A GENERALIZATION OF SOME FIXED POINT THEOREMS OF K.M. GHOSH

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ABSTRACT. This note establishes the following result. Let T be a selfmap of a normed linear space E . For $0 < \lambda \leq 1$, define $T_\lambda x = \lambda x + (1 - \lambda)Tx$ for each x in E . If, in addition, $S = TT_\lambda$ satisfies any contractive definition strong enough to guarantee that S has a unique fixed point u in E , and, if $TT_\lambda u = T_\lambda Tu$, then u is the unique fixed point for T .

KEY WORDS AND PHRASES. Fixed Point, Mean Value Iteration.

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1. INTRODUCTION.

The purpose of this note is to formalize the underlying principle in Theorems 1 and 4 of [1].

To prove the result stated in the abstract, observe that $T(T_\lambda Tu) = T(TT_\lambda u) = Tu$. Hence Tu is also a fixed point of $S = TT_\lambda$. Since the fixed point of S is unique, $u = Tu$.

For uniqueness, since the fixed point set for T_λ , $F(T_\lambda)$, contains $F(T)$, with equality when $\lambda \neq 1$, it is clear that $F(TT_\lambda) \supset F(T)$.

Example 4 of [1], corrected to read $Tx = 1$ for $x \in [0,1)$, $T(1) = 0$, shows that the condition $TT_\lambda u = T Tu$ is necessary.

REFERENCE

1. GHOSH, M.K. A generalization of contraction principle, Inter. J. Math. & Math. Sci. 4 (1981) 201-206.