RESEARCH NOTES

PIS FOR n-COUPLED NONLINEAR SYSTEMS

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ABSTRACT. A numerical algorithm dealing with solutions of equations with one variable may not be extended to solve nonlinear systems with n unknowns. Even when such extensions are possible, properties of these two similar algorithms are, in general, different. In [2] a perturbed iterative scheme (PIS) has been developed to solve nonlinear equations with one variable. Its properties with regard to nonlinear systems were analyzed in [1]. Here these properties were extended to n-coupled nonlinear systems.

KEY WORDS AND PHRASES. Perturbed numerical iterations, nonlinear equations.


1. THE ALGORITHM.

In [2] a simple functional iterative scheme has been developed to solve nonlinear equations with one variable by adding a unique perturbation parameter to Picard's iterations. The algorithm was directly extended to nonlinear systems...
[1] and convergence properties were analyzed using a special mapping called D-mapping [1]. We will study here that for n-coupled nonlinear systems analysis of convergence may be done in a similar way.

Let us consider a nonlinear system having n-coupled equations:

\[ x^i = F_i(x^1, x^2, \ldots, x^n) \]  \hspace{1cm} (1.1)

where,

\[ x^i = (x^i_1 \ x^i_2 \ \ldots \ x^i_n)^T \in D^i \subset R^n \]

\[ F^i = (F^i_1 \ F^i_2 \ \ldots \ F^i_n)^T \in D^i \subset R^n, \]

\( i = 1, 2, \ldots, n \) and \( R^n = \text{real n-dimensional space} \). Thus, each \( F^i: D^1 \times D^2 \times \ldots \times D^n \subset R^n \times R^n \times \ldots \times R^n \times D^i \). Let \( R = R^n \times R^n \times \ldots \times R^n \) and \( D = D^1 \times D^2 \times \ldots \times D^n \). As before [1] we assume that (1.1) has a solution in \( D \) given by \( x^* = (x^*_1, x^*_2, \ldots, x^*_n)T \in D^i \). Hence,

\[ x^i,* = F^i(x^*_1, x^*_2, \ldots, x^*_n) \]  \hspace{1cm} (1.2)

In the element form, PIS is:

\[ x^i_{j,k} = w^i_{j,k} + F^i_j(x^l_{j,k}, \ldots, x^i_{j-1,k}, x^i_{j,k} \ldots x^n_{j-1}) \]  \hspace{1cm} (1.3)

\( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, n \).

To compute the perturbation parameters \( w^i_{j,k} \) we assume that they are small and their squares may be neglected; also the functionals \( F^i_j \) are \( \partial F^i_j / \partial x^i_j \neq 1 \), and \( \partial^2 F^i_j / \partial x^i_j \partial x^i_j \) are bounded \( \psi x^i \in D^i \). Then assuming convergence after \((k-1)\) iterations we have:

\[ w^i_{j,k} + F^i_{j,k} = F^i_j(x^l_{j,k}, \ldots, x^i_{j-1,k}, x^i_{j,k} \ldots x^n_{j-1}) \]  \hspace{1cm} (1.4)

where \( F^i_{j,k} \) is given by the second term of the right side of (1.3). Expanding the right hand side of (1.4) by Taylor's theorem and using the above assumptions we have:
where
\[ w_{j,k} = \left( p_{j,k} - p_{j,k}^0 \right) / \left( 1 - \beta_{j,k} \right) \] (1.5)

and
\[ \beta_{j,k} = F_{j,k}^0 \]

Once \( w_{j,k} \) is known from (1.5) we use PIS to get:
\[ x_{j,k} = w_{j,k} + p_{j,k} \] (1.6)

for \( i = 1, 2, \ldots, n \); \( j = 1, 2, \ldots, n \). Comparing this equation with (1.2) it is clear that a necessary condition for convergence is:
\[ \lim_{k \to \infty} |w_{j,k}| = 0, \forall i,j \] (1.7)

If we write, \( X^k = (x^i,k \ldots x^n,k)^T \in D \) and \( W^k = (w^1,k \ldots w^n,k)^T \in R \) then (1.6) may be expressed as:
\[ X^k = W^k + F(X^k, X^{k-1}) \] (1.8)

where
\[ F : D \times D \subseteq R \times R \times D \]

Now it is clear that \( X^* = F(X^*, X^*) \) giving \( X^* \) as the fixed image of \( F \) on \( D \times D \).
Also, if \( F \) is a D-mapping [1] on \( D \times D \), (1.7) will be both necessary and sufficient condition for convergence of PIS.

Hence, after studying the convergence analysis of PIS in [1], it is now easy to see the same concept being extended for n-coupled nonlinear systems.

REFERENCES

