Research Article

TEM Response of a Large Loop Source over the Multilayer Earth Models

A. K. Tiwari, S. P. Maurya, and N. P. Singh

Department of Geophysics, Faculty of Science, Banaras Hindu University, Varanasi 221005, India

Correspondence should be addressed to N. P. Singh; singhnphbu@yahoo.co.in

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The general expression of TEM response of large loop source over the layered earth models is not available in the literature for arbitrary source-receiver positions, except for the case of central loop and coincident loop configurations over the homogeneous earth model. In the present study, an attempt is made to present the TEM response of a large loop source over the layered earth model for arbitrary receiver positions. The frequency domain responses of large loop source over the layer earth model for arbitrary receiver positions are converted into the impulse (time derivative of magnetic field) TEM response using Fourier cosine or sine transform. These impulse TEM responses in turn are converted into voltage responses for arbitrary receiver positions, namely, central loop, arbitrary in-loop, and offset-loop TEM responses over the layered earth models. For checking the accuracy of the method, results are compared with the results obtained using analytical expression over a homogeneous earth model. The complete matching of both of the results suggests that the present computational technique is capable of computing TEM response of large loop source over the homogeneous earth model with high accuracy. Thereafter, the technique is applied for computation of TEM response of a large loop source over the layered earth (2-layer, 3-layer, and 4-layer) models for the central loop, in-loop, and offset-loop configurations and the results are presented in voltage decay form. The results depict their characteristic variations. These results would be useful for modeling and inversion of large loop TEM data over the layer earth models for all the possible configurations resulting from a large loop source.

1. Introduction

A large horizontal loop on or above the earth is one of the most widely used sources in TEM and airborne TEM (ATEM) methods. Using a large loop source, one can measure at the center of the loop, at any arbitrary point inside the loop, coincident loop, and at arbitrary offset-loop points. A number of studies on methods of computation of TEM response and modeling can be found in the literature [1–13]. All these studies are based on considering transmitter as a large circular loop and receiver either at the center of the loop and/or coincident with the loop, over the simple layer/homogeneous earth models, and majority of them are based on the computation of TEM responses from their frequency responses only for the central and/or coincident loop configurations, because of the computational intricacies associated with the frequency domain response computation of large loop TEM source for arbitrary receiver positions

(except for central and coincident loop configurations) over the layer earth models which involve product of two Basel functions that make the integral unstable and nonconvergent. Singh and Mogi [14, 15] for the first time presented a new computational method for computation of EM responses of large loop source over the layer earth models for arbitrary receiver positions. Thereafter, a number of researches appeared in the literature [16–24]. Thus, there was scarcity of EM modeling due to large loop sources over the layer earth models even in frequency domain, which resulted in a lack of studies in TEM response of large loop sources over the layer earth models for arbitrary receiver positions, except for the case of central loop and coincident loop configurations that were too over the homogeneous half-space because of nonavailability of analytical expression for arbitrary receiver positions, and sophisticated computational methods for computing the frequency domain responses from which TEM responses are usually derived. Moreover,

during the process of converting the frequency domain EM response into time domain, even a small error of less than 1% in frequency response can produce a considerable difference in time domain EM response of large loop source over the layered earth models for all the configurations. Therefore, with the view of overcoming this drawback and filling the gap in the existing literature for TEM response of large loop sources, in this study, an attempt is made to present a reliable method for computation of TEM response of a large loop source over the layer earth models for any arbitrary receiver positions using the frequency response computation method described in [14, 15]. The computed TEM voltage response is compared with the available TEM responses over the homogeneous earth model for checking the accuracy and reliability of the method.

2. Theoretical Background

The plan view of large loop TEM method with central loop, in-loop, and offset-loop configurations over a layer earth is shown in Figure 1. The large loop presents a source loop and the small loop at the center of the source loop (P_1) represents receiver position corresponding to the central loop configuration. Similarly, the large loop presents a source loop and small loop at arbitrary position inside the source loop (P_2) represents receiver position corresponding to the in-loop configuration and small loop at arbitrary point outside the source loop (P_3) represents receiver position corresponding to arbitrary offset-loop configuration.

The frequency domain expressions of EM field components at a point on or above the surface of an n -layered earth due to a finite horizontal circular loop of radius a , carrying a current $Ie^{i\omega t}$ and placed at the height $z = -h$ above the surface of layered earth model, can be found in [9]. The expression of H_z field component at a measurement point on the surface of n -layered earth (i.e., at $z = 0$) can be written as

$$H_z(\omega, \rho, h) = \frac{Ia}{2} \int_0^\infty \frac{[e^{-u_0 h} (1 + r_{TE})] \lambda^2}{u_0} J_1(\lambda a) J_0(\lambda r) d\lambda, \quad (1)$$

where $r_{TE} = (Y_0 - \hat{Y}_1)/(Y_0 + \hat{Y}_1)$ with $Y_0 = u_0/i\omega\mu_0$ (intrinsic admittance of free space) and $\hat{Y}_1 = H_y^{TE}/E_x^{TE} = -H_x^{TE}/E_y^{TE}$ (surface admittance at $z = 0$).

