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ON THE EXISTENCE OF TACHYONS IN A BRADYON - DOMINATED UNIVERSE

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ABSTRACT. In the present paper, the question has been raised as to whether tachyons exist in our universe. The gravitational interaction of tachyons with bradyons is discussed by employing the method of geodesic deviation of a tachyon-trajectory from a bradyon-trajectory. It is noted that in certain cases, tachyons are attracted by bradyons, but in most cases they are repelled from bradyons. It is speculated that, due to their superluminal speed as well as gravitational repulsion, free tachyons would have fled out of this universe and inhabited a Meta-Universe.

KEY WORDS AND PHRASES. Tachyons, bradyons, trajectories, universe models.
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1. INTRODUCTION.

Tachyons are superluminal particles. Contrary to general belief, it has been shown by various scientists such as Bilanuik, Deshpande, and Sudarshan [1], Recami and Mignani [2] and others that the existence of such particles does not violate the principles of relativity theory. Now one can ask a very natural question, "Where do they exist?" At this juncture, physicists have different opinions. Some physicists opine that tachyons exist in a different universe which is similar to ours. Others think that tachyons exist in our universe but, due to their superluminal speed, it has been difficult to detect them. Still others, such as Narlikar and Sudarshan [3], assume that tachyons were created at the epoch of the big-bang

alongwith other particles of bradyonic matter (bradyons are particles moving slower than light).

Here, we also assume that tachyons were created at the epoch of big-bang. Now the question arises, 'Where are these primordial tachyons?'. If they exist in our universe, they should manifest themselves through some observable effects, but so far, no such effects have been noticed. Here we analyze this question.

If tachyons were produced alongwith bradyons, they would have been living with bradyons in this universe up to some time. Hence, it is necessary to study the gravitational interaction of tachyons and bradyons. For the purpose, we employ the method of the geodesic deviation of a tachyon-trajectory from a bradyon-trajectory and study $\frac{d^2(\delta r)}{dt^2}$ where (r,θ,ϕ) is the position of a bradyon at time t and $(r+\delta r,\theta,\phi)$ is the position of a tachyon at time $(t + \delta t)$ on their respective trajectories relative to a cosmic observer at the origin as shown in Figure 1.

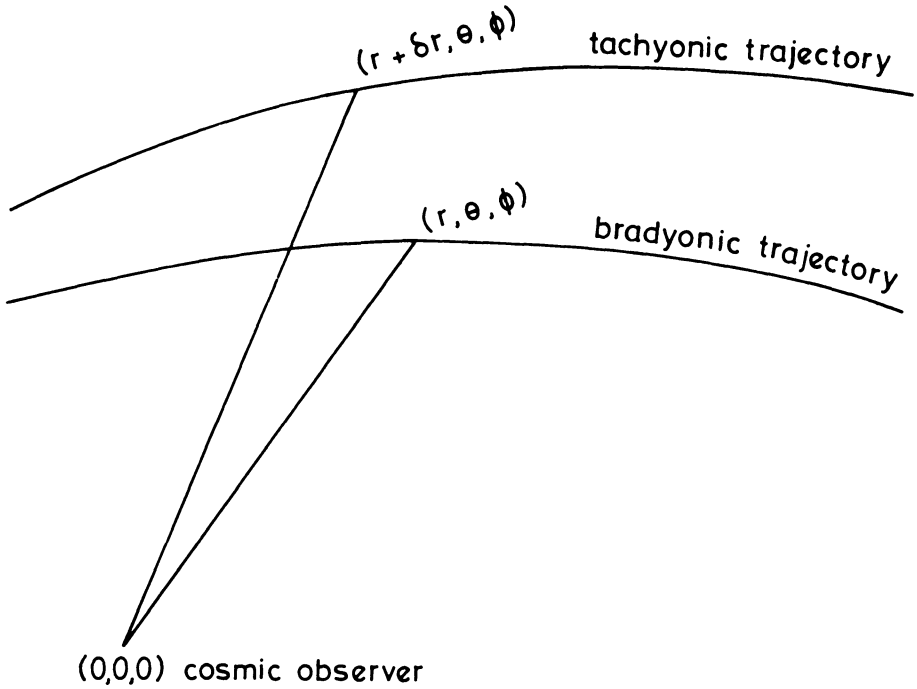

Fig. 1

We interpret the second derivative of δr (the connecting vector between the tachyon and bradyon trajectories) with respect to time as the acceleration of the tachyon in the bradyon field. We consider θ and ϕ as constants.

