EFFECT OF VACUUM POLARIZATION ON THE EXCITATION OF HYDROGEN ATOM BY ELECTRON IMPACT

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ABSTRACT. The vacuum polarization potential is included in the interaction Hamiltonian for 1S - 2S excitation of the hydrogen atom by electron impact. The excitation amplitude calculated field theoretically is found to be lowered by

$$0.47t^2 / (t^2 + 9)^3$$

where $t^2 = 4|P-Q|^2$, P and Q being the momenta of the incident and scattered electrons respectively.

KEY WORDS AND PHRASES. Polarization Potential, Atomic and Molecular Collisions.

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1. INTRODUCTION.

This paper is accessible to readers familiar with our reference [1]. The generalist reader interested in this area may start with the bibliography given there and in [2].

The cross-section for excitation of ground state hydrogen atoms to 2S metastable state on electron impact has been the subject of extensive theoretical investigation. Very recently it has been calculated by Kingston, Fen and Burke [3]

with close-coupling approximation showing good agreement with the measurements of Williams and Willis [4]. Callaway, McDowell and Morgan [5] did the calculation in a hybrid pseudostate close-coupling distorted wave model. Here also agreement with the experimental result is good. Roy and Bhattacharyya [2] tried the problem in a field theoretic way and compared the result with the then available experimental result of Stebbing, Fite, Hummer, and Brackmann [1]. Even now this simple scattering problem in atomic physics is not a closed chapter. There is room for theorization and approximation.

Although vacuum polarization gives a very small correction to the Coulomb potential, it is of interest to see its influence on the excitation amplitude. In the muonic atom the effect of vacuum polarization has been extensively studied and found to affect the energy levels [6]. In non-relativistic approximation vacuum polarization is known to affect the hydrogen S- states and in particular the 2S level is lowered by 27 MHz relative to the 2P - level. As such it is not out of place to look for the effect of vacuum polarization on the excitation amplitude of the hydrogen atom.

2. MATHEMATICAL FORMALISM.

Quantum electrodynamics predicts deviation from Coulomb potential behavior near a point charge. The potential energy of a negative charge $-e$ located at a distance r from another point charge is not given entirely by the Coulomb expression λ :

$$V_1 = e^2 \int \frac{\rho(r) \rho(r')}{|r - r'|} d^3r d^3r' . \quad (2.1)$$

There is a correction of order α , the Uehling potential. It arises from the polarization of the vacuum of the virtual electrons of mass m and is given by [6]

$$V_2 = e^2 \int \frac{\rho(r) \rho(r')}{|r - r'|} \left\{ \frac{\alpha}{\pi} Z_0(r - r') \right\} d^3r d^3r' \quad (2.2)$$

$$\text{where} \quad Z_0(R) = \frac{2}{3} \{ \ln(mR) + \gamma \} + \frac{5}{2} - \frac{1}{2} \pi(mR) \quad (2.3)$$

here $\gamma = 0.57721$ is the Euler's constant

and $\rho(r)$ is the charge density of the electron field

$$\rho(r) = \int \psi^*(r) \dot{\psi}(r) d^3r \quad (2.4)$$

The amplitude for 1S - 2S excitation of the hydrogen atom by electron impact due to static Coulomb interaction V_1 as computed by Roy and Bhattacharyya [2] is

$$M_{fi} = 4 \int \sum_{ab,cd} g_{cd}^{*rr'}(\ell\ell'k_3) g_{ab}^{ss'}(kk'k_3) \frac{\delta^3(k-\ell + k'-\ell')}{|k' - \ell'|} \times \\ \chi_r^* \chi_s \chi_r^* \chi_s, d^3k d^3\ell d^3k' d^3\ell' d^3k_3 \quad (2.5)$$

which can be written as

$$M_{fi} = 4 \int \sum_{ab,cd} g_{cd}^{*rr'}(\ell\ell'k_3) g_{ab}^{ss'}(kk'k_3) \delta^3(k-\ell + k'-\ell') \frac{\exp\{i(k' - \ell')R\}}{|R|} \times \\ \chi_r^* \chi_s \chi_r^* \chi_s, d^3k d^3\ell d^3k' d^3\ell' d^3k_3 d^3R \cdot \lambda \quad (2.6)$$

We can proceed in a similar way to compute contributions of the vacuum polarization term V_2 towards the amplitude

$$M'_{fi} = \int 4 \sum_{ab,cd} g_{cd}^{*rr'}(\ell\ell'k) g_{ab}^{ss'}(kk'k_3) \delta^3(k-\ell + k'-\ell') \\ \times \frac{\alpha}{\pi} Z_0(R) \frac{\exp\{i(k' - \ell')R\}}{|R|} \chi_r^* \chi_r \chi_s \chi_s, \\ \times d^3k d^3\ell d^3k' d^3\ell' d^3R d^3k_3 \quad (2.7)$$

The total amplitude W for the process in the co-ordinate space is obtained by adding M_{fi} to M'_{fi} , and integrating over k, k', ℓ, ℓ' , and k_3 we get as in the previous paper [2]

$$W = 4 \int \sum_{ab,cd} \phi_{cd}(X, X+R) \phi_{ab}(X, X+R) \frac{1}{|R|} \left(1 + \frac{\alpha}{\pi} Z_0(R)\right) \delta^3(P_c - Q_c) d^3X d^3R \quad (2.8)$$

$\phi_{ab}(X, X+R)$ and $\phi_{cd}(X, X+R)$ are initial and final state solutions respectively of the Schrodinger equation for the combined system of hydrogen atom and a free electron.