For an n -layer case, the surface admittances are given by the recurrence relation

$$\begin{aligned} \hat{Y}_1 &= Y_1 \frac{\hat{Y}_2 + Y_1 \tanh(u_1 h_1)}{Y_1 + \hat{Y}_2 \tanh(u_1 h_1)}, \\ \hat{Y}_n &= Y_n \frac{\hat{Y}_{n+1} + Y_n \tanh(u_n h_n)}{Y_n + \hat{Y}_{n+1} \tanh(u_n h_n)}, \end{aligned} \quad (2)$$

and $\hat{Y}_n = Y_n$ with $Y_n = u_n/i\omega\mu_n$, $u_n = (k_x^2 + k_y^2 - k_n^2)^{1/2} = (\lambda^2 - k_n^2)^{1/2}$, and $k_n^2 = \omega^2 \mu_n \epsilon_n - i\omega \mu_n \sigma_n$.

Central loop configuration $OP_1 = 0.0$
In-loop configuration $OP_2 = a/2$
Offset loop configuration $OP_3 = 2a$
Loop radius = a

FIGURE 1: Schematic diagram showing all the possible configurations of large loop TEM system over a 3-layer earth model.

Here, r is source-receiver offset (measured from the center of the loop). For calculation purposes, $\tanh(u_n h_n)$ is used in its exponential form for stability reasons [5].

Thereafter, the time derivative of vertical magnetic field ($\partial h_z/\partial t$) is obtained by transforming the frequency domain solution of vertical magnetic field computed using [14] into the time domain solution using the Fourier cosine and/or sine transform as given in [25].

$$\begin{aligned} \frac{\partial h_z(t)}{\partial t} &= \frac{-2}{\pi} \int_0^\infty \text{Re}[H_z(\omega, \rho, h)] \cos(\omega t) d\omega, \\ \frac{\partial h_z(t)}{\partial t} &= \frac{2}{\pi} \int_0^\infty \text{Im}[H_z(\omega, \rho, h)] \sin(\omega t) d\omega, \end{aligned} \quad (3)$$

where $\text{Re}[H_z(\omega, \rho, h)]$ and $\text{Im}[H_z(\omega, \rho, h)]$ are the real and imaginary parts of the vertical magnetic field over a layered earth model in frequency domain. The components of vectors ρ and h are the resistivities and thickness of different layers of the layered earth model, and ω is the angular frequency.

In general, the data collected from a large loop TEM system consist of vertical voltage measurements made at various time intervals after the current in the transmitter is turned off. These voltage measurements are related to the time derivatives of vertical magnetic field ($\partial h_z/\partial t$) in accordance with the following relation:

$$V(t) = -\mu_0 \left(\frac{\partial h_z}{\partial t} \right) M, \quad (4)$$

where M is the area-turns product of the receiver coil. These voltage data can be further transformed into the apparent resistivity because sometimes it is preferable to use apparent resistivity transformation to have a direct relation with the geoelectrical section, as well as an initial estimate of the layer resistivities, which are often required in nonlinear inversion for interpretation of TEM data.

FIGURE 2: Comparison of results computed using the present method for voltage response of vertical magnetic field at the center of a large circular loop source of radius, $a = 50$ m, placed over the surface of a homogeneous earth model of conductivity ($\sigma = 0.01$ S/m) with the results generated using the analytical expression given by ward and Hohmann (1988) and Singh et al. (2009).

Therefore, starting with the computation of $H_z(\omega, \rho, h)$ field (see (1)), using the method and algorithm described in [14, 15], we have computed the time derivative of vertical magnetic field using the Fourier cosine and sine transforms (see (3)) [8, 25–27]. Thereafter, the transient voltage response is computed using (4). Computations are performed for the homogeneous, 2-layer, 3-layer, and 4-layer resistive and conductive earth models and the results are presented in the following section.

3. Results and Discussions

3.1. Check for Validity of the Method. For checking the validity and accuracy of the method, we applied it for the computation of TEM (impulse) response of a large circular loop source over the surface of a homogeneous earth model for central loop configurations, and the result is compared with the published results for TEM response of a large loop source generated using the central loop analytical expression for impulse response [8, 9]. The analytic expression for the impulse response of the h_z field at the center of a large loop source of radius (a) over the homogeneous earth model of conductivity σ can be written as [8, 9]

$$\begin{aligned} h_{z|\text{Impulse}} &= \frac{\partial h_{z|\text{Step}}}{\partial t} \\ &= \frac{I}{\mu_0 \sigma a^3} \left[3 \operatorname{erf}(\theta a) - \frac{2}{\pi^{1/2}} \theta a (3 + 2\theta^2 a^2) e^{-\theta^2 a^2} \right], \end{aligned} \quad (5)$$

where $\theta = (\mu_0 \sigma / 4t)^{1/2}$, erf indicates the error function, and t means the delay time after the current is turned off.