We take the Robertson-Walker metric as the background model

$$ds^2 = dt^2 - \frac{R^2(t)}{(1 + \frac{kr^2}{4})^2}[dx^2 + dy^2 + dz^2] \tag{1.1}$$

where $r^2 = x^2 + y^2 + z^2$ and k is a curvature constant. Also, for our convenience, we choose c = 1. The dot over the variable denotes the first derivative with respect to time.

In the last section, we discuss that, due to their superluminal speed and gravitational repulsion, tachyons would have fled away to a Meta-Universe or a Tachyon-Universe.

2. BRADYONIC AS WELL AS TACHYONIC TRAJECTORIES.

The geodesic equations in the model chosen can be written as

$$\frac{d^2t}{ds^2} + \frac{R\dot{R}}{(1 + \frac{kr^2}{4})^2} \sum_{j=1}^{3} \left(\frac{dx^j}{ds}\right)^2 = 0 \tag{2.1}$$

and

$$\frac{d}{ds}\left[\frac{R^2}{(1 + \frac{kr^2}{4})^2}\frac{dx^2}{ds}\right] + \frac{kR^2x^i}{2(1 + \frac{kr^2}{4})^3} \sum_{j=1}^{3} \left(\frac{dx^j}{ds}\right)^2 = 0 \tag{2.2}$$

where suffices i and j run from 1 to 3 and we identify x,y,z by x^1, x^2, x^3 respectively. Changing these equations into polar coordinates through transformations $x^1 = r \sin\theta \cos\phi$, $x^2 = r \sin\theta \sin\phi$ and $x^3 = r \cos\theta$, we have

$$\frac{d^2t}{ds^2} + \frac{R\dot{R}}{(1 + \frac{kr^2}{4})^2}\left(\frac{dr}{xs}\right)^2 = 0 \tag{2.3}$$

and

$$\frac{d}{ds}\left[\frac{R^2}{(1 + \frac{kr^2}{4})^2}\frac{dr}{ds}\right] + \frac{kR^2r}{(1 + \frac{kr^2}{4})^3}\left(\frac{dr}{ds}\right)^2 = 0 . \tag{2.4}$$

On integrating (2.4), we have

$$\frac{R^2}{(1 + \frac{kr^2}{4})}\frac{dr}{ds} = R_m \tag{2.5}$$

where R_m is a constant.

From (1.1) and (2.5), we have

$$\left(\frac{dt}{ds}\right)^2 = \frac{R^2 + R_m^2}{R^2} . \tag{2.6}$$

Connecting equations (2.4), (2.5), and (2.6), we have

$$\left(\frac{R^2+R_m^2}{R^2}\right)\frac{d^2r}{dt^2} - \frac{\dot{R}R_m^3(1 + \frac{kr^2}{4})}{R^4\sqrt{R^2 + R_m^2}} + \frac{2\dot{R}R_m(1 + \frac{kr^2}{4})\sqrt{R^2+R_m^2}}{R^4} - \frac{krR_m^2(1+\frac{kr^2}{4})}{2R^4} = 0. \tag{2.7}$$

This equation gives the trajectory of a bradyon.

For tachyons, we have $ds = id\sigma$. Equations (2.5) and (2.6) for tachyons can be written as

$$\frac{R^2}{(1+\frac{kr^2}{4})}\frac{dr}{d\sigma} = S_m = iR_m \tag{2.8}$$

and

$$\left(\frac{dt}{d\sigma}\right)^2 = -\left(\frac{R^2+R_m^2}{R^2}\right) . \tag{2.9}$$

From (2.8) and (2.9), we compute

$$\frac{d}{d\sigma}(\delta r) = \frac{iR_m\, kr\, \delta r}{2R^2} \tag{2.10}$$

and

$$\frac{d}{d\sigma}(\delta t) = -\frac{i\, R_m^2\, \dot{R}\, \delta t}{R^2\sqrt{R^2+R_m^2}} . \tag{2.11}$$