Writing these in a slater determinant form, equation (2.8) becomes

$$W = 4\delta^3(P_c - Q_c) \int \frac{1}{2} \sum_{ab,cd} \begin{vmatrix} \phi_{2S}(X) \chi_c & \phi_{2S}(X+R) \chi_d \\ \psi_f^*(X) \chi_c & \psi_f^*(X+R) \chi_d \end{vmatrix} \times \\ \times \begin{vmatrix} \phi_{1S}(X) \chi_a & \phi_{1S}(X+R) \chi_b \\ \psi_i(X) \chi_a & \psi_i(X+R) \chi_b \end{vmatrix} \frac{1}{|R|} \left(1 + \frac{\alpha}{\pi} Z_0(R)\right) d^3X d^3R \quad (2.9)$$

where

$\phi_{1S}(X)$, $\phi_{2S}(X)$ are respectively the ground state and 2S excited state wave functions of the hydrogen atom. $\psi_1(X)$ and $\psi_f(X)$ are the plane wave solutions for the incident and scattered electrons respectively. After integrating over X and R (all quantities are in atomic units) and taking for Pauli spin vector

$$\sum_{ab} \chi_a \chi_b = \sum_{ab} \delta_{ab} = 2,$$

we obtain

$$W = 4 \left[(A_0 + B_0 + C_0 + D_0) + \frac{\alpha}{\pi} (A_1 + B_1 + C_1 + D_1) \right] \quad (2.10)$$

With P and Q as the momenta of the incident and scattered electrons, and

$$t^2 = 4|P - Q|^2, \quad I_0 = 0.375 I_D(t^2 + 9)/t^2, \quad I_D = 512t^2/\sqrt{2}(t^2 + 9)^3,$$

we obtain

$$\begin{aligned} A_0 &= (4\pi I_D) \frac{4}{t^2}, & B_0 &= \frac{4\pi I_D}{Q^2 + 1}, \\ C_0 &= 4\pi I_D \left[\frac{4}{4P^2 + 9} - \frac{9(t^2 + 9)}{t^2(t^2 + 9)^2} \right], \\ D_0 &= 4 I_D \left[1.444 - 1.034 \frac{(t^2 + 9)}{t^2} \right]; \\ A_1 &= I_D \int \exp \left[i(P - Q)R \right] \frac{Z_0^{(R)}}{|R|} d^3R, \\ B_1 &= I_D \int \exp \left[iQR - |R| \right] \frac{Z_0^{(R)}}{|R|} d^3R, \\ C_1 &= \int (I_D - I_0 R) \exp \left[iPR - \frac{3}{2} |R| \right] \frac{Z_0^{(R)}}{|R|} d^3R, \\ D_1 &= \int (I_D - I_0 R) \exp \left[-\frac{3}{2} |R| \right] \frac{Z_0^{(R)}}{|R|} d^3R \end{aligned} \quad (2.11)$$

Putting $Z_0(R)$ from (2.3) in A_1 and knowing that R varies from 0 to λ (the Compton wave length for electrons), integral A_1 becomes

$$A_1 = 4\pi I_D \frac{\lambda^2}{3} \cdot \frac{1}{2}(2\ell n\lambda - 1) \quad (2.12)$$

λ being very small (45.778×10^{-3} in a.u.) we have approximated for low energy encounter

$$\cos (P - Q)\lambda = 1 \quad \text{and} \quad \sin (P - Q)\lambda = (P - Q)\lambda$$

To integrate B_1 , C_1 , and D_1 we have taken only the first term in the expansions of $\exp(-R)$ and $\exp(-\frac{3}{2}|R|)$ and we find

$$A_1 = B_1 = C_1 = D_1 \quad (2.13)$$

Hence the total amplitude for the process

$$W = 4 \left[A_0 + B_0 + C_0 + D_0 + \frac{8\lambda^2\alpha}{3} (2\ell n - 1)I_D \right] \quad (2.14)$$

3. CONCLUSION.

From equation (2.14) we find that vacuum polarization reduces the Coulomb amplitude for 1S - 2S excitation of hydrogen atom by

$$10.6\lambda^2\alpha (2\ell n\lambda - 1)I_D = -0.47 \frac{t^2}{(t^2 + 9)^3}$$

For small momentum transfer this reduction becomes $-0.47 t^2/729$ while for high momentum transfer its value is $-0.47/t^4$. Thus change in the amplitude for low momentum transfer is greater than that for high momentum transfer. These changes are very small and give a correction of the order of 0.4 to 0.5 per cent to the differential cross-section. So there is every possibility for these to escape detection during experiment. But with the increase in the precision of the experiment we hope that these small changes may be detected in the future.

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