* s_1^* - \lambda_1^* p_1^* s_2^*) + 4\mu\lambda_1^* \lambda_2^* \lambda_4^* (p_2^* - p_1^*) = 0, \quad (28)$$

where

$$\begin{aligned} \lambda_1^{*2} + \lambda_2^{*2} &= 2 - c^2(a_1 + a_3 + a_4), \\ \lambda_1^{*2} \lambda_2^{*2} &= 1 - c^2(a_1 + a_3 + a_4) + a_3 a_4 c^4, \\ p_i^* &= \frac{\eta^2 \{\rho c^2 - (\lambda + 2\mu)(1 - \lambda_i^{*2})\}}{\beta}, \quad (i = 1, 2), \\ s_i^* &= \lambda(1 - \lambda_i^{*2}) - 2\mu\lambda_i^{*2} + \frac{\beta p_i^*}{\eta^2}, \quad (i = 1, 2). \end{aligned} \quad (29)$$

Equation (28) is the frequency equation of Rayleigh wave in an isotropic thermoelastic solid.

5.2. Isotropic Elastic Case. In the case where thermal parameters are neglected, the frequency equation (28) is reduced to

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\sqrt{1 - \frac{c^2}{c_1^2}} \sqrt{1 - \frac{c^2}{c_2^2}}, \quad (30)$$

which is the frequency equation of Rayleigh wave for an isotropic elastic case.

6. Numerical Example

The speed of propagation of Rayleigh wave is computed for the following physical constants of the model: $\lambda = 2.17 \times 10^{10} \text{ N}\cdot\text{m}^{-2}$, $\mu = 3.278 \times 10^{10} \text{ N}\cdot\text{m}^{-2}$, $\rho = 1.74 \times 10^3 \text{ kg}\cdot\text{m}^{-3}$, $aT_0 = 1.8 \times 10^6 \text{ J}\cdot\text{m}^{-3} \text{ deg}^{-1}$, $\beta = 2.68 \times 10^6 \text{ N}\cdot\text{m}^{-2} \text{ deg}^{-1}$, $K = 1.7 \times 10^2 \text{ W}\cdot\text{m}^{-1} \text{ deg}^{-1}$, $K_1 = 2 \times 10^{10} \text{ W}\cdot\text{m}^{-1}$, $K_2 = 0.1 \times 10^{10} \text{ W}\cdot\text{m}^{-1}$, $K_3 = 0.4 \times 10^{10} \text{ W}\cdot\text{m}^{-1}$, $K_4 = 0.3 \times 10^{10} \text{ W}\cdot\text{m}^{-1}$, $K_5 = 0.5 \times 10^{10} \text{ W}\cdot\text{m}^{-1}$, $K_6 = 0.7 \times 10^{10} \text{ W}\cdot\text{m}^{-1}$, $b = 1.3849 \times 10^{10} \text{ N}$, $T_0 = 298 \text{ K}$, and $\chi = 3.58 \times 10^{11} \text{ s}^{-1}$.

The non-dimensional speed $\sqrt{\rho c^2/\mu}$ of Rayleigh wave is computed and plotted against frequency for the range $2 \text{ Hz} \leq \chi \leq 30 \text{ Hz}$. It increases very sharply with the increase of frequency as shown in Figure 1. The non-dimensional speed $\sqrt{\rho c^2/\mu}$ of Rayleigh wave is also computed for certain ranges of microtemperature constants. The non-dimensional speed $\sqrt{\rho c^2/\mu}$ of Rayleigh wave remains almost constant at different values of microtemperature constants K_1 and K_3 . The non-dimensional speed $\sqrt{\rho c^2/\mu}$ of Rayleigh wave decreases very sharply with the increase of K_2 in the range $0.01 \leq K_2 \leq 1 \text{ W}\cdot\text{m}^{-1}$. This variation of non-dimensional speed against K_2 is shown in Figure 2. The variation of

FIGURE 1: Variation of the nondimensional speed $(\rho c^2/\mu)^{1/2}$ of Rayleigh wave versus frequency χ .

FIGURE 2: Variation of the nondimensional speed $(\rho c^2/\mu)^{1/2}$ of Rayleigh wave versus microtemperature constant K_2 .

the non-dimensional speed $\sqrt{\rho c^2/\mu}$ of Rayleigh wave is similar against microtemperature constants K_4 , K_5 , and K_6 . It first decreases very sharply to its minimum value and thereafter it increases with the increase of K_4 , K_5 and K_6 values in the range $0 \leq K_4, K_5, \text{ or } K_6 \leq 1 \text{ W}\cdot\text{m}^{-3}$ as shown in Figures 3, 4, and 5. The non-dimensional speed of Rayleigh wave significantly depends on the frequency and microtemperature constants, as evident from Figures 1 to 5.

FIGURE 3: Variation of the nondimensional speed $(\rho c^2/\mu)^{1/2}$ of Rayleigh wave versus microtemperature constant K_4 .

FIGURE 5: Variation of the nondimensional speed $(\rho c^2/\mu)^{1/2}$ of Rayleigh wave versus microtemperature constant K_6 .

FIGURE 4: Variation of the nondimensional speed $(\rho c^2/\mu)^{1/2}$ of Rayleigh wave versus microtemperature constant K_5 .

7. Conclusion

The appropriate solutions of all the governing equations of thermoelastic medium with microtemperatures are applied at the boundary conditions at a thermally insulated free surface of a half-space to obtain the frequency equation of Rayleigh wave. From the frequency equation of Rayleigh wave, it is observed that the phase speed of Rayleigh wave depends on various material parameters including the microtemperature

parameters. The dependence of numerical values of non-dimensional speed of propagation on the frequency and microtemperature parameters is shown graphically for a particular material representing the model. The problem though is theoretical but it can provide useful information for experimental researchers working in the field of geophysics and earthquake engineering and seismologist working in the field of mining tremors and drilling into the Earth crust. The study on wave propagation phenomenon in thermoelasticity with microtemperature is at its early stage. Recently, Steeb et al. [21] introduced the plane waves in such material. The present paper studied the propagation of Rayleigh wave in thermoelastic half-space with microtemperature. Based on theoretical results obtained by Steeb et al. [21] and in this paper, it is quiet early to predict possible specific applications of this phenomenon. The rapid advancement of MEMS/NEMS technology needs design and fabrication of microstructures. The possible applications of such studies may be in development of microtemperature sensors.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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