The tachyonic trajectory passes through $(r+\delta r, \theta, \phi, t+\delta t)$; hence,

$$\frac{d^2(r+\delta r)}{d\sigma^2} + 2\left[\frac{\dot{R}}{R} + \frac{R\ddot{R}-\dot{R}^2}{R^2}\delta t\right]\frac{d(t+\delta t)}{d\sigma}\frac{d(r+\delta r)}{d\sigma} - \frac{k(r+\delta r)}{2[1+\frac{k(r+\delta r)^2}{4}]}\left[\frac{d}{d\sigma}(r+\delta r)\right]^2 = 0 . \tag{2.12}$$

With the help of (2.10) and (2.11), equation (2.12) is rewritten as

$$-\left(\frac{R^2+R_m^2}{R^2}\right)\frac{d^2(r+\delta r)}{dt^2} - \frac{\dot{R}\, R_m^3(1+\frac{kr^2}{4})\sqrt{R^2+R_m^2}}{R^2} - \frac{\dot{R}\, R_m^3\, kr\sqrt{R^2+R_m^2}}{R^6}\delta r$$

$$+ 2\left[\frac{\dot{R}}{R} + \frac{R\ddot{R}-\dot{R}^2}{R^2}\delta t\right]\left[-\frac{R_m(1+\frac{kr^2}{4})\sqrt{R^2+R_m^2}}{R^2} + \frac{R_m^3\,\dot{R}(1+\frac{kr^2}{4})\,\delta t}{R^6\sqrt{R^2+R_m^2}}\right.$$

$$+ \frac{R_m\, k^2 r\sqrt{R^2+R_m^2}}{2R^3}\delta r + \frac{R_m^3\,\dot{R}\, kr\, \delta t\, \delta r}{2R^4\sqrt{R^2+R_m^2}}$$

$$\left. + \frac{4k\, R_m^2\, r}{R^4}\left\{(1+\frac{kr^2}{4})^2 - \frac{kr}{4}\right\} + \frac{k^2r^2}{R^4}R_m^2\,\delta r\right] = 0 \tag{2.13}$$

where terms containing $(\frac{1}{\delta r})$ and higher orders of $(\frac{1}{\delta r})$ have been neglected; owe-ing to the superluminal speed of tachyons, $(r+\delta r)$ will increase rapidly with respect to the cosmic observer, implying that δr should increase rapidly (r gives the position of bradyon).

From (2.7) and (2.14), we obtain

$$\frac{d^2}{dt2}(\delta r) = -\frac{R_m^3\, H(1+\frac{kr^2}{4})}{R(R^2+R_m^2)^{3/2}} + \frac{2R_m\, H(1+\frac{kr^2}{4})}{R\sqrt{R^2+R_m^2}} - \frac{kr\, R_m^2(1+\frac{kr^2}{4})}{R^2(R^2+R_m^2)}$$

$$- \frac{H R_m^3 (1 + \frac{kr^2}{4})}{R^3 \sqrt{R^2 + R_m^2}} - \frac{kr R_m^2 (1 + \frac{kr^2}{4})}{2R^2 (R^2 + R_m^2)} + \frac{kr R_m^2 (1 + \frac{kr^2}{4})}{R^2 + R_m^2}$$

$$+ \frac{4R_m^2 kr}{R^2 (R^2 + R_m^2)} \left\{ (1 + \frac{kr^2}{4})^2 - \frac{kr}{4} \right\} + \left\{ \frac{R_m k^2 Hr}{R\sqrt{R^2 + R_m^2}} + \frac{R_m^2 k^2 r^2}{R^2 (R^2 + R_m^2)} \right\} \delta r \,. \quad (2.14)$$

Since δt is small, terms containing δt have been neglected. Here H is the Hubble's constant.

3. CLOSED MODEL OF R-W UNIVERSE.

Taking k = +1 for the closed model of a R-W Universe, (2.14) yields

$$\frac{d^2}{dt^2}(\delta r) = -0(\frac{1}{t})0(\frac{1}{R^4}) + 0(\frac{1}{t})0(\frac{1}{t^3}) - 0(\frac{1}{R^4}) - 0(\frac{1}{t})0(\frac{1}{R^2}) + 0(\frac{1}{R^2}) + 0(\frac{1}{R^4}) + 0(\frac{1}{R^2})0(\frac{1}{t})\delta r + 0(\frac{1}{R^4})\delta r \,. \quad (3.1)$$

