OBSERVATION ON THE FOUNDATION OF KINETIC THEORY OF GASES

U. BASU

S.N. Bose Institute of Physical Sciences
Calcutta University
92, Acharya Prafulla Chandra Road
Calcutta-700009

and

M. DUTTA

Department of Applied Mathematics and Centre of
Advanced Study in Applied Mathematics
Calcutta University
92, Acharya Prafulla Chandra Road
Calcutta-700009

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ABSTRACT. This paper deals with the study of the mathematical foundation of the Kinetic theory of gases. The significance of the Boltzmann model for a system of particles treated by classical mechanics is briefly explained. An apparent, but inherent difficulty in this model is pointed out.

KEY WORDS AND PHRASES. *Mathematical foundation, Model, Classical mechanical particle system.*

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1. BOLTZMANN MODEL IN KINETIC THEORY.

For a quantitative explanation of the behaviour of fluids, the basic equation of kinetic theory as derived by Boltzmann [1] is given by

$$\frac{\partial f(\underline{x}, \underline{v}, t)}{\partial t} + (\underline{v} \cdot \text{grad}_{\underline{x}} f) + (\underline{F} \cdot \text{grad}_{\underline{v}} f) \\ = N \int \dots \int \{f(\underline{x}, \underline{v}', t) f(\underline{x}, \underline{v}_1', t) - f(\underline{x}, \underline{v}_1, t) \cdot f(\underline{x}, \underline{v}, t)\} g \cos \theta \, s^2 d\omega d\underline{v}_1, \dots, \quad (1.1)$$

where N is the number of gas-molecules in a closed volume V , $F(\underline{x}, \underline{v}, t)$ is the distribution function of the molecules, $\underline{F}$ is the external force on the gas-molecules, $\underline{g} = \underline{v} - \underline{v}_1$ is the relative velocity of the molecule moving with velocity $\underline{v}$ with respect to the other moving with velocity $\underline{v}_1$, $s^2 d\omega$ is the surface element on the surface of the former molecule subtending a solid angle $d\omega$ at its center, and θ is the angle between the normal $\underline{x}$ to the surface-element and the direction of relative velocity. $\underline{v}$ and $\underline{v}_1$ are such that after the collision, the former will have velocity $\underline{v}$.

For calculation of the terms indicating the effects of collision on the right hand side of equation (1.1), Boltzmann considered the rigid volume of each particle. For calculation of the terms of the left hand side of (1.1) indicating the effect due to velocity, the constituents are treated as simple particles and fluxes due to the continuous change in coordinates and in velocities are taken into account. Thus, there are some apparent paradoxical features in the model introduced by Boltzmann. The mathematical system of Boltzmann is a system of particles of classical mechanics. So, for the clarification of the above, the mathematical foundation of the classical mechanics of a system of particles is useful and so its basic postulates and some relevant analysis are presented here.

2. MODEL OF A CLASSICAL MECHANICAL PARTICLE SYSTEM.

A classical mechanical particle system is represented by $\{P, T, E_3, \underline{x}: P \times T \rightarrow E_3, m: P \rightarrow R_1, \vec{\partial}, \mathcal{S}\}$, if and only if it satisfies the following postulates [2,3,4].

- (i) POSTULATE ON EXISTENCE. There exists a system of particles indicated by $P = \{p_i\} \neq \emptyset$, where p_i is called a particle and $i = 1, 2, \dots, n$.
- (ii) POSTULATE OF TIME. There exists $T = \{t\} = I_{ab} \subset R_1, I_{ab}$ is an interval (open, semi-open, or closed) where the end points a and b ; may be infinite and t refers to an instant of time.
- (iii) POSTULATE OF SPACE. In classical mechanics, the physical space is isomorphic to E_3 .

(iv) POSTULATE OF KINEMATICS. There exists a mapping $\underline{x}$ such that $\underline{x}: P \times T \rightarrow E_3$, where $\underline{x}$ is an one to one mapping; $\underline{x}(p_i, t)$ as a single valued function of p_i implies the position of p_i at time t . The existence of an inverse function is equivalent to the principle of impenetrability of matter. As a function of t , it is bounded in any finite interval of time and has sectionally smooth derivatives. Sectionally smooth means that the entire interval can be divided into a finite number of sections in which $\underline{x}(p_i, t)$ has continuous derivatives with respect to t . A finite discontinuity of $\underline{x}(p_i, t)$ corresponds to a collision.

(v) POSTULATE OF GEOMETRY OF MASS. There is an additive mapping m such that $m: P \rightarrow R_1$.

(vi) POSTULATE OF FORCE. There exists an F where F is a class of mapping $\underline{F}$ such that $\underline{F}: P \times T \rightarrow E_3$, then $\underline{F}$ is called the external force.

(vii) POSTULATE OF INTERACTION. There is a class of mapping $I = \{I\}$ such that $\underline{I}: P \times P \times T \rightarrow E_3$ where $\underline{I}(p_i, p_j, t) = -\underline{I}(p_j, p_i, t)$, then $\underline{I}$ is called the interaction force.

(viii) POSTULATE OF CONNECTION. The postulate of connection is given by the following relation:

$$m(p_i) \ddot{\underline{x}}(p_i, t) = \underline{F}(p_i, t) + \sum_{i \neq j} \underline{I}(p_i, p_j, t).$$

NOTE: To introduce the notion of collision in classical mechanics of particles, as notion of consecutiveness is absent in the continuum E_3 , interaction forces $\underline{I}(p_i, p_j, t)$ should be such that

$$||\underline{I}(p_i, p_j, t)|| \rightarrow \infty \text{ when } |\underline{x}(p_i, t) - \underline{x}(p_j, t)| \rightarrow d_{ij} + 0,$$

where d_{ij} represents a fixed number for particles p_i and p_j , signifying the distance between the centers of the particles p_i and p_j . This idea leads to a minimum distance of approach for each pair of particles.

3. CONCLUSION.

From the above discussion, it is clear that the notion of a minimum distance of approach of two particles, which means the existence of rigid exclusion volume, is necessary in the classical mechanics of particles, and so in the Boltzmann model of the kinetic theory of gases.

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