CORRIGENDUM

ON LINEAR ALGEBRAIC SEMIGROUPS III

MOHAN S. PUTCHA

School of Physical and Mathematical
Sciences, Department of Mathematics,
North Carolina State University,
Raleigh, North Carolina 27650

There are some errors in the above paper. There is a line missing at the bottom of page 672. Also, pages 681-683 are organized incorrectly.

These errors are corrected as follows:

Replace the last sentence on page 672 with:

" $\bigcup_{g \in G} gE(Y)g^{-1} = E(S)$. In particular $E(Y) \cap J \neq \emptyset$ for all $J \in \mathcal{U}(S)$. Moreover the length of any maximal chain in $\mathcal{U}(S)$ equals $\dim Y$. "

Replace page 681 beginning from line 14 (from the top), the entire page of 682 and the first seven lines (from the top) of page 683 with:

"PROOF. We can assume that e is the identity element of S (otherwise we work with eSe). By Lemma 1.1 we are reduced to the case when f is the zero of S . By Corollary 1.5, we are reduced to the case when S is also a d -semigroup. By Lemma 2.2 and Theorem 2.7, we can assume that S is as in Theorem 2.7, with $e = (1, \dots, 1)$, $f = (0, \dots, 0)$. Let $V_1 = \{(\omega_1(a, \dots, a), \dots, \omega_n(a, \dots, a)) \mid a \in K\}$, $S_1 = \overline{V_1}$. Then $e, f \in S_1$, $\dim S_1 = 1$, $S_1 \subseteq S$. Define $\theta: K \rightarrow S_1$ as $\theta(a) = (\omega_1(a, \dots, a), \dots, \omega_n(a, \dots, a))$. Then θ is a $*$ -homomorphism. So S_1 is connected. This proves the theorem.

3. POLYTOPES

If $X \subseteq \mathbb{R}^n$, then we let $C(X)$ denote the convex hull of X (see[4]). The convex hull of a finite set in $\mathbb{R}^n$ is called a polytope [4]. If the vertices of P are rational, then P is said to be a rational polytope. If $X \subseteq P$, then X is said to be a face of P [4; p. 25] if for all $a, b \in P$, $\alpha \in (0,1)$, $\alpha a + (1 - \alpha)b \in X$ if and only if $a, b \in X$. Let $X(P)$ denote the set of all faces of P . Then [4; p. 21], $(X(P), \subseteq)$ is a finite lattice. Dimension of P is defined to be the dimension of the affine hull of P [4; p.3]. Then dimension of $P = (\text{length of any maximal chain in } X(P)) - 1$. Two polytopes P_1, P_2 have the same combinatorial type if $X(P_1) \cong X(P_2)$ (see [4; p. 38]). By [4; p. 244], every polytope of dimension ≤ 3 has the same combinatorial type as some rational polytope. However this is not true in general [4; p. 94]. If $u = (\alpha_1, \dots, \alpha_n), v = (\beta_1, \dots, \beta_n) \in \mathbb{R}^n$ then let $u \cdot v = \sum_{i=1}^n \alpha_i \beta_i$ denote the inner product of u and v .

Let S be a semigroup. An ideal I of S is said to be semiprime if for all $a \in S$, $a^2 \in I$ implies $a \in I$. I is prime if for all $a, b \in S$, $ab \in I$ implies $a \in I$ or $b \in I$. Let

$$I(S) = \{\text{All ideals of } S\}$$

$$A(S) = \{\text{All principal ideals of } S\}$$

$$\Gamma(S) = \{\text{All semiprime ideals of } S\} \cup \{\emptyset\}$$

$$\Lambda(S) = \{\text{All prime ideals of } S\} \cup \{\emptyset\}.$$

$$X(S) = \{S \setminus I \mid I \in \Lambda(S)\}.$$

$$\Omega(S) = \text{Maximal semilattice image of } S.$$

It is easy to see that $(\Lambda(S), \subseteq) \cong (\Lambda(\Omega(S)), \subseteq)$ is a complete lattice. If S is finitely generated, then $\Omega(S)$ is finite and so $(\Lambda(S), \subseteq)$ is a finite lattice.

THEOREM 3.1. Let S be a connected d -semigroup with zero. Define $\alpha: I(S) \rightarrow \Gamma(\Phi(S))$ as $\alpha(I) = \{\chi \mid \chi \in \Phi(S), \chi(a) = 0 \text{ for all } a \in I\}$. Define $\beta: \Gamma(\Phi(S)) \rightarrow I(S)$ as $\beta(W) = \{a \mid a \in S, \chi(a) = 0 \text{ for all } \chi \in W\}$. Then α, β are inclusion reversing bijections and $\beta = \alpha^{-1}$. Moreover $\alpha(A(S)) = \Lambda(\Phi(S))$.

PROOF. Clearly α, β are inclusion reversing. Let $I \in A(S)$. Then $I = eS$ for some $e \in E(S)$. So $\alpha(I) = \{\chi \mid \chi \in \Phi(S), \chi(e) = 0\}$. It follows that $\alpha(I) \in \Lambda(\Phi(S))$. Clearly $I \subseteq \beta(\alpha(I))$. We claim that $I = \beta(\alpha(I))$. Suppose not. Then there exists $a \in \beta(\alpha(I))$ such that $a \notin I$. Let $a = hf$, $f \in E(S)$. Then $f \in I$, $f \in \beta(\alpha(I))$. So $e \not\leq f$. By Lemma 2.1 (2), there exists $\chi \in \Phi(S)$ such that $\chi(f) = 1$, $\chi(e) = 0$. So $\chi \in \alpha(I)$ and $f \notin \beta(\alpha(I))$, a contradiction. So

$$\text{for all } I \in A(S), \alpha(I) \in \Lambda(\Phi(S)) \text{ and } \beta(\alpha(I)) = I \quad (12)$$

Let $P \in \Lambda(\Phi(S))$. We claim that $\beta(P) \in A(S)$ and $\alpha(\beta(P)) = P$. By Lemma 2.1, this is true for $P = \Phi(S)$. So assume $P \neq \Phi(S)$. Then $F = \Phi(S) \setminus P$ is a subsemigroup of $\Phi(S)$. By Lemma 2.2 we can assume that S is a closed submonoid of some $(K^n, \cdot)$, $0 = (0, \dots, 0) \in S$ and that $\Phi(S) = \langle \chi_1, \dots, \chi_n \rangle$ where χ_i is the i^{th} projection of S into K , $i = 1, \dots, n$. Let $A = \{\chi_i \mid \chi_i \in F\}$. Then $\langle A \rangle = F$. Let $e = (e_1, \dots, e_n)$ where $e_i = 1$ if $\chi_i \in A$, $e_i = 0$ if $\chi_i \notin A$. We claim that $e \in S$. Suppose not. Then by Lemma 2.3, there exist $u, v \in F(\chi_1, \dots, \chi_n)$ such that $u(a) = v(a)$ for all $a \in S$ and $u(e) \neq v(e)$. Since $u(e)^2 = u(e)$ and $v(e)^2 = v(e)$ we can assume that $u(e) = 1$, $v(e) = 0$. Clearly $u(\chi_1, \dots, \chi_n) = v(\chi_1, \dots, \chi_n)$. Since $u(e) = 1$, $u(\chi_1, \dots, \chi_n)$