If this model contains a dust-like perfect fluid, we have from the Einstein field equation that

$$R \sim t^{2/3} \,. \quad (3.2)$$

Substituting R from (3.2) in (3.1), we have

$$\frac{d^2}{dt^2}(\delta r) \approx -0(t^{-11/3}) + 0(t^{-4}) - 0(t^{-8/3}) - 0(t^{-7/3}) + 0(t^{-4/3}) + 0(t^{-7/3})\ r + 0(t^{-8/3})\delta r \,. \quad (3.3)$$

If this model contains radiation, we have

$$R \sim t^{1/2} . \quad (3.4)$$

Hence,

$$\frac{d^2}{dt^2}(\delta r) \approx -0(t^{-3}) + 0(t^{-4}) - 0(t^{-2}) + 0(t^{-1}) + 0(t^{-2})\ \delta r \,. \quad (3.5)$$

If this model contains super-dense matter, we have

$$R \sim t^{1/3} \,. \quad (3.6)$$

Hence,

$$\frac{d^2}{dt^2}(\delta r) \approx -0(t^{-7/3}) + 0(t^{-4}) + 0(t^{-4/3}) - 0(t^{-5/3}) + 0(t^{-2/3}) + 0(t^{-5/3})\delta r + 0(t^{-4/3})\delta r \,. \quad (3.7)$$

We observe from (3.3), (3.5), and (3.7) that, when $t < 1$ sec., $\frac{d^2}{dt^2}(\delta r)$ is negative, showing attraction of tachyons to bradyons. But when t increases beyond 1 sec., the bradyon field becomes more and more repulsive for tachyons because $\frac{d^2}{dt^2}(\delta r)$ becomes more and more positive with increasing dominance of the terms $0(t^{-4/3})$, $0(t^{-1})$ and $0(t^{-2/3})$ in (3.3), (3.5), and (3.7) respectively. The more interesting point is that the sequences formed by $0(t^{-1})$ in (3.5) and in (3.7) are divergent as time increases, while the sequence $0(t^{-4/3})$ is convergent. Hence, we may have a consequence that, as time increases, tachyons are repelled from bradyons.

Also, on comparing (3.3), (3.5), and (3.7), we find that the repulsive fields are related as

$$\overline{F}_s > \overline{F}_r > \overline{F}_d \tag{3.8}$$

where s, r, d denote super-dense matter, radiation, and dust, respectively.

4. OPEN MODEL OF R-W UNIVERSE.

In the open model, k = -1; hence, we have

$$\frac{d^2}{dt^2}(\delta r) = -0(\tfrac{1}{t})\,0(\tfrac{1}{R^4}) + \frac{r^2}{4}\,0(\tfrac{1}{t})\,0(\tfrac{1}{R^4}) - \frac{r^3}{4}\,0(\tfrac{1}{R^4}) + \frac{r}{2}\,0(\tfrac{1}{R^2})$$
$$- \frac{r^3}{8}\,0(\tfrac{1}{R^4}) - r0(\tfrac{1}{R^2}) + \frac{5}{4}\,r^3 0(\tfrac{1}{R^2}) - r^2 0(\tfrac{1}{R^4}) + \{r0(\tfrac{1}{R^2}) + r^2 0(\tfrac{1}{R^4})\}\,\delta r\,. \tag{4.1}$$

Substituting R from (3.2) into this equation, we have

$$\frac{d^2}{dt^2}(\delta r) \approx (\frac{r^2}{2} - 2)0(t^{-11/3}) + (\frac{r^3}{4} - \frac{r}{2})0(t^{-4/3}) + (\frac{5}{8}r^3 - r^2 - r^5)\,0(t^{-8/3})$$
$$+ \{r\,0(t^{-4/3}) + r^2\,0(t^{-8/3})\}\,\delta r\,. \tag{4.2}$$

Substituting R from (3.4) into equation (4.1), we have

$$\frac{d^2}{dt^2}(\delta r) \approx (\frac{r^2}{2} - 2)\,0(t^{-3}) + (\frac{5r^3}{8} - r^2 - r^5)\,0(t^{-2}) + (\frac{r^3}{4} - \frac{r}{2})\,0(t^{-1})$$
$$+ \{r\,0(t^{-1}) + r^2\,0(t^{-2})\}\,\delta r\,. \tag{4.3}$$

Substituting R from (3.6) into (4.1), we have

$$\frac{d^2}{dt^2}(\delta r) \approx (\frac{r^2}{2} - 2)\,0(t^{-7/3}) + (\frac{5}{8}r^3 - r^2 - r^5)\,0(t^{-4/3}) + (\frac{r^3}{4} - \frac{r}{2})\,0(t^{-2/3})$$
$$+ \{r\,0(t^{-2/5}) + r^2\,0(t^{-4/3})\}\,\delta r\,. \tag{4.4}$$

Thus, we find that in the open model, when r is very small and t < 1 sec., the field is attractive for tachyons. But as t increases more and more beyond 1 sec. and r also increases, the field becomes more and more repulsive for tachyons. As with the closed model, we note that in the open model

$$\overline{F}_s > \overline{F}_r > \overline{F}_d\,.$$

5. FLAT R-W UNIVERSE.

In the case of the flat model, k = 0; hence, we can write (2.14) as

$$\frac{d^2}{dt^2}(\delta r) \approx -0(\tfrac{1}{t})\,0(\tfrac{1}{R^4})\,. \tag{5.1}$$

Substituting R from (3.2) into (5.1), we obtain

$$\frac{d^2}{dt^2}(\delta r) \approx -0(t^{-11/3})\,. \tag{5.2}$$

Substituting R from (3.4) into (5.1), we obtain

$$\frac{d^2}{dt^2}(\delta r) \approx -0(t^{-3})\,. \tag{5.3}$$

Substituting R from (3.6) into (5.1), we obtain

$$\frac{d^2}{dt^2}(\delta r) \approx -O(t^{-7/3}) . \tag{5.4}$$

Here, we find that in the flat model, the field is always attractive for tachyons. On comparing $\frac{d^2}{dt^2}(\delta r)$ from equations (5.2), (5.3), and (5.4), we find for the attractive field $\bar{F}$ that

$$\bar{F}_d > \bar{F}_r > \bar{F}_s$$

when $t < 1$ sec. But as t increases beyond 1 sec., we find that

$$\bar{F}_s > \bar{F}_r > \bar{F}_d .$$

6. DISCUSSION.

Here we are using the cosmic time; hence, in the light of the above investigations, we can say that, after the epoch of the big-bang, tachyons were attracted by bradyons up to one second, but beyond this time they were repelled. With the passage of time, the repulsive force became stronger and stronger in the curved space-time. But in the case of flat space, tachyons were attracted by bradyons. Since flatness is a local property, we can say that tachyons which were very near bradyons became gravitationally coupled with bradyons and could not remain free. Free tachyons would have fled away due to gravitational repulsion and superluminal speed. During this phenomenon, there is the possibility of collisions of tachyons with bradyons. Corben [4] has proposed that, if a free tachyon of meta-mass μ and a free bradyon of rest mass m_o collide with each other, the rest mass of the resulting free particle is given by $m = [m_o^2 - \mu^2]^{1/2}$. Now three cases arise:

(1) $m_o = \mu$ (2) $m_o > \mu$ (3) $m_o < \mu$

In the first case, we find that the rest-mass of the resulting particle is zero; hence, we can say that this particle is a photon. Here, in contrast to the usual bradyon physics, we find that an electrically neutral free tachyon and an electrically neutral bradyon annihilate to photons on collision. In the second case, we find that the resulting free particle will be a bradyon. In the third case, the rest-mass of the resulting particle is imaginary; hence, it is a tachyon.

However, from the above discussion, we find that free tachyons would have fled away from this universe. We ask the question, 'Where have the tachyons gone?'. From the Lorentz Transformations, we find that tachyons are superluminal relative to bradyons, but they are subluminal relative to themselves. This means that tachy-

ons become bradyons in a superluminal frame of reference. It is well known that bradyons attracts bradyons. Hence, in a superluminal frame, where tachyon becomes bradyon, one tachyon will attract another tachyon. Now we are in a position to speculate that tachyons would have fled away to a Meta-Universe or Tachyon-Universe [5] which is richly populated by tachyons.

Thus, we find that tachyons would live in our universe for a short period of time only. This result is in tune with Narlikar and Sudarshan's result that there would be a maxima of time for existence of tachyons in this universe. They hope that, if some tachyons existed in this universe now, their mass would be less than even the mass of an electron. According to our theory, if such tachyons exist, they will also flee away to the Meta-Universe.

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