3.2. Illustration of TEM Response over Layer Earth Models. For illustrating the accuracy, applicability, and efficiency of the program for generating voltage response due to large loop TEM methods over the layer earth models for arbitrary receiver positions, we applied it for computing the TEM response over the homogeneous earth model of conductivity 0.01 S/m and compared the results with the published results [8, 9] (Figure 2). From Figure 2, it is evident that the computed results match very well the published results computed using the analytical expressions [9], thereby indicating the accuracy and validity of the method for computation of TEM response of large loop source over the homogeneous earth models.

Further, the method is applied for computation of TEM responses of a large loop source over the 2-layer, 3-layer, and 4-layer earth models for arbitrary receiver positions, that is, central loop, in-loop, and offset-loop configurations, for different loop sizes, and the results are presented in Figures 3–5.

Figure 3 presents TEM responses of a large loop source over a two-layer earth model (as shown in the inset of the figure) for central loop, in-loop, and offset-loop configurations, respectively, for source loop radii 100 m and 200 m. The computed results depict characteristic features of TEM response over the two-layer earth models.

Figure 4 presents TEM response of a large loop source over the three-layer earth model (as shown in the inset of the figure) for loop sizes of radii 100 m and 200 m, for central loop, in-loop, and offset-loop configurations. The computed results depict characteristic response over the three-layer earth models.

Figure 5 depicts TEM response of a large loop source over the four-layer earth model (as shown in the inset

FIGURE 3: Transient voltage response due to large circular loop source of radii 100 m and 200 m over the 2-layer earth model (as shown in the inset of the figure) for (a) central loop configuration, $r = 0$, (b) arbitrary in-loop configuration, $r = a/2$, and (c) arbitrary offset-loop configuration, $r = 2a$.

of the figure) for source loop sizes of radii 100 m and 200 m, for central loop, in-loop, and arbitrary offset-loop configurations, respectively. The results depict characteristic response of large loop sources over the four-layer earth models.

From these figures (Figures 3–5), it is clear that, with the increase in loop size, the TEM response shows smooth and well-defined characteristics. Moreover, it is also noticed that the central loop TEM responses are more regular as compared to the offset-loop and arbitrary in-loop responses.

4. Conclusion

The present article describes a simple and sophisticated method for computation of TEM response of a large loop source over layered earth model for arbitrary receiver

positions, that is, at the center of the loop, at arbitrary in-loop, and at arbitrary offset-loop points. The method is based on conversion of frequency domain results computed using EMLCLTR program of Singh and Mogi [15] into the time derivative of the vertical magnetic field, that is, impulsive TEM responses. The method is simple and suitable for computation of TEM response of a large loop source at any arbitrary receiver locations, that is, central loop, in-loop, and offset-loop configurations. During its frequency domain computation, it has the option of computing frequency domain responses with or without the inclusion of displacement current factor and hence is a more reliable and accurate one.

For illustrating the nature and characteristics of large loop TEM responses over the layer earth models at arbitrary receiver positions, results are presented for the TEM response

FIGURE 4: Transient voltage response of large circular loop source of radii 100 m and 200 m over the 3-layer earth model (as shown in the inset of the figure) for (a) central loop configuration, $r = 0$, (b) arbitrary in-loop configuration, $r = a/2$, and (c) arbitrary offset-loop configuration, $r = 2a$.

at source-receiver offsets $r = 0$, $r = a/2$, and $r = 2a$ pertaining to the central loop, in-loop, and offset-loop configurations over 2-layer, 3-layer, and 4-layer earth models. The results depict their characteristic variations. From the results, it is noticed that the voltage response curves are more regular for central loop, followed by offset loop, and the least for the in-loop configurations. Further, it is also observed that, with the increase in source loop size, the voltage responses become smoother.

This study would enable the prospect, development, and use of loop-in-loop method (in-loop method) and loop-offset-loop method (offset-loop method) along with the well-developed central loop and coincident loop methods using large loop sources, and thus it would enhance the applicability and cost-effectiveness of the large loop source transient

electromagnetic method for exploration and applied geophysics applications.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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FIGURE 5: Transient voltage response of large circular loop source of radii 100 m and 200 m over the 4-layer earth model (as shown in the inset of the figure) for (a) central loop configuration, $r = 0$, (b) arbitrary in-loop configuration, $r = a/2$, and (c) arbitrary offset-loop configuration, $r = 2a$.

providing the opportunity and facility for completion of this work